Ocean transport and marine biological dynamics from Finite-Size Lyapunov Exponents

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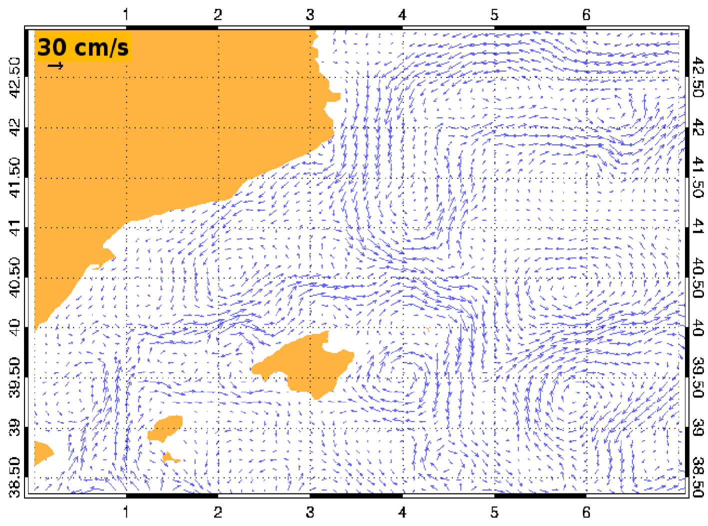
OUTLINE

- **Introduction**: the dynamical systems approach to fluid transport (hyperbolic points, manifolds, Lyapunov …)
- **Finite-size Lyapunov exponents (FSLEs).** Impact of flow structures on marine ecosystems:
  - Phytoplankton
  - Top predators: Frigatebirds
- **Robustness and reliability of FSLES for ocean dynamics**:
  - Error in the data.
  - Noise in the particle trajectories.
  - Multifractality..

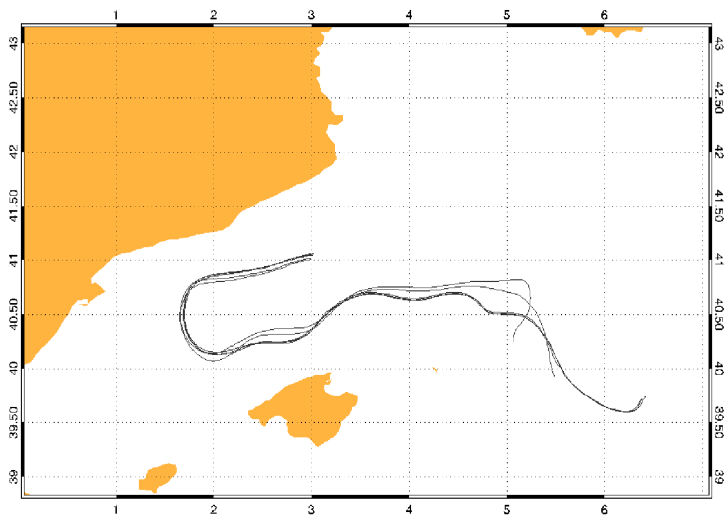
This talk is about some applications of Lagrangian techniques to Oceanic processes.

Eulerian description



One deals at any time with velocity field at any spatial point in the fluid.

Lagrangian description



One deals with trajectories of fluid particles.

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$

$$\mathbf{x}(t_0) = \mathbf{x}_0$$

Connection between the Eulerian and Lagrangian description

The dynamical systems approach to fluid transport

Lagrangian dynamical system:

deduction of the phase portrait from the velocity field

Phase portrait of the velocity field ⟷ ***Physical meaning in fluid dynamics***

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$

ODEs

Phase portrait

KAM tori, stable/unstable manifolds

Chaotic sets

.....

=>

Velocity field

Transport organization.

Transport barriers, eddies, fronts and avenues

Mixing regions

.....

- Lagrangian dynamical system: **particle trajectories** in a given velocity field.

- **Incompresibility: symplectic structure,**

- Phase space = physical space

- Global behavior in phase space is organized by some relevant lines

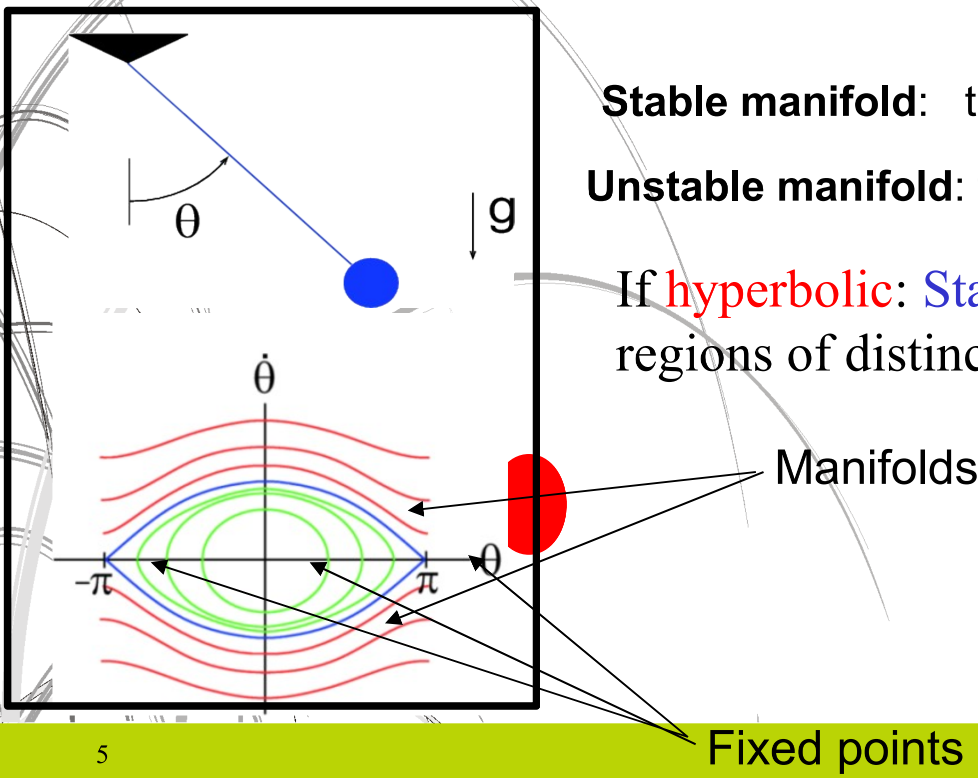
$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$

WHICH ARE THE RELEVANT LINES?

Trajectories of two-dimensional time-independent flows are organized by the fixed points of the dynamical system $\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t))$

Global flow geometry understood by studying **invariant manifolds** of the **fixed points**.

Pendulum



Stable manifold: trajectories asymptote the fixed point $t \to \infty$

Unstable manifold: trajectories asymptote the fixed point $t \to -\infty$

If hyperbolic: Stable and unstable manifolds $\to$ separate regions of distinct motion

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How to find separatrices in time-dependent flows?
Separatrices divide regions of qualitatively different dynamics

We measure stretching with Lyapunov exponents

This is one of the ways

Separatrices → Lagrangian Coherent Structures (LCS)

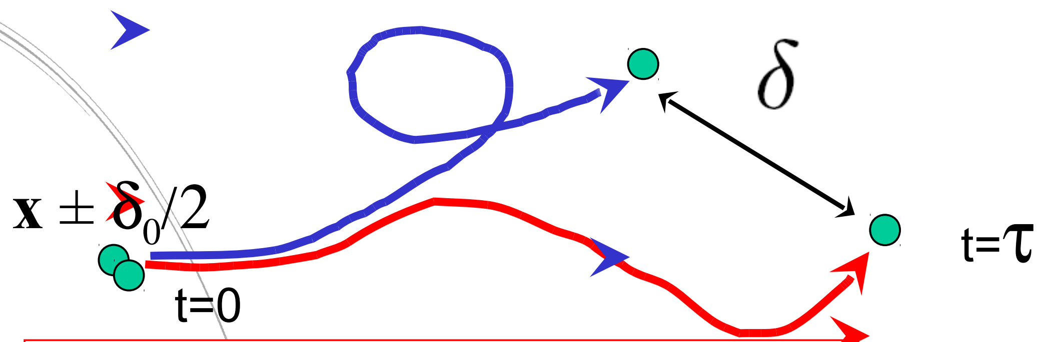
PARTICLE TRAJECTORIES CANNOT CROSS THEM, SO THEY ARE A TEMPLATE FOR TRANSPORT.

LYAPUNOV EXPONENTS

$$\lambda(t) = \lim_{\|\delta(0)\| \to 0} \frac{1}{t} \ln \frac{\|\delta(t)\|}{\|\delta(0)\|}$$ Finite-**time** Lyapunov exponent

$$\lambda = \lim_{t \to \infty} \lambda(t)$$ Lyapunov exponent

Finite-size Lyapunov exponent (FSLE)



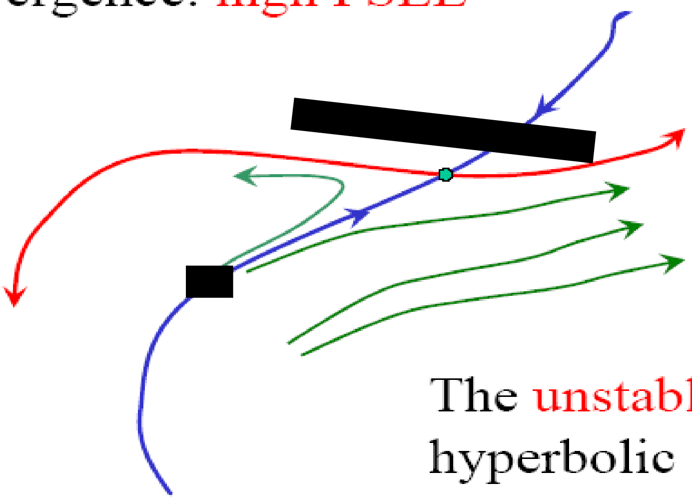
$$\lambda(\delta_0, \delta_f) \equiv \frac{1}{\tau} \log \frac{\delta_f}{\delta_0}$$

All the quantities are also functions of the initial position and time: $\lambda(\mathbf{x}, t, \delta_0, \delta_f)$

Aurell, Boffetta, Celani, Cencini, Crisanti, Falcioni Paladin, Vulpiani: **PRL 96, J. Phys A 1997, Phys. Rep. (2002), Chaos (2000)**

Catching Lagrangian Coherent structures with ridges (maxima) of FSLEs

The idea is that initial conditions close to the stable manifold of a hyperbolic trajectory or set will show strong divergence: high FSLE



Stable and unstable manifods are also called Lagrangian Coherent Structures

The unstable manifold of hyperbolic sets would be marked by high FSLE in the time backwards direction

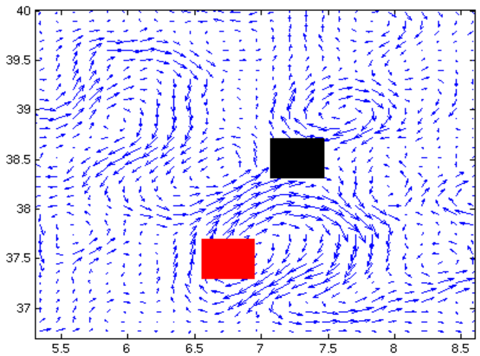
Other types of Lyapunov exponents would display similar information, but FSLE is less afected by saturation

Rigouros results are needed

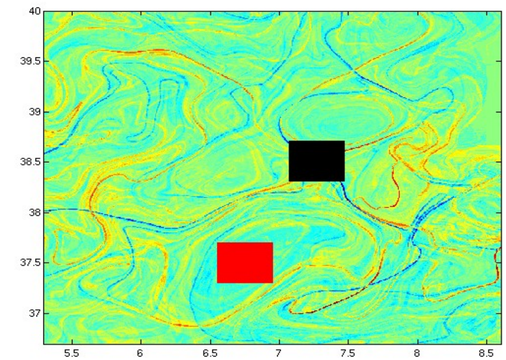
The spatial dependence of the FSLE allows the detection of stable and unstable manifolds of hyperbolic objects

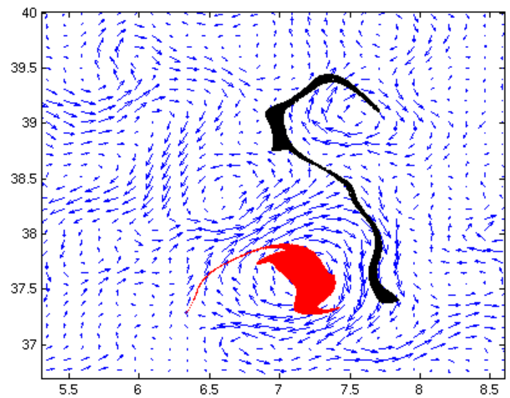
Velocity field

FSLEs

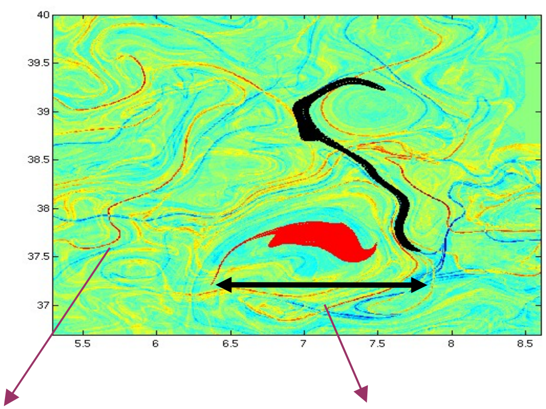


Initial conditions





After several days

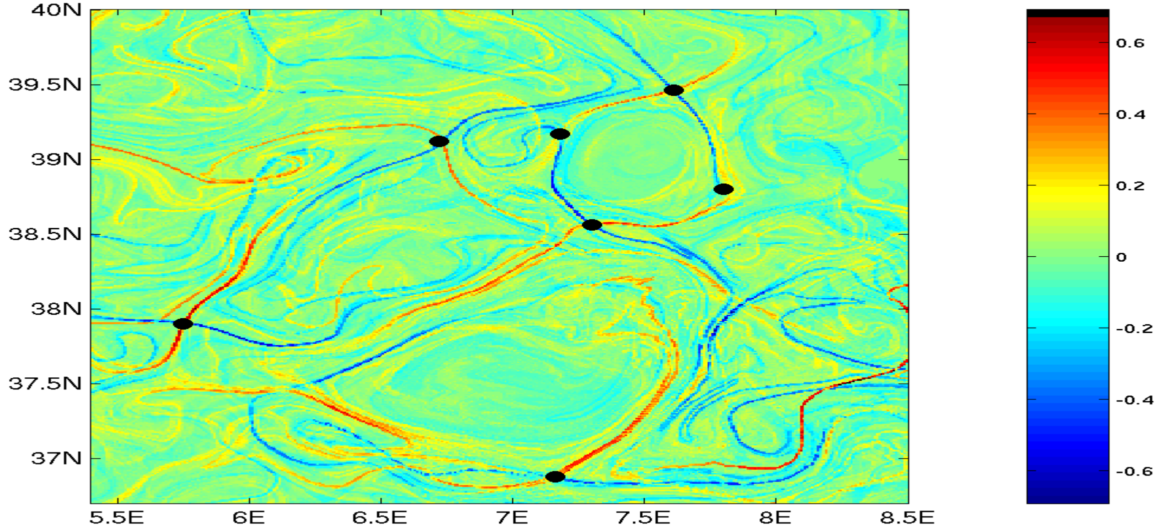


Submesoscale filament

Mesoscale eddy

The strongest lines are seen to organize tracer flow

Intersection of stable and unstable manifolds: hyperbolic points



A measure of mixing strength: counting the number of hyperbolic points in a given area

$\lambda_+(\mathbf{x},t)$: FSLE from forward integration

$\lambda_-(\mathbf{x},t)$: FSLE from backwards integration

$$M_\pm(t) \equiv \left\langle \sqrt{\lambda_+\lambda_-} \right\rangle_A$$

Lagrangian diagnostics needed because:

- Able to reveal the dynamical structures in the flow which strongly organises fluid motion (transport barriers, avenues, etc..).

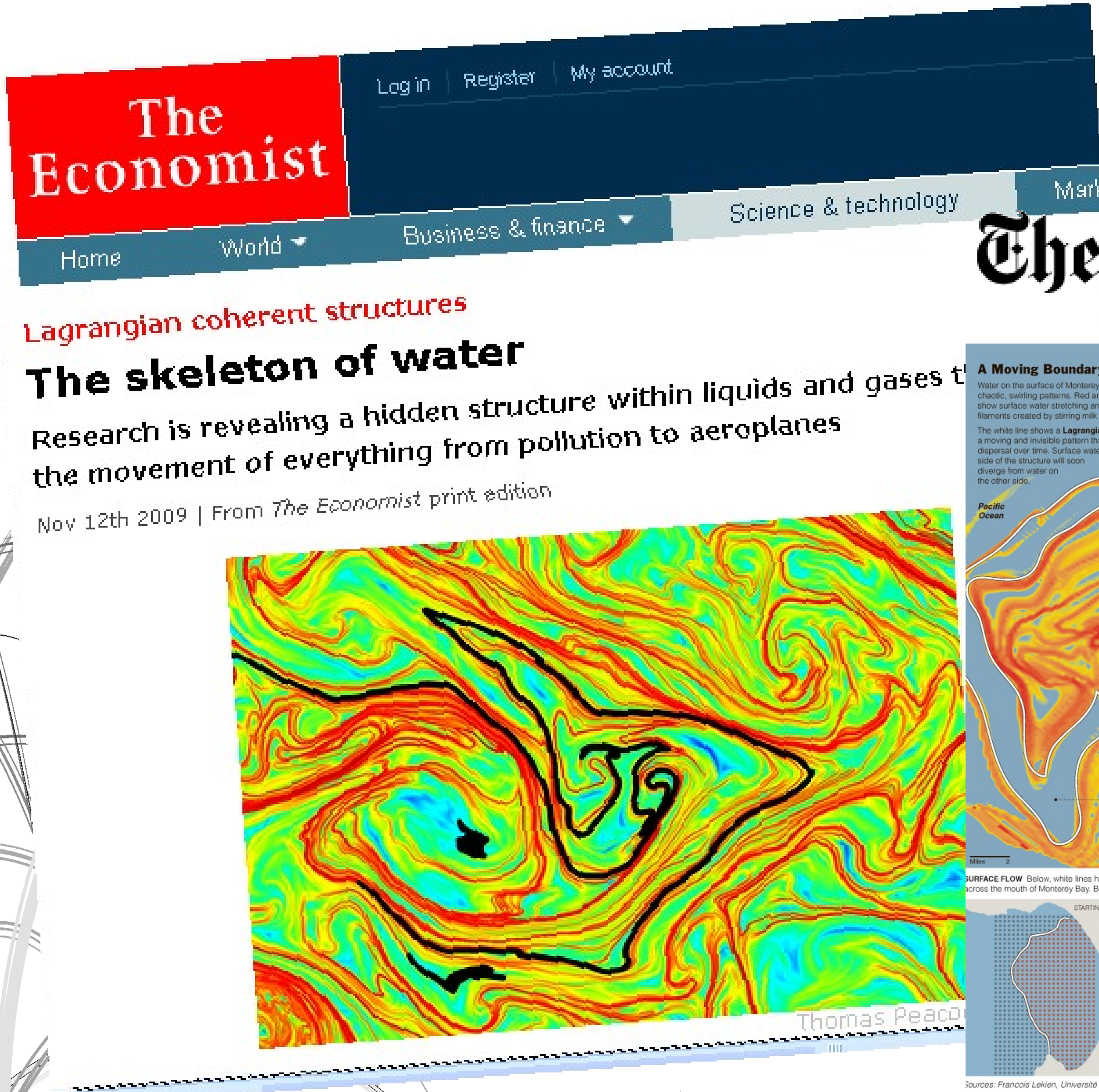
-- Simple enough to be applied in a practical way to complex, aperiodic, and huge velocity data sets.

-- Give additional information of oceanographic interest: time scales, mixing strength…

- Is able to reveal oceanic structures under the resolution of the velocity field.

- Natural framework to study the interaction: hydrodynamics/biological tracers.

Lagrangian Coherent Structures



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Lagrangian coherent structures

The skeleton of water

Research is revealing a hidden structure within liquids and gases t

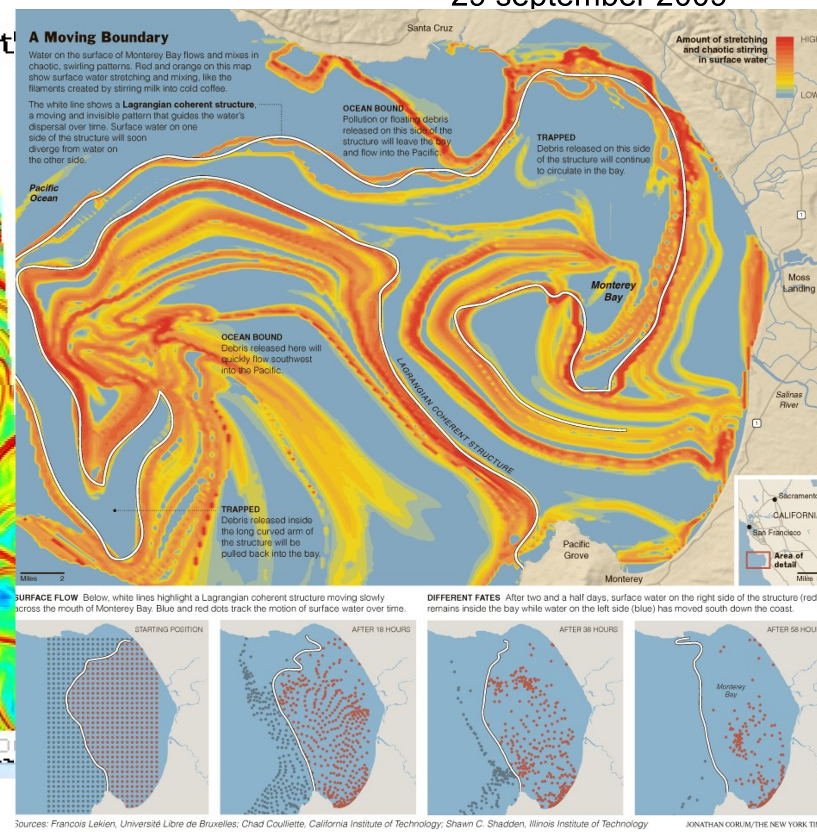
the movement of everything from pollution to aeroplanes

Nov 12th 2009 | From *The Economist* print edition

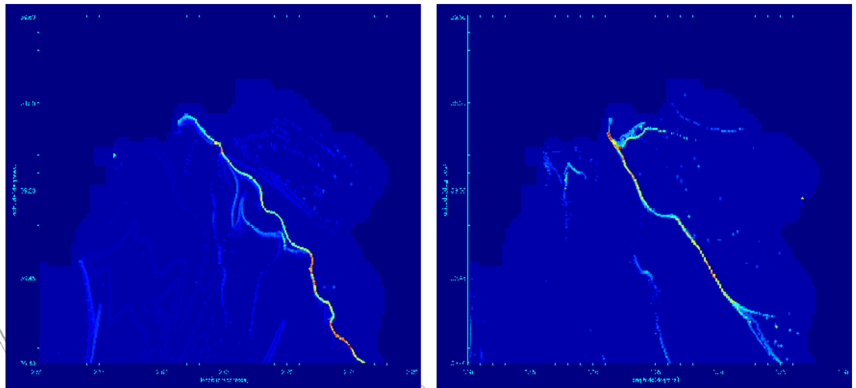
Thomas Peaco

The New York Times

29 september 2009



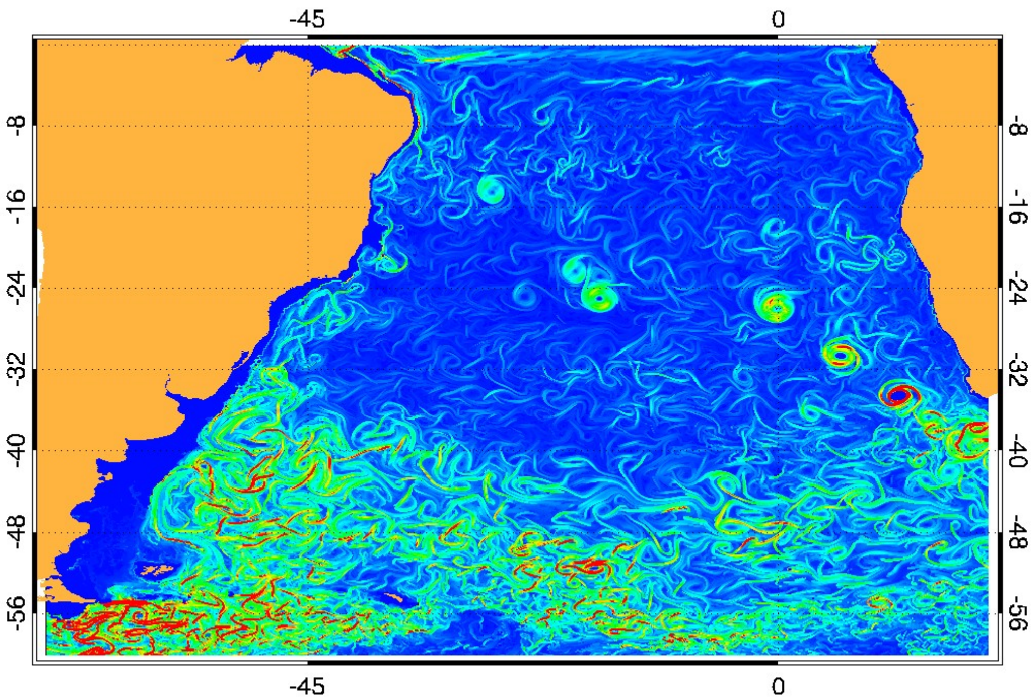
FSLE diagnosis: from coastal to global scales



(a) October 8 at 19:00h (GMT), 2008 (b) October 9 at 19:00h (GMT), 2008

Coastal scales: Barriers in the Bay of Palma

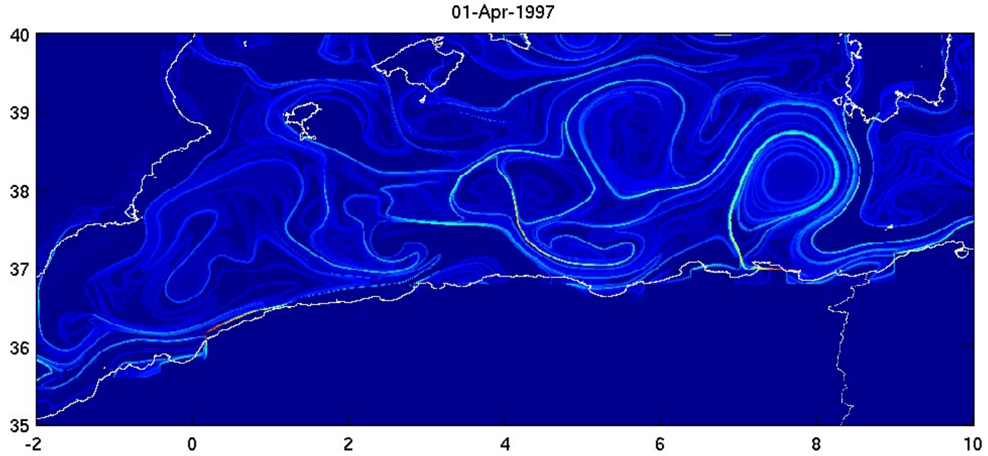
Velocity data from numerical simulations



Global scales

Mesoscales (1-100 km and 1day- 1year) are crucial for oceanic processes

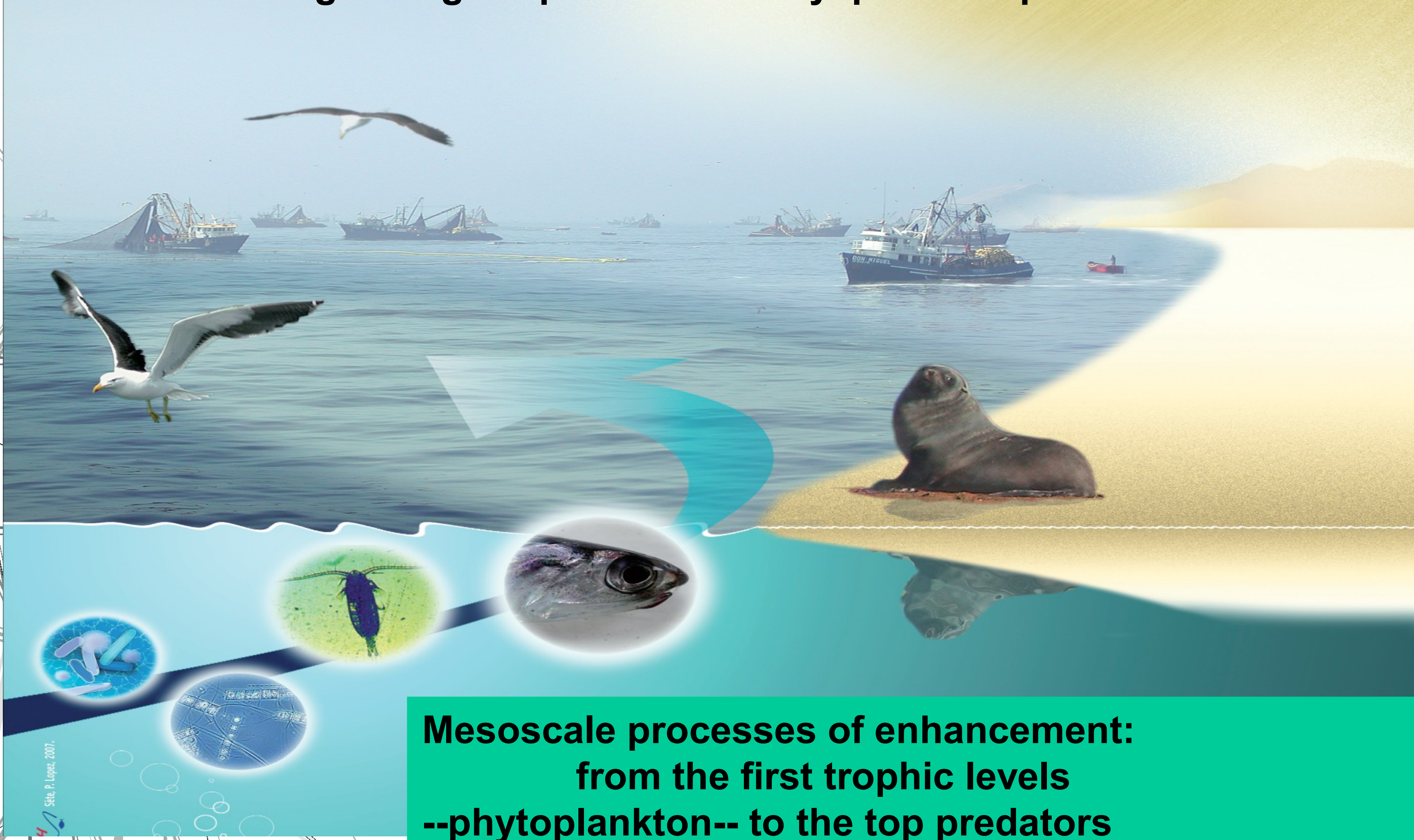
VELOCITY DATA FROM ALTIMETRY DATA (SATELLITE)



Note the MULTIFRACTAL character, and the access to SUB-MESOSCALE

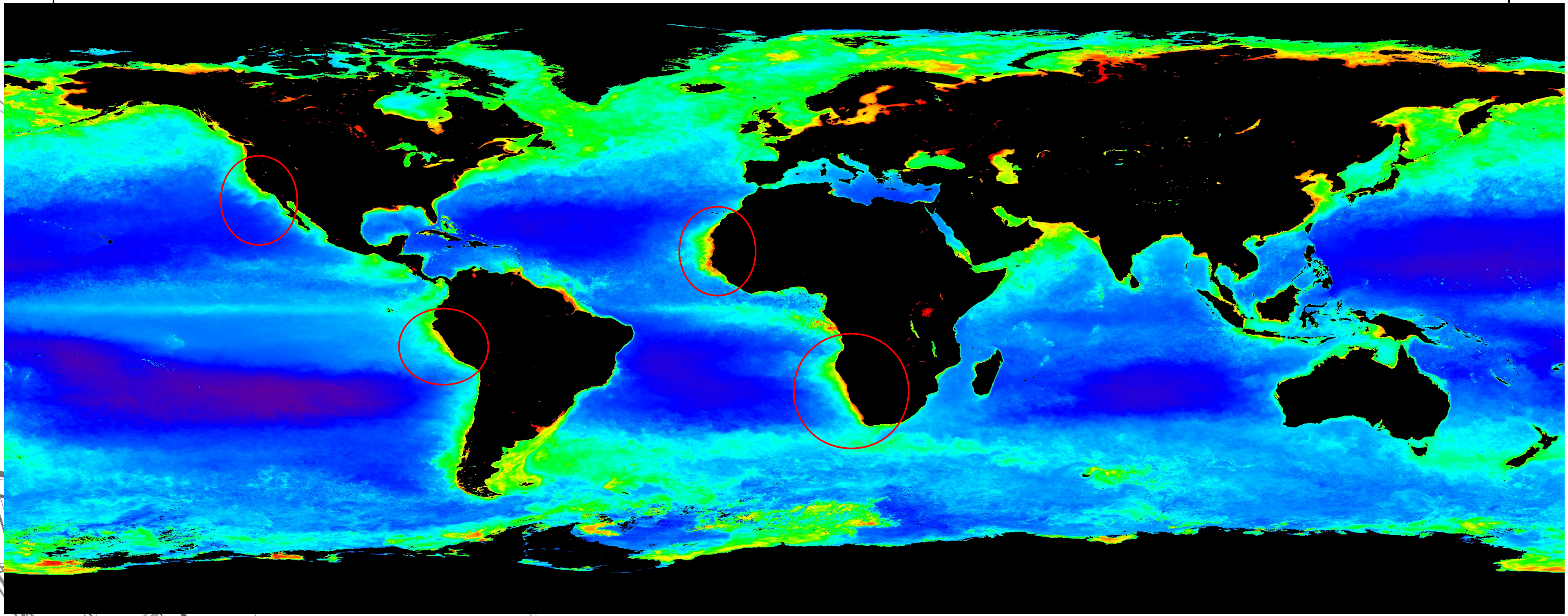
APPLICATIONS TO MARINE ECOSYSTEMS

Describing biological process with Lyapunov exp…



Mesoscale processes of enhancement:
from the first trophic levels
--phytoplankton-- to the top predators

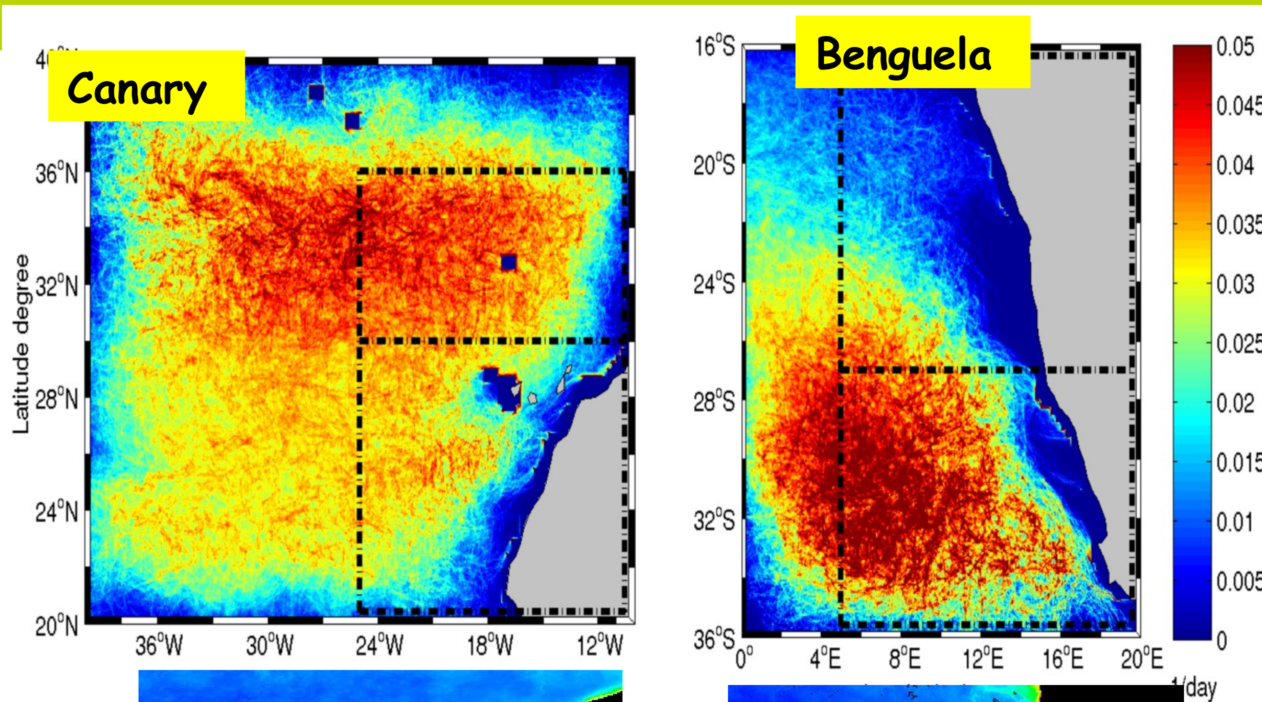
Chlorophyll-a (≈ phytoplankton) from space



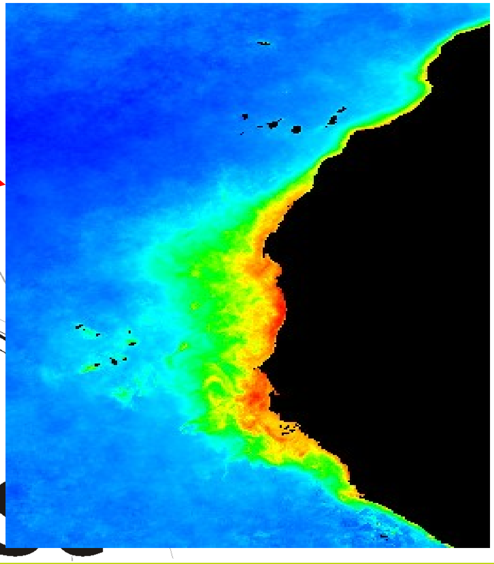
- **Importance of upwelling areas due to:**
  - **large contribution in the world ocean productivity and biomasses.**
  - **several and intense human activities (about half of the world fisheries).**
- **High variability of the physico-chemical properties of the ocean.**
- **Vulnerability, especially in a global climate change context.**

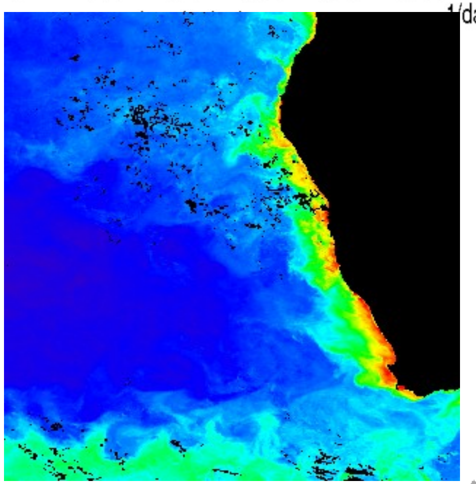
Eastern Bounday Upwelling areas

Backward FSLE (λ-): Temporal average **(a measure of horizontal MIXING)** from June 2000 till June 2005



Phytoplankton





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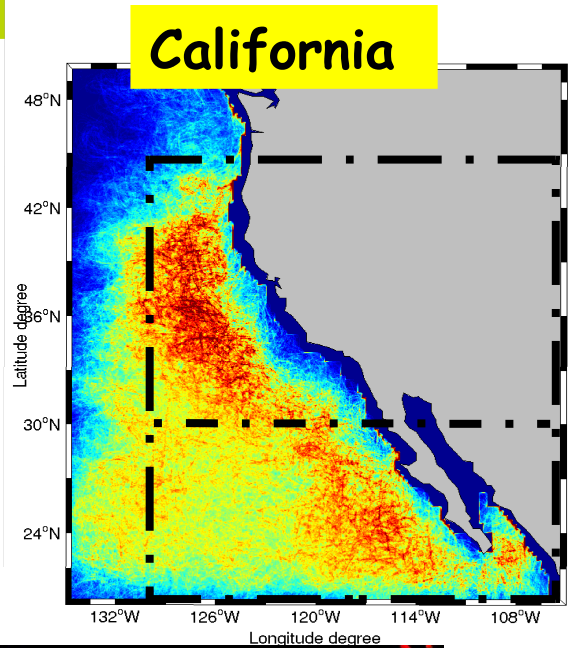
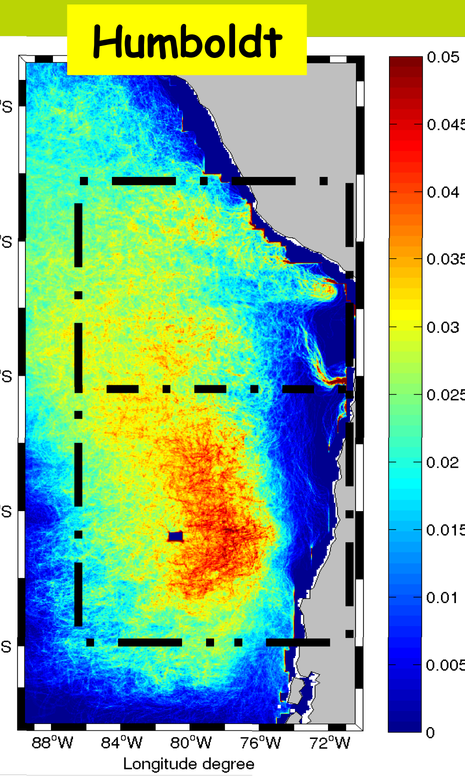
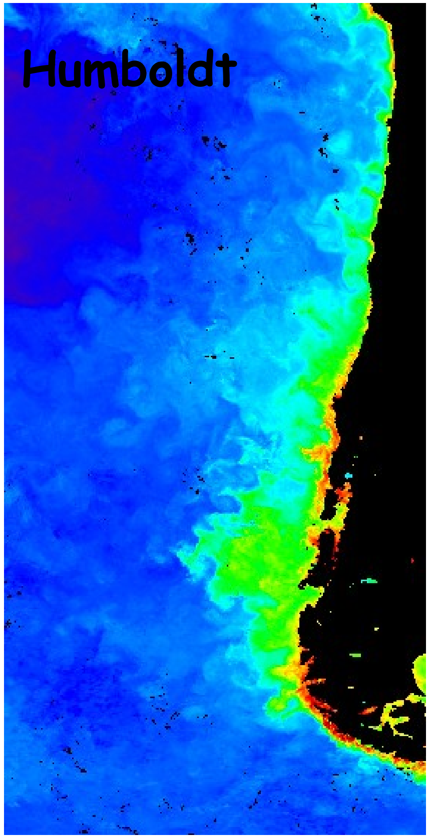
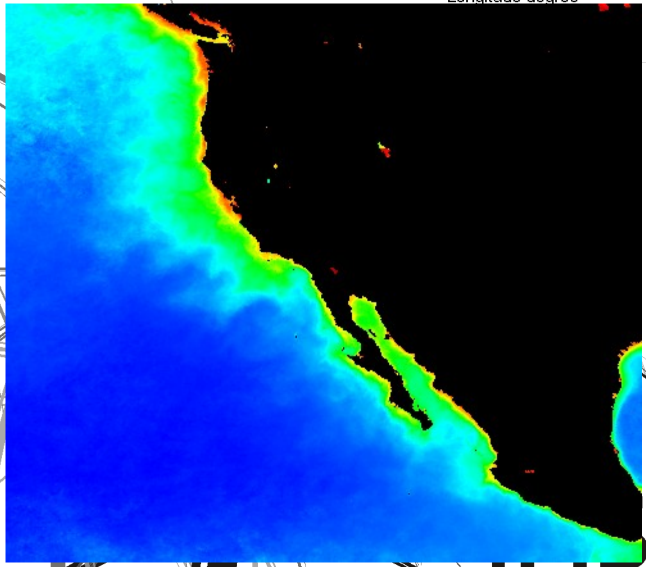
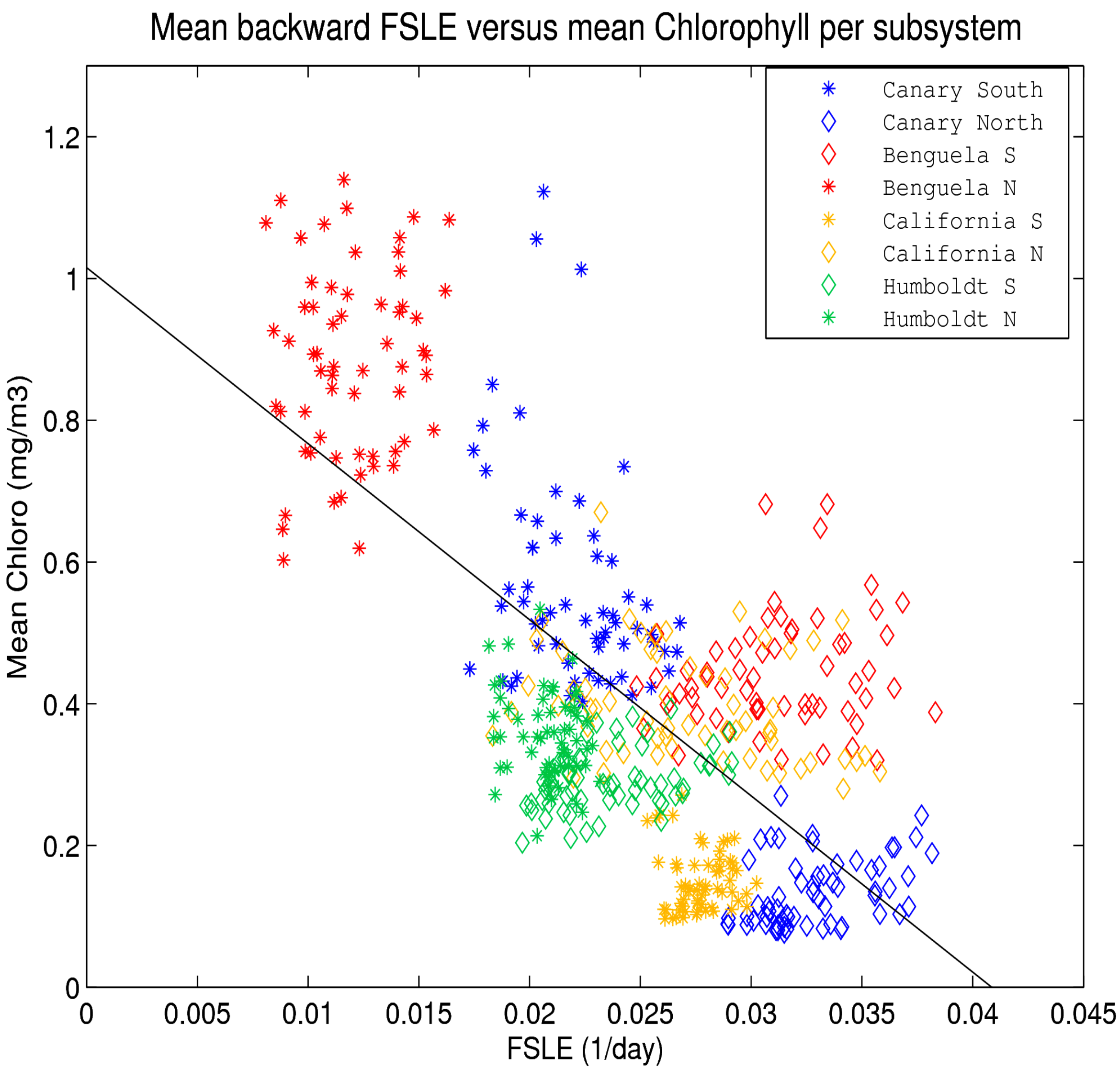








Backward FSLE (λ-): Temporal average
(a measure of horizontal MIXING)
from June 2000 till June 2005

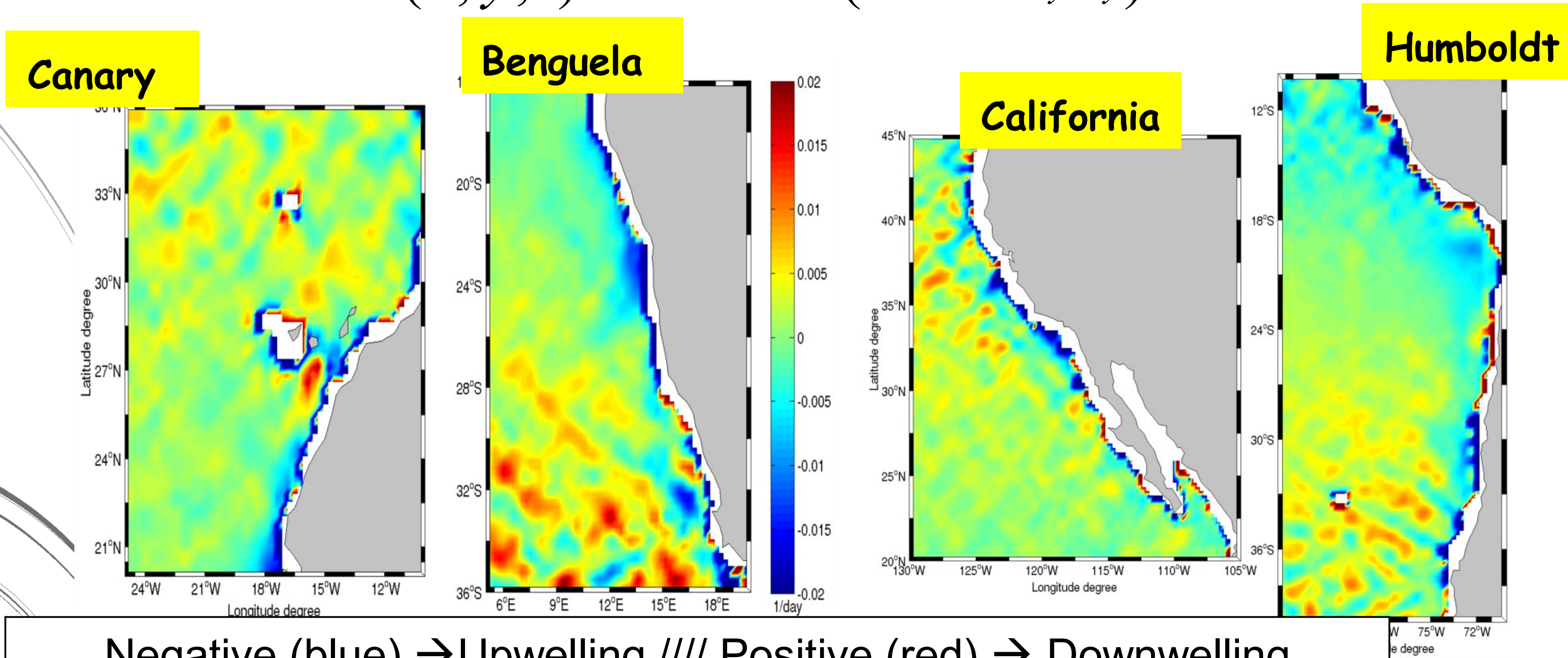


- Negative correlation
- Less turbulent systems are characterized by: LOW FSLE / HIGH CHLOROPHYLL.
- Most turbulent systems: HIGH FSLE / LOW CHLOROPHYLL.

Opposite to behavior seen in less enriched systems

Temporal averages of vertical velocities from incompressibility condition

$$\Delta(x,y,t) \equiv \partial_z V_z = -(\partial_x V_x + \partial_y V_y)$$



Negative (blue) →Upwelling //// Positive (red) → Downwelling

• Dominance of (small) upwelling vertical velocities in the less turbulent subsystem.

• Thus, probably the influence of horizontal stirring on plankton is only indirect: need to understand ***the 3d flow structure***.

- Lagrangian Coherent Structures give the skeleton of horizontal transport
- This certainly influences abiotic quantities: temperature, nutrients, …(not shown in this talk).
- This certainly influences plankton distribution
- From there, impact is expected in plankton consumers, their predators, … cascades up along the food chain …

Do top marine predators track Lagrangian Coherent structures?

Great Frigatebirds : Fregata minor

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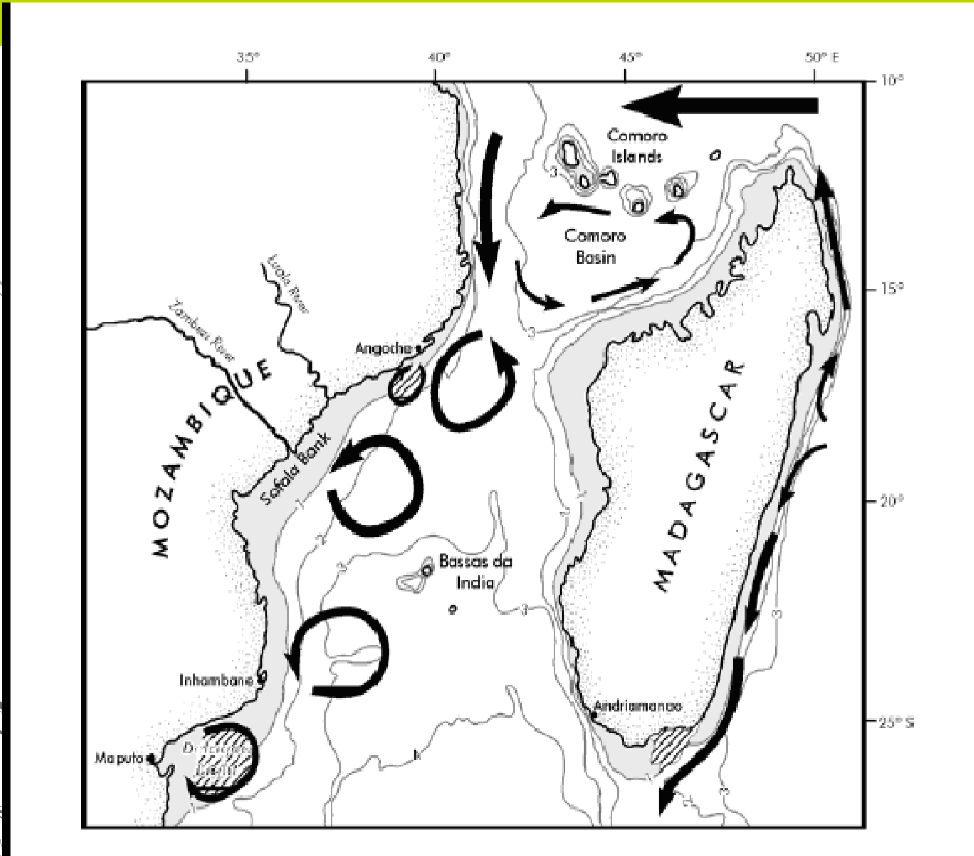
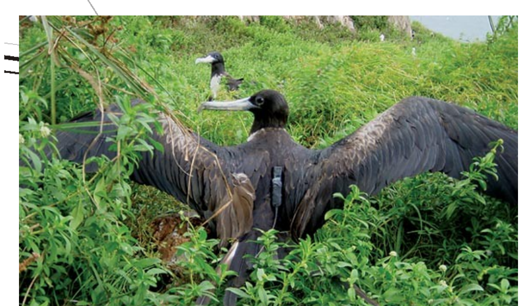
DATA

Dynamics in the Mozambique channel (MC)



Satellite transmitter and altimeter : total weight : 1 to 3% mass of adults (maxi 45 g)



-Strong mesoscale activity in the MC.

- Surface velocity data in the channel.

- 8 birds from Europa island fitted with satellite transmitters (august-september 2003).

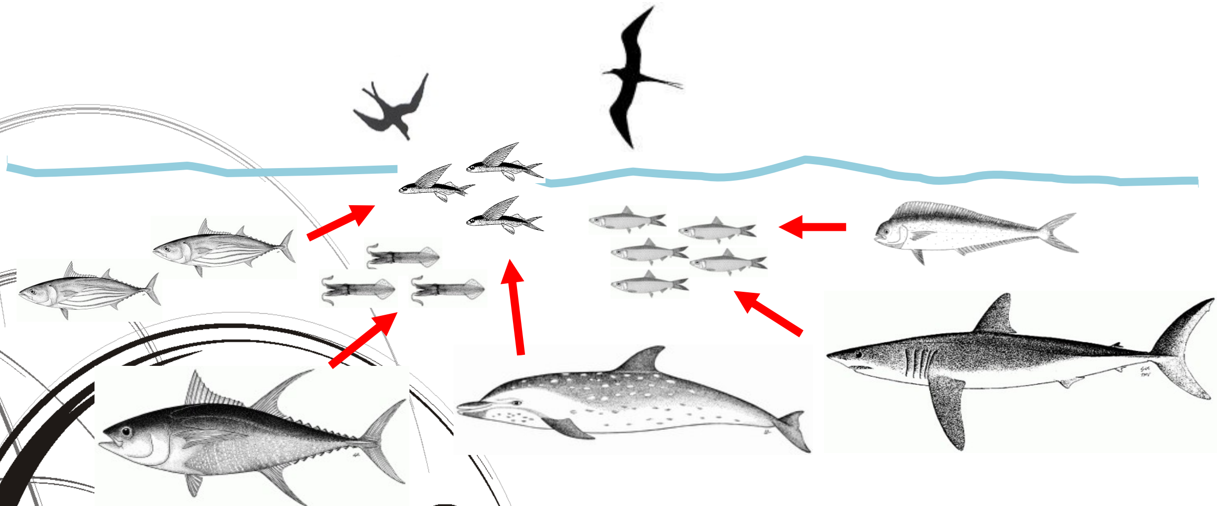
- Foraging trips:

50 trips : 17 long trips (> 614 km), 33 short trips

1600 Argos positions

Great frigatebird (*fregata minor*):

- Large seabirds (light weight < 5 kg and large wings > 2m). Use thermals to soar before gliding over long distances and time (days/nights over weeks).
- Traveling at high altitudes to locate patches of prey and come close to surface to feed (reduced flight speed indicates foraging).
- Feeding occurs only during daytime (peaks in the morning and evening).
- Unable to dive or rest on the water surface (permeable plumage) → in association with subsurface predators (tuna, …): **fisheries indicators**



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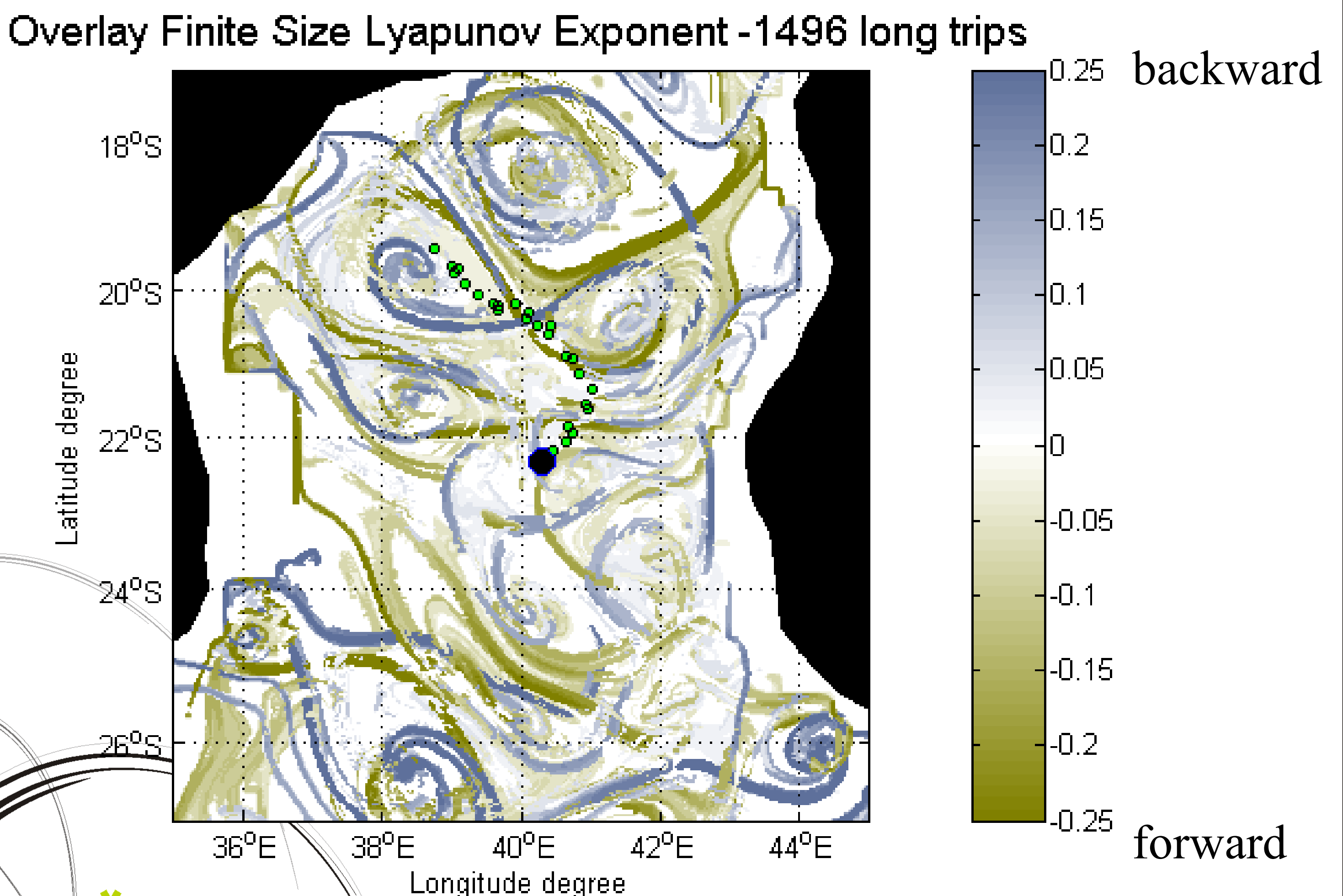
Overlay Finite Size Lyapunov Exponent -1496 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
0.15
0.1
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-0.05
-0.1
-0.15
-0.2
-0.25
backward
forward

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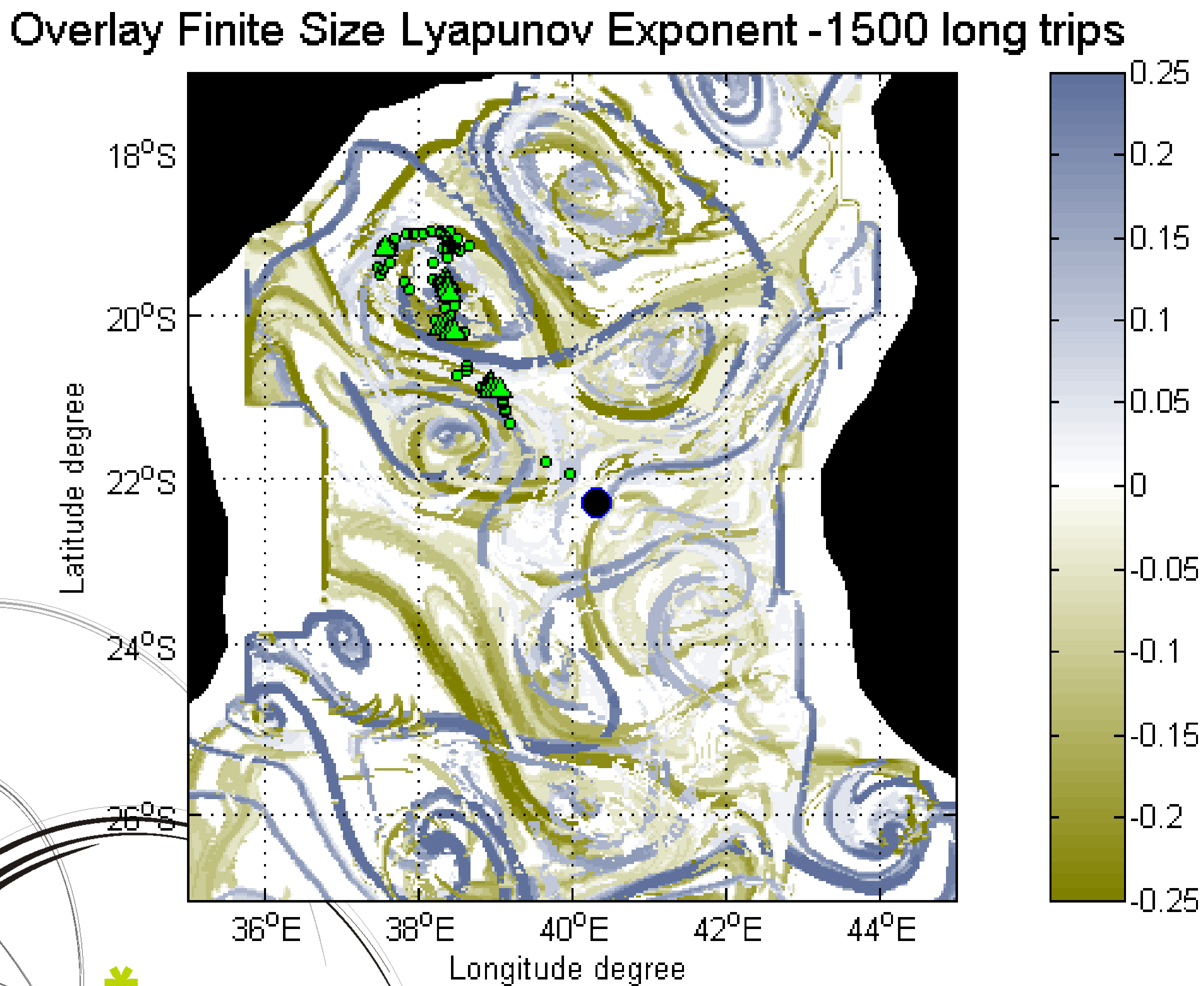
Overlay Finite Size Lyapunov Exponent -1500 long trips
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20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
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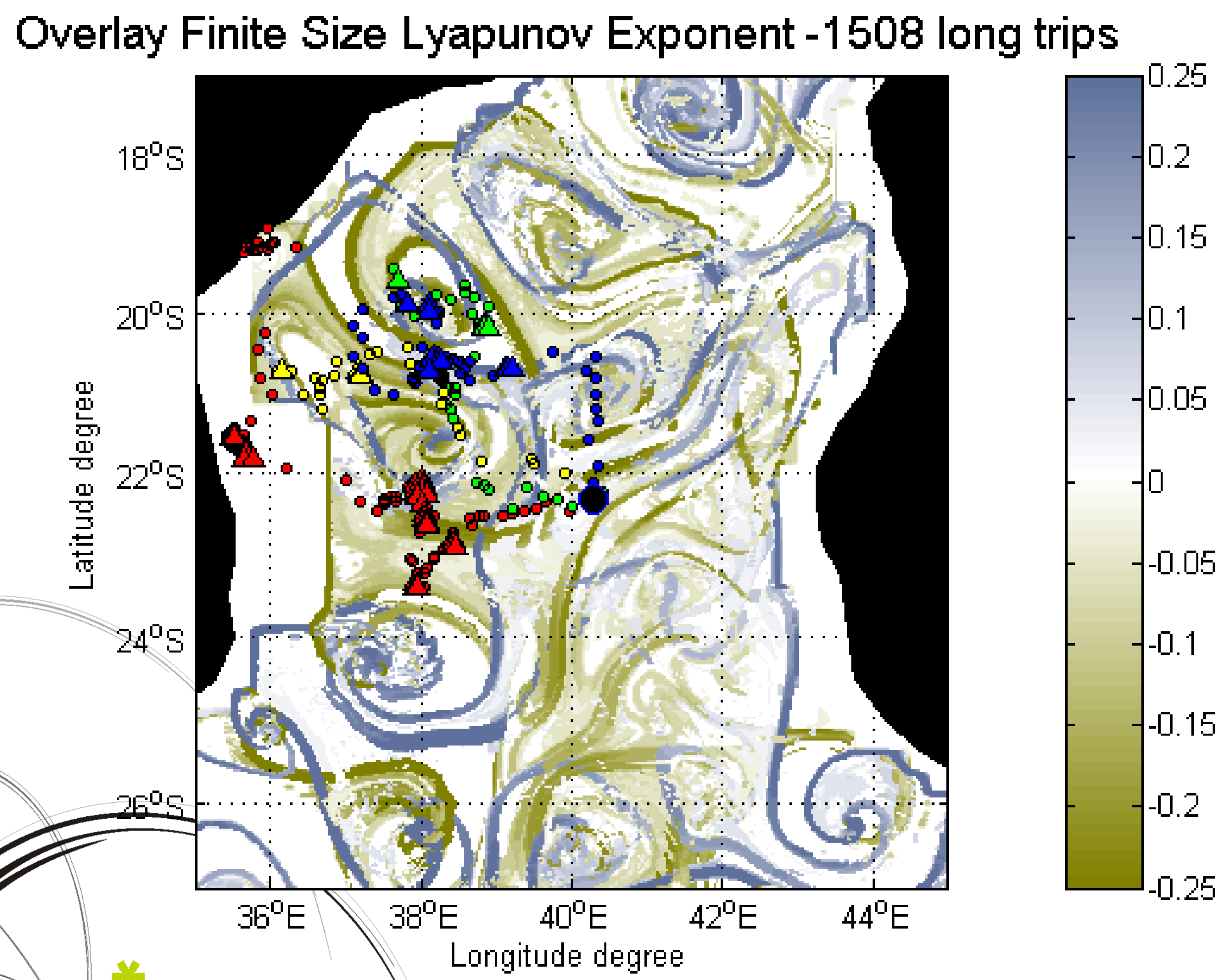
Overlay Finite Size Lyapunov Exponent -1508 long trips
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20°S
22°S
24°S
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Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
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-0.2
-0.25

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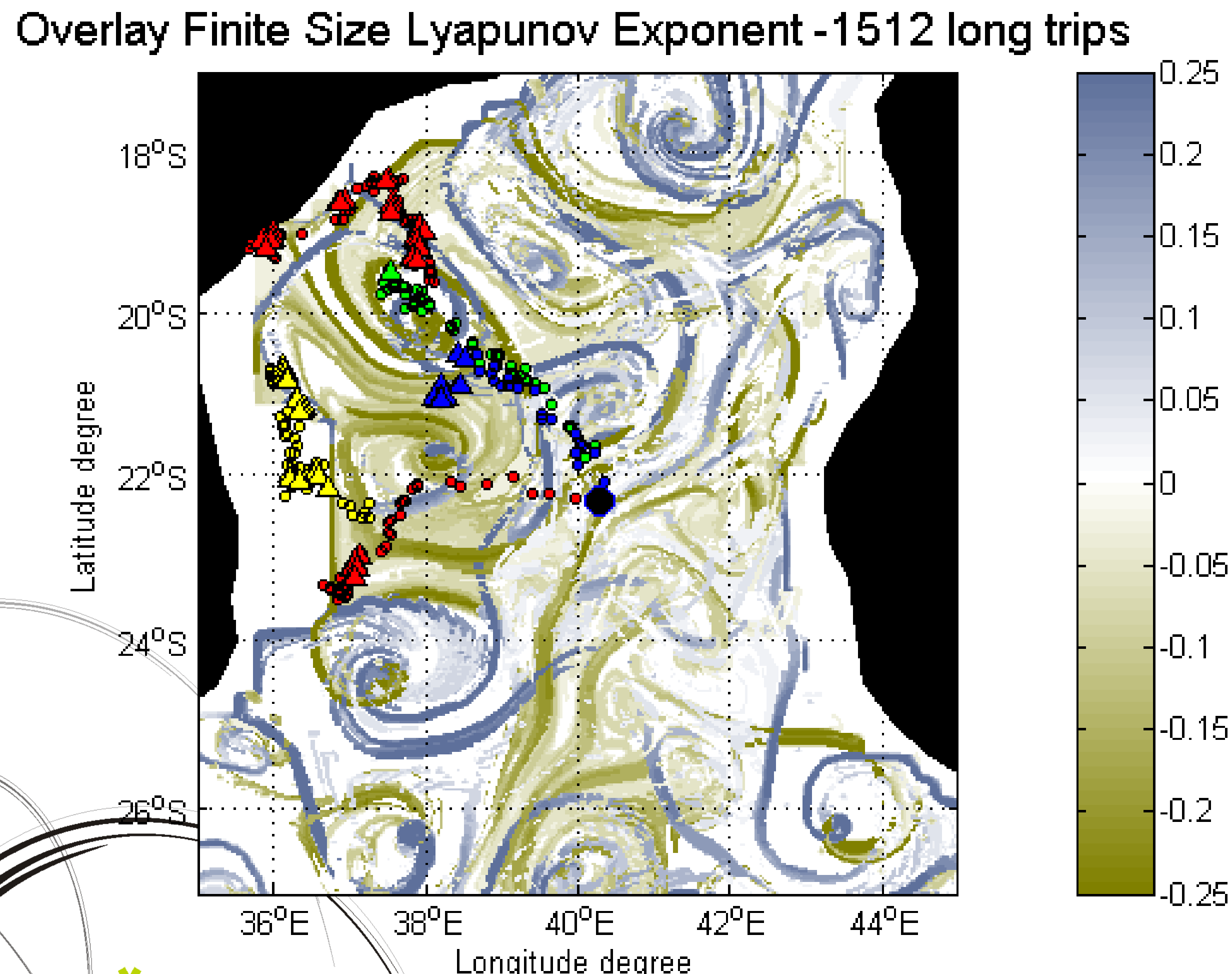
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26°S
Latitude degree
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42°E
44°E
Longitude degree
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-0.2
-0.25

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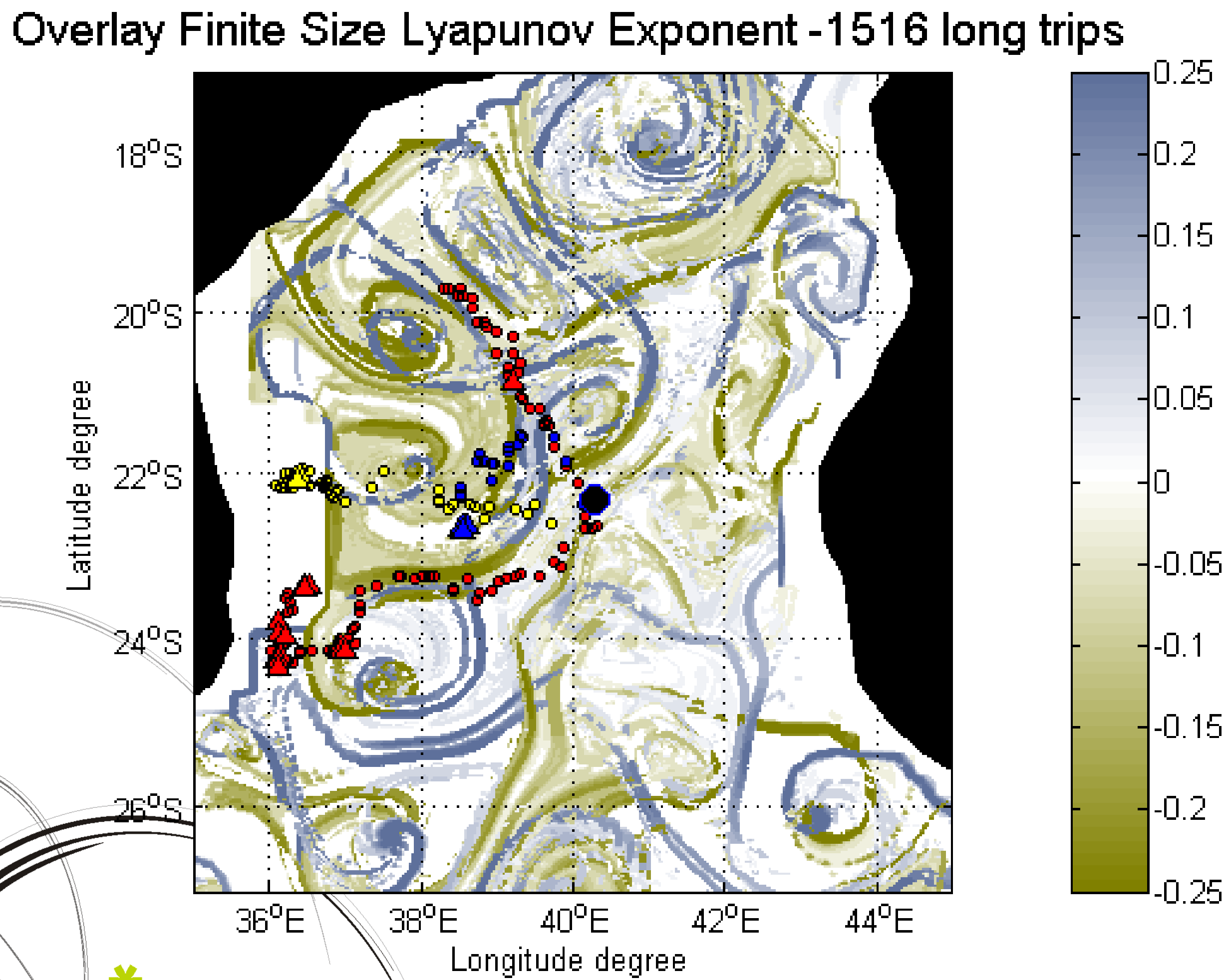
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44°E
Longitude degree
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-0.2
-0.25

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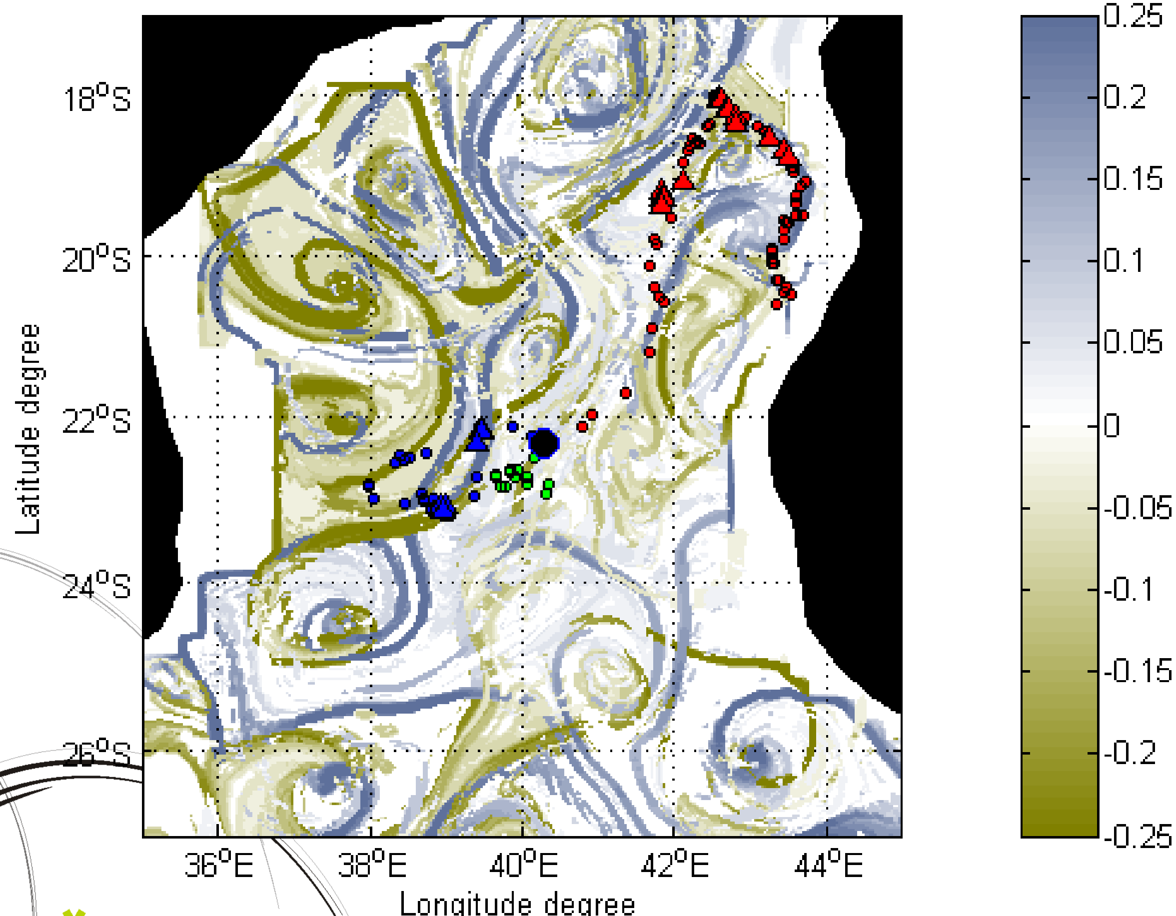
Overlay Finite Size Lyapunov Exponent -1520 long trips



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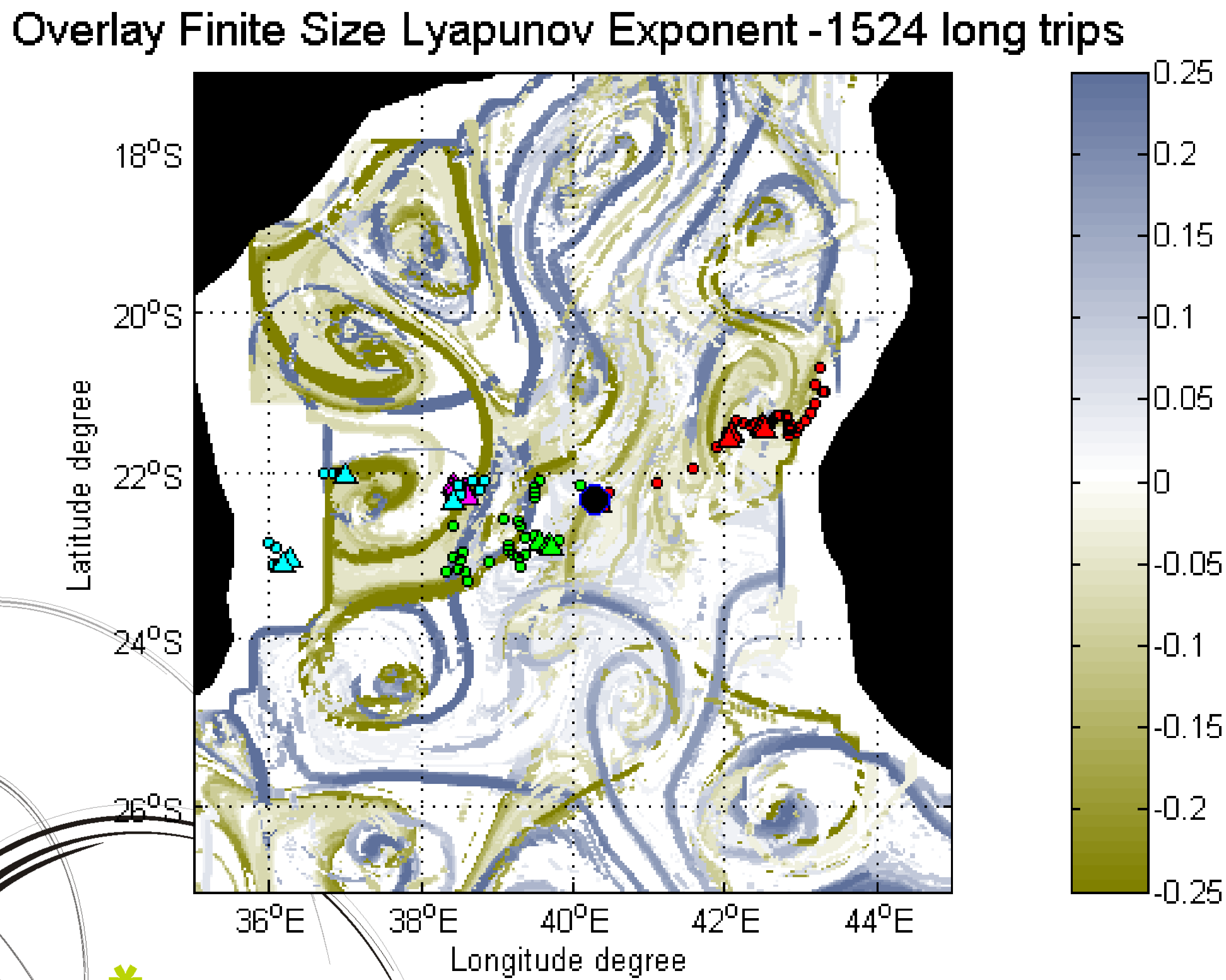
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26°S
Latitude degree
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40°E
42°E
44°E
Longitude degree
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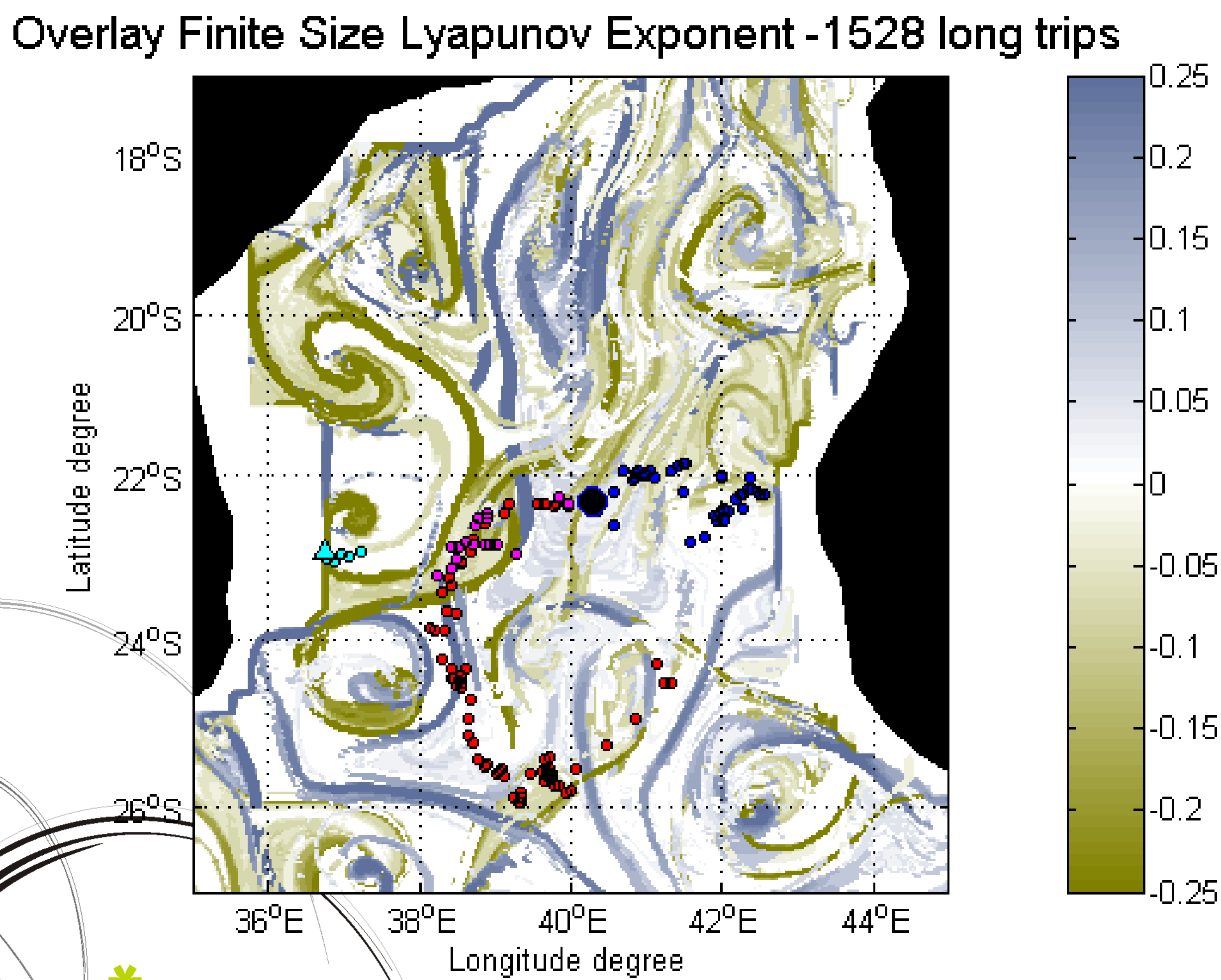
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26°S
Latitude degree
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42°E
44°E
Longitude degree
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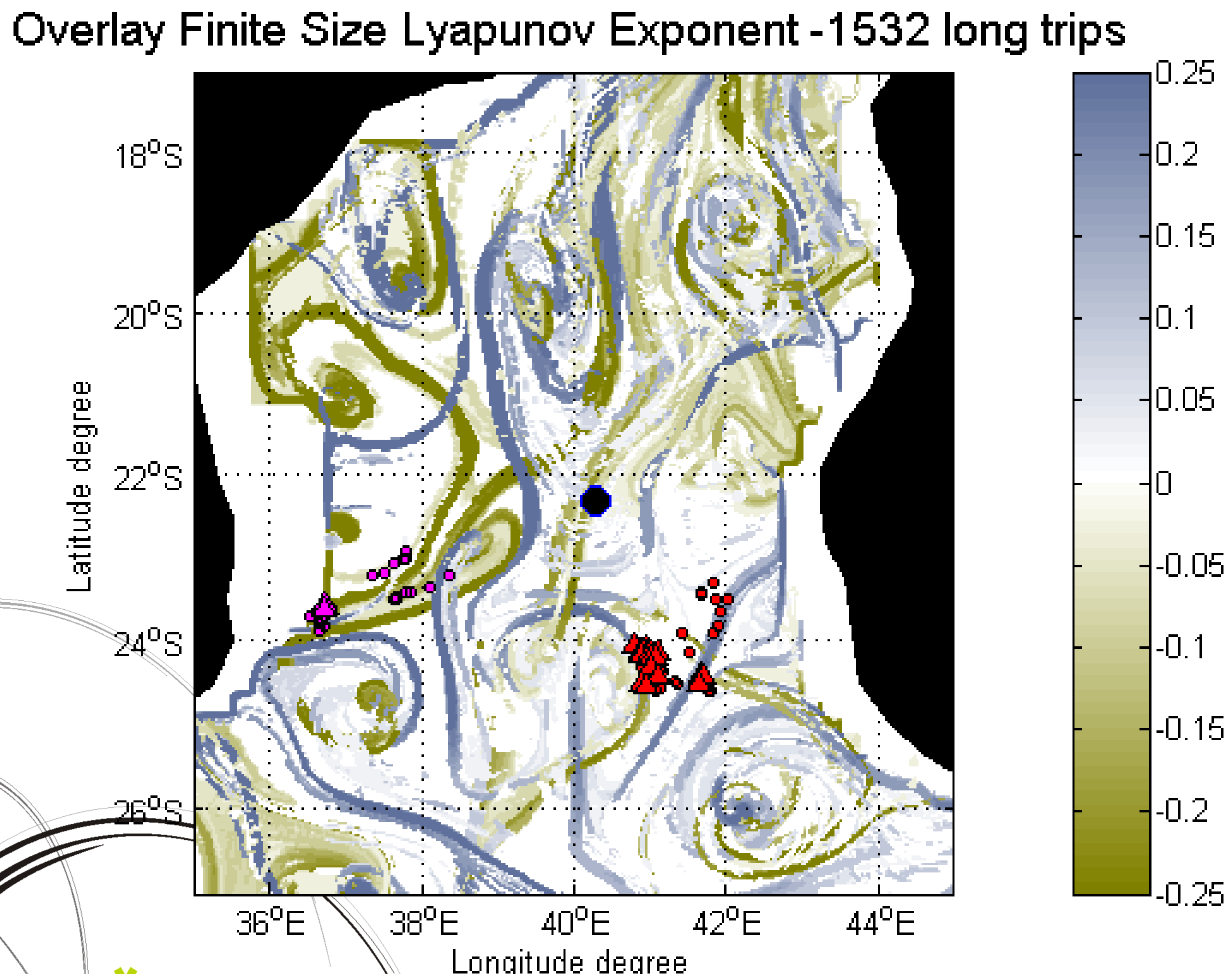
-0.1
-0.15
-0.2
-0.25



Overlay Finite Size Lyapunov Exponent -1532 long trips
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20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.25
0.2
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0.1
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-0.05
-0.1
-0.15
-0.2
-0.25

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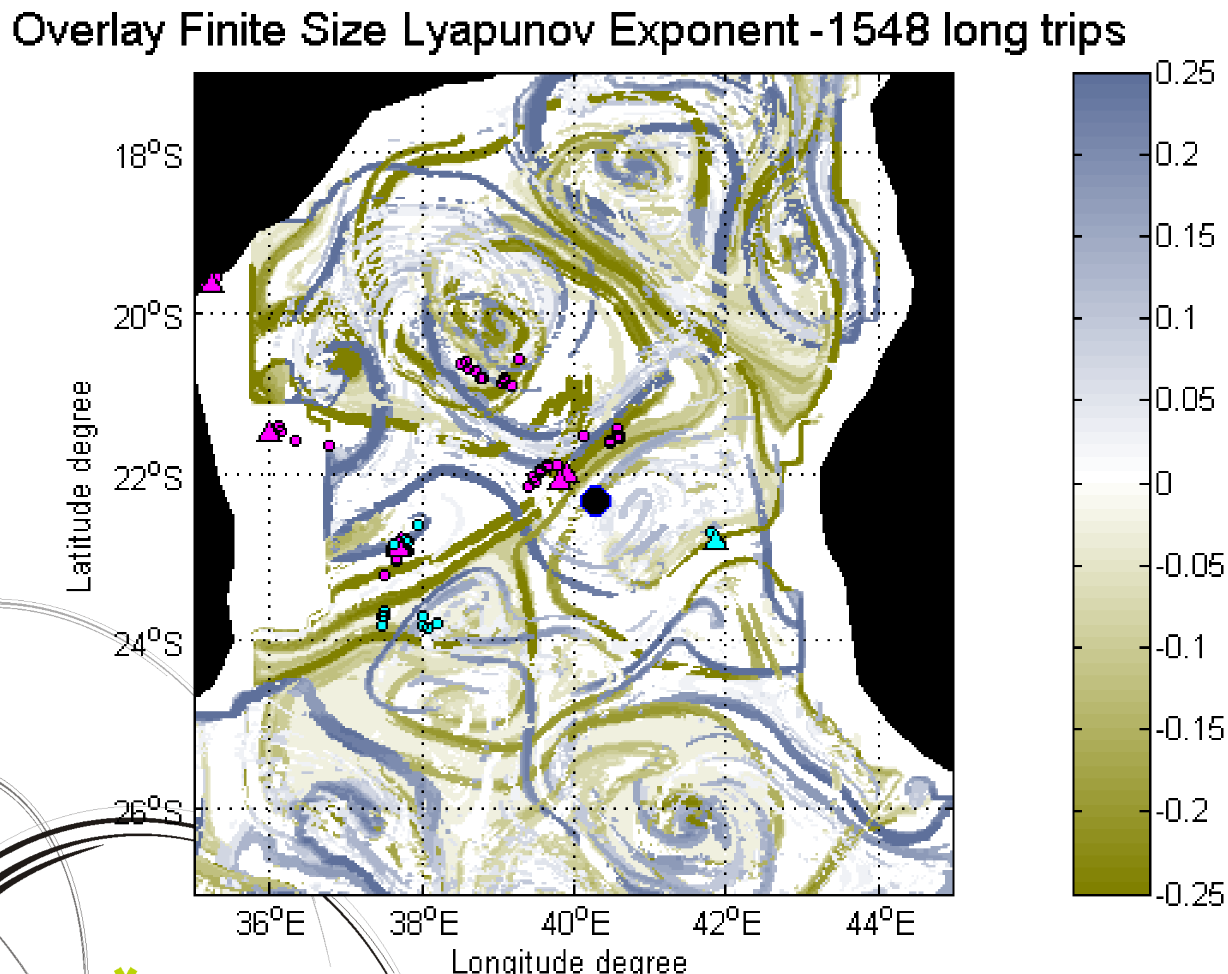

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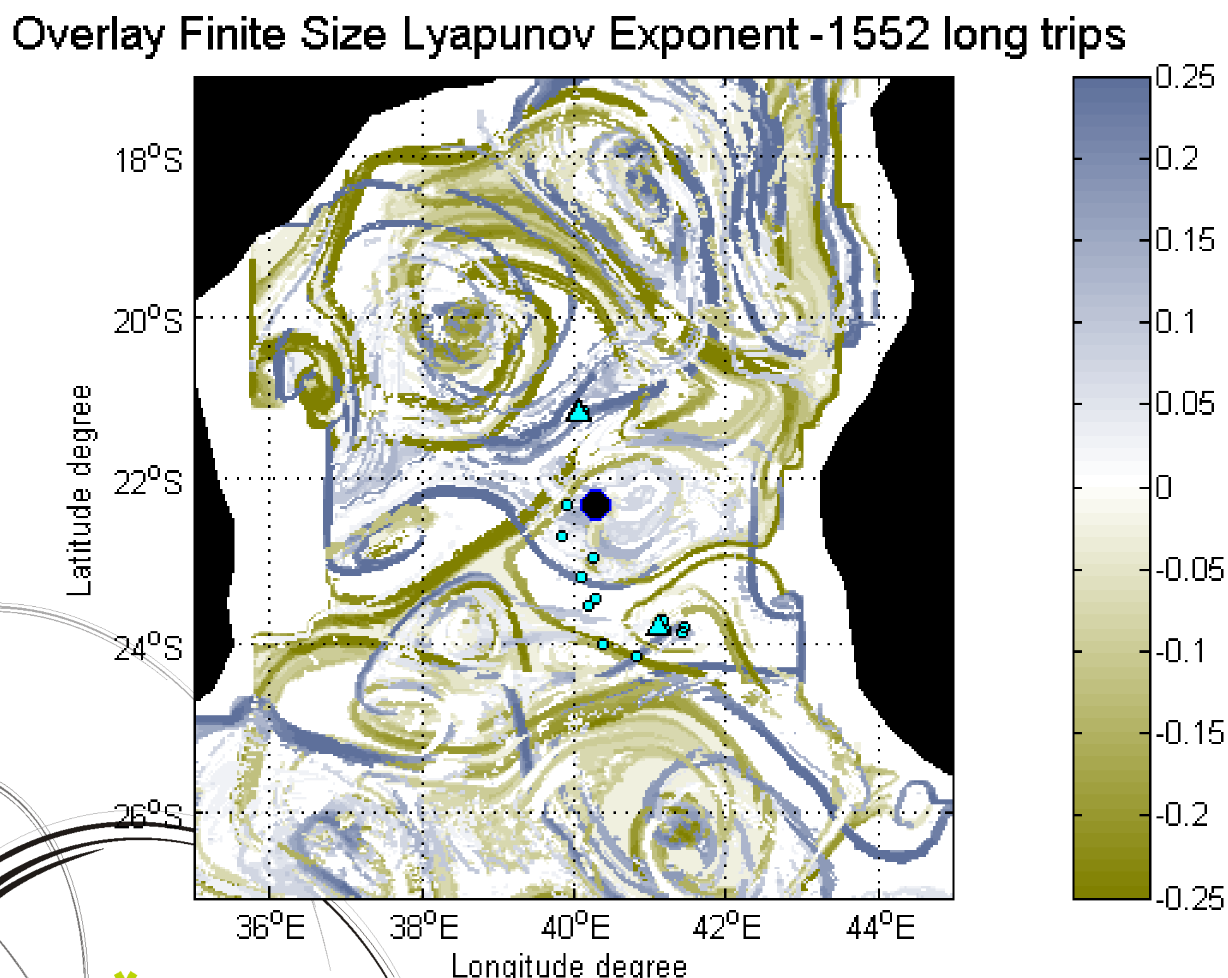

Overlay Finite Size Lyapunov Exponent -1552 long trips
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20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Latitude degree
Longitude degree
0.25
0.2
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0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

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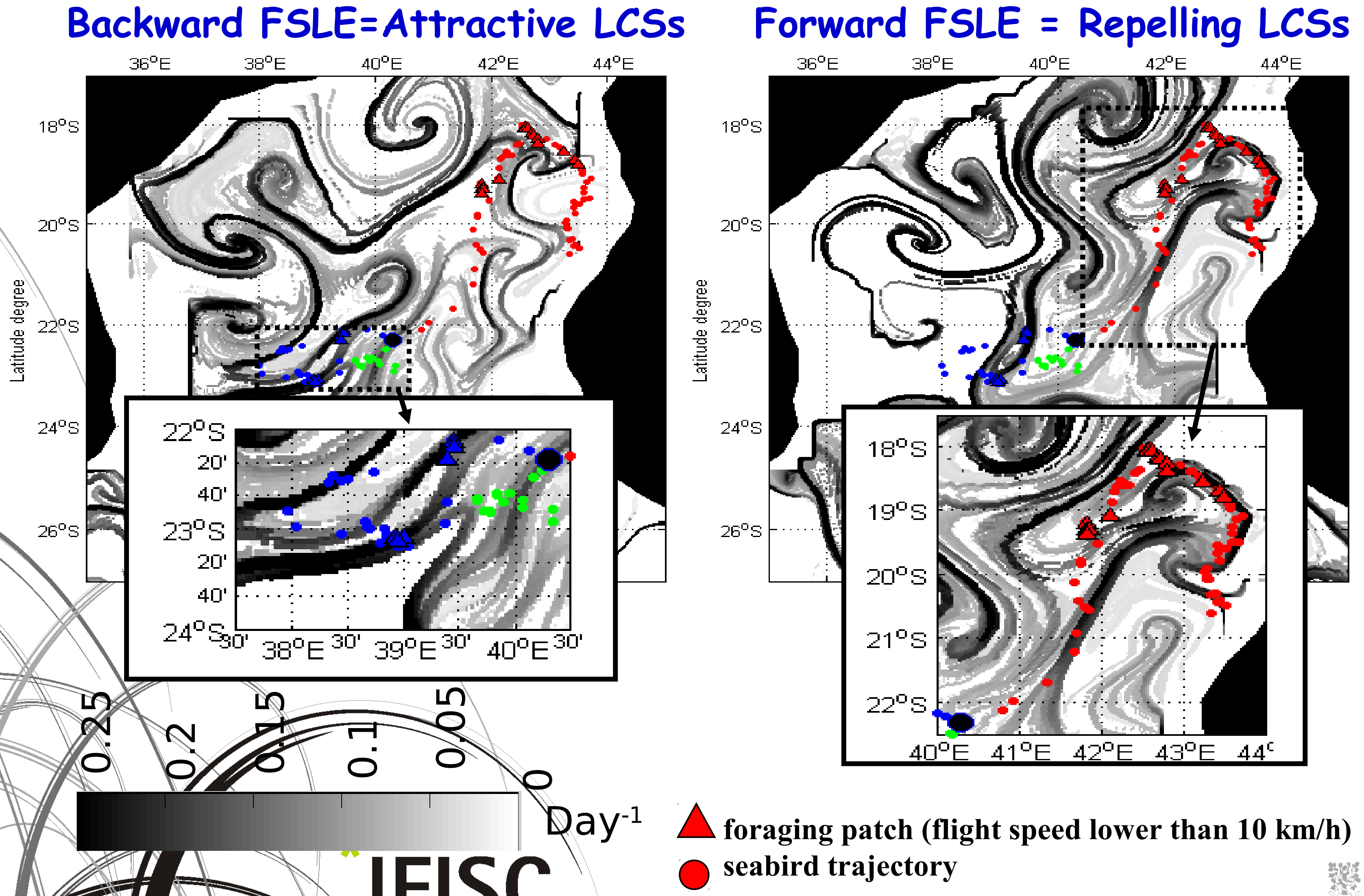
Week of September 24, 2003



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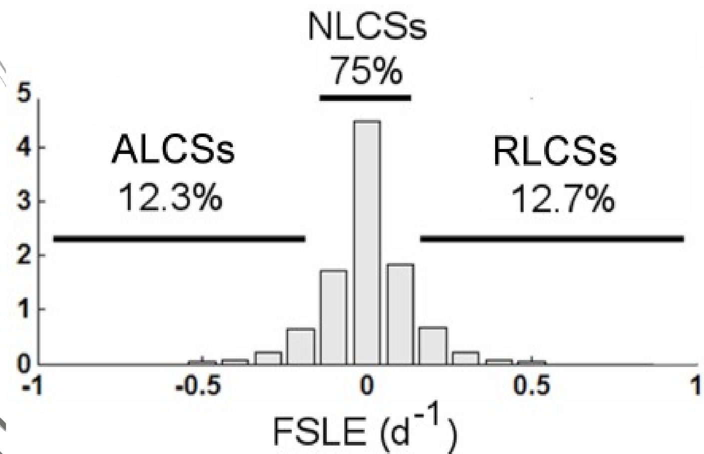
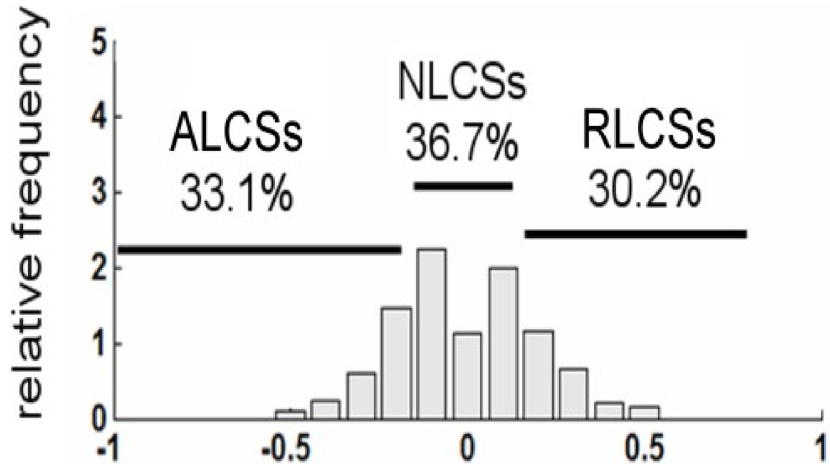
Histograms of FSLE values

On the whole area



On the birds positions



ALCS: attracting LCS, i.e. FSLE (backwards) < - 0.1 day^{-1}

RLCS: repelling LCS, i.e. FSLE (forwards) > 0.1 day^{-1}

NLCS: not LCS (small FSLE)

Despite LCS occupy only 25% of space, 63% of bird’s positions are on them

Results of statistical tests:

- Frigate birds fly on top of LCSs both for travelling as for foraging
- No significant difference between day and night positions
- No significant difference between come and return trip

Frigatebirds ‘follow’ LCSs not only to find there prey, but as biological corridors which bring them to foraging places

HOW AND WHY?

Aggregation of prey on LCSs? or aggregation of subsurface predators?

Olfatory clues (DMS produced by zooplankton) ? thermal air currents?

Puzzling issue: no significant difference between attracting and repelling LCSs

- Tangencies between manifolds?
- Interleaving between them?
- 3d dynamics associated both to ALCS and RLCS?
- Do they simply avoid low FSLE regions?

FINITE-SIZE LYAPUNOV EXPONENT FIELDS

- Able to reveal globally the dynamical structures in the flow: main hyperbolic trajectories, their manifolds, …
- Simple enough to be applied in a practical way to real and complex ocean velocity fields.
- On the negative side: rigorous results are missing.
- Reveals impact of fluid flow on biological dynamics at all scales: from plankton to top predators
- Relationship with 3d dynamics desirable

Robustness of FSLE

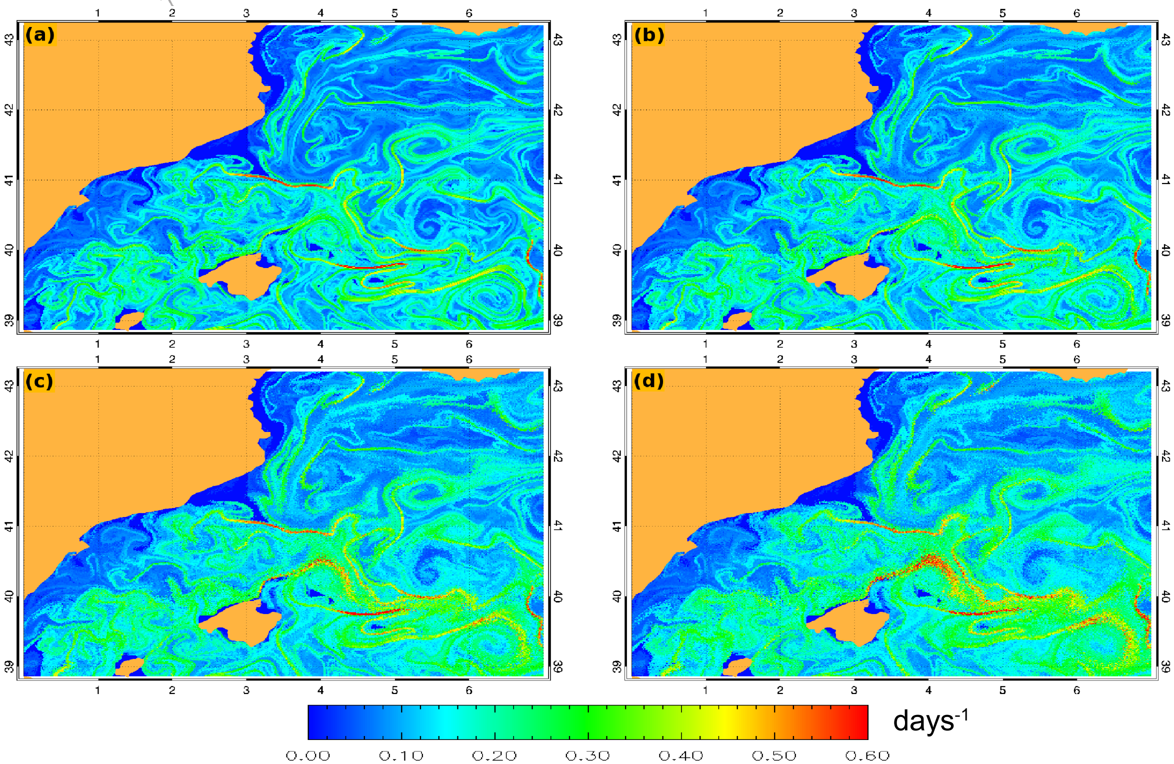
Error in the data

We get a perturbated velocity data (u',v') , by introducing a small random number in the original velocity (u,v).

$$u'(x,t) = u(\mathbf{x},t)(1+\alpha\eta_x(\mathbf{x},t))$$
$$v'(\mathbf{x},t) = v(\mathbf{x},t)(1+\alpha\eta_y(\mathbf{x},t))$$

$\{\eta_x(\mathbf{x},t),\eta_y(\mathbf{x},t)\}$: noise

Sets of Gaussians random number with mean zero and variance one



α is the relative size of perturbation

a) $\alpha = 0$
b) $\alpha = 2$
c) $\alpha = 6$
d) $\alpha = 10$

$\Delta_0 = 1/8^0$ $\delta_0 = 1/64^0$

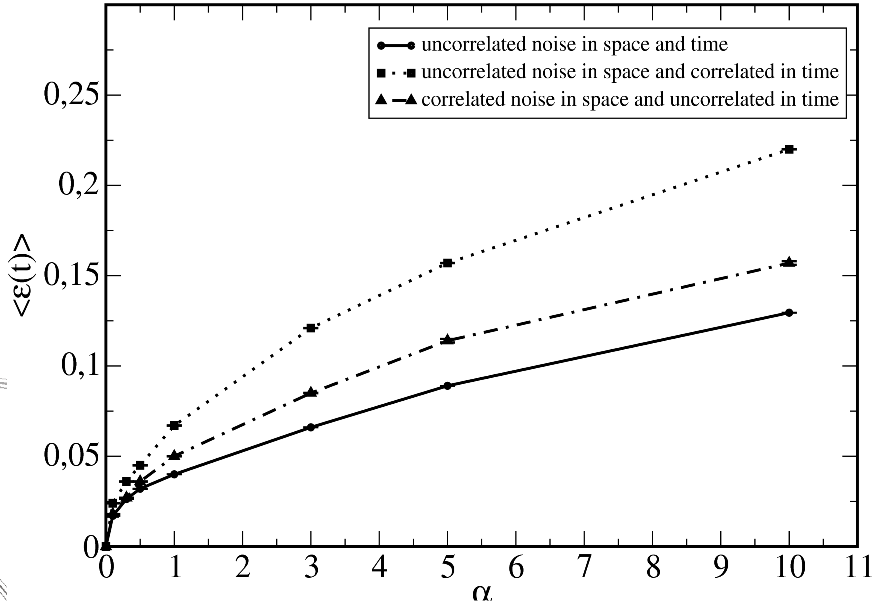
The Lagrangian structures look rather the same despite $\alpha = 10$

- **Relative error:** $\epsilon(t_i) = \sqrt{\frac{1}{N}\sum_{\mathbf{x}} \frac{|\Lambda^\alpha(\mathbf{x},t_i) - \Lambda(\mathbf{x},t_i)|^2}{|\Lambda(\mathbf{x},t_i)|^2}}, \quad < \epsilon(t) > \equiv \frac{1}{s}\sum_{i=1}^{s} \epsilon(t_i)$

Λ FSLE with data unperturbated

Λ^α FSLE with data perturbated

s = 100 snapshots, and N is the total points in the FSLE field



$\delta_0 = 1/8^0$

FSLE are robust to relatively large amount of error in the velocity data.
The average effect produced when computing FSLE by integrating over trajectories, make them robust against several kinds of uncorrelated noise in the velocity data

Noise in the particle's trajectories

We include unresolved small scales in the computation of FSLE.

$$\frac{d\phi}{dt} = \frac{u(\phi,\lambda,t)}{R\cos(\lambda)} + \frac{\sqrt{2D}}{R\cos(\lambda)}\xi_1(t),$$

$$\frac{d\lambda}{dt} = \frac{v(\phi,\lambda,t)}{R} + \frac{\sqrt{2D}\xi_2(t)}{R}$$

$< \xi_i(t)\xi_j(t') >= \delta_{ij}\delta(t-t')$ Gaussian white noise

Diffusivity: $D(l) = 2.055\ 10^{-4}\ l^{1.15}$
Okubo,1971

l = length scale = spatial resolution

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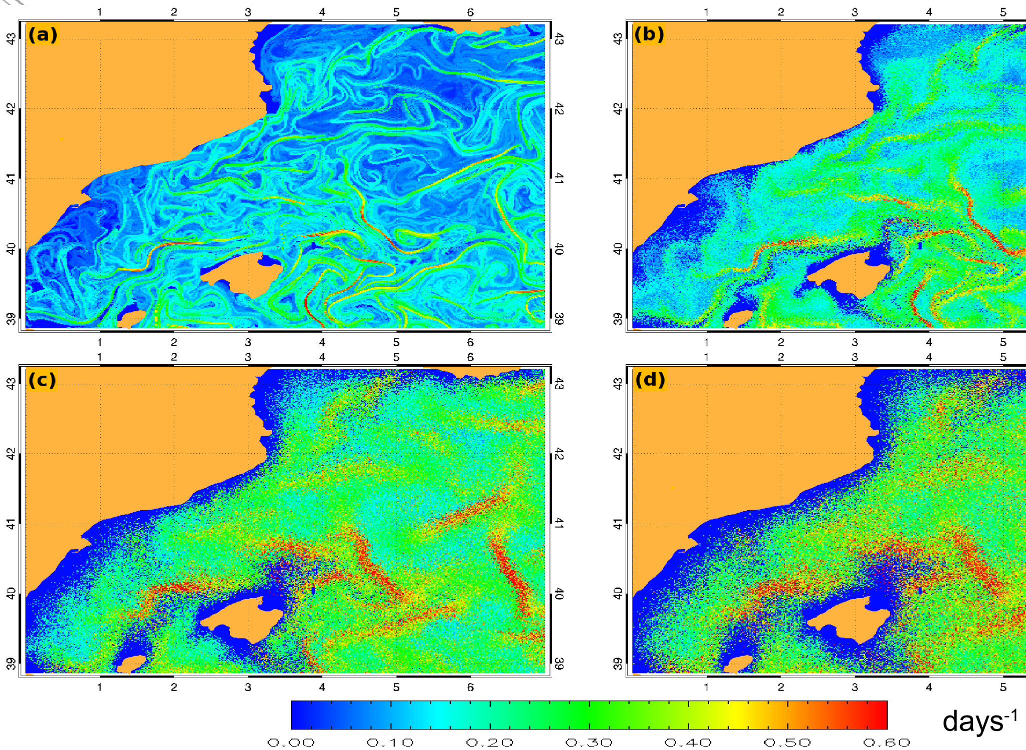
Noise in the particle's trajectories



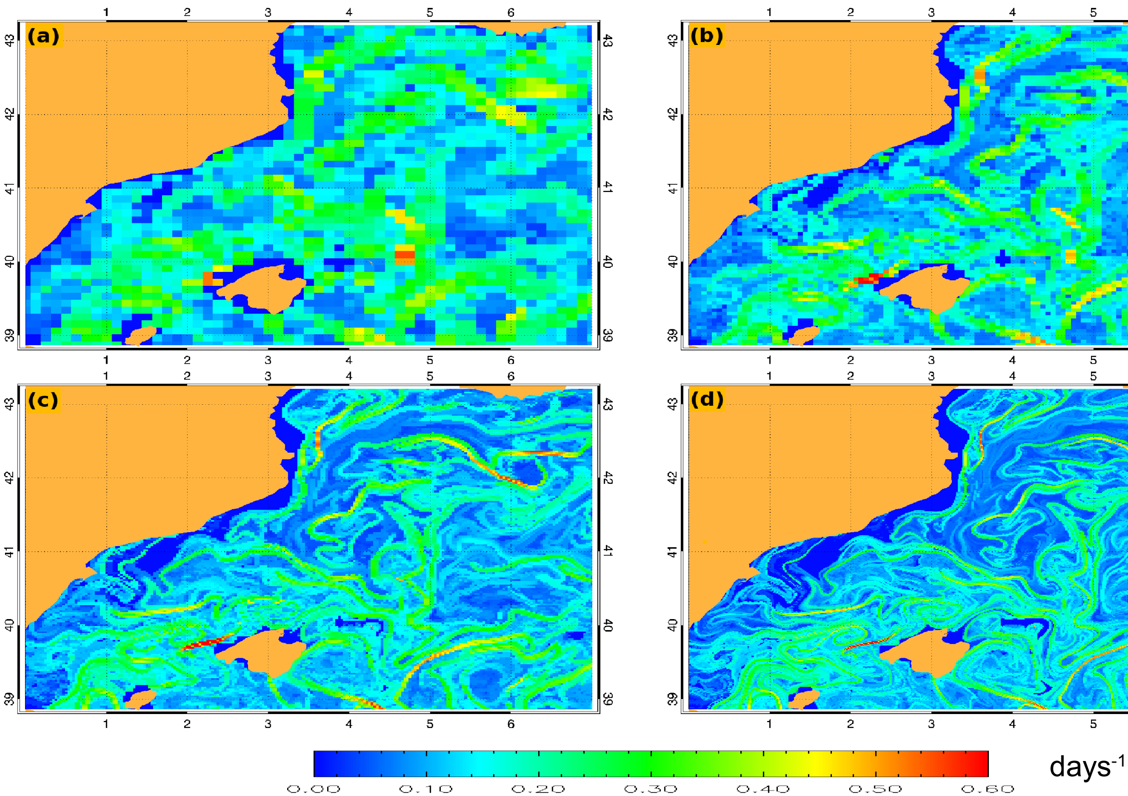
a) D = 0 m^2/s
b) D = 0.9m^2/s
c) D = 10m^2/s
d) D = 17m^2/s

$\Delta_0 = 1/8^0$ $\delta_0 = 1/64^0$

The mesoscale structures are maintained with eddy diffusivity (D)

Scale invariance properties of FSLE

FSLE at different spatial scales (spatial resolutions, δ_0)



a) $\delta_0 = 1/8^0$ $\Delta_0 = 1/8^0$

b) $\delta_0 = 1/16^0$ $\Delta_0 = 1/8^0$

c) $\delta_0 = 1/32^0$ $\Delta_0 = 1/8^0$

d) $\delta_0 = 1/64^0$ $\Delta_0 = 1/8^0$

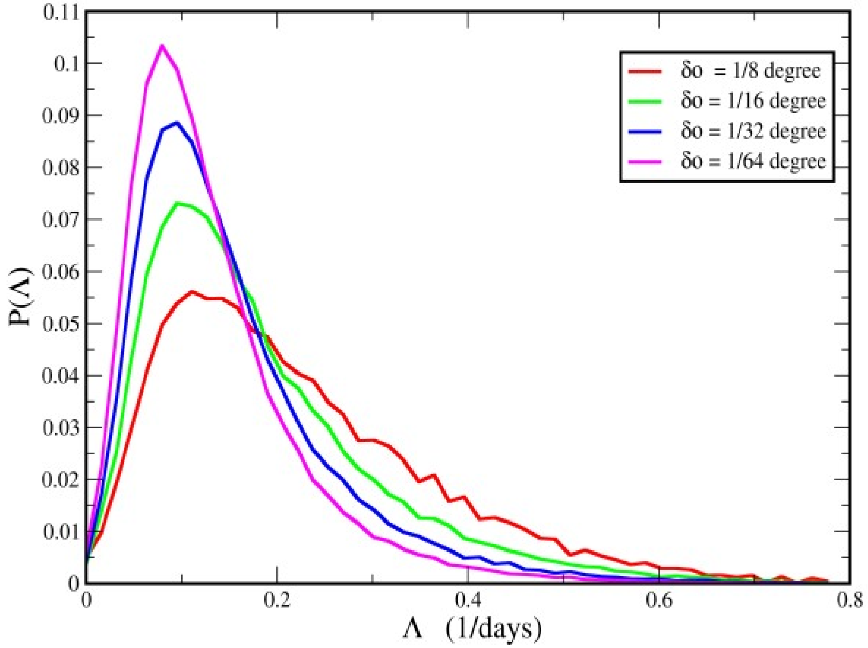
Note:

δ_0 = spatial scale of FSLE.

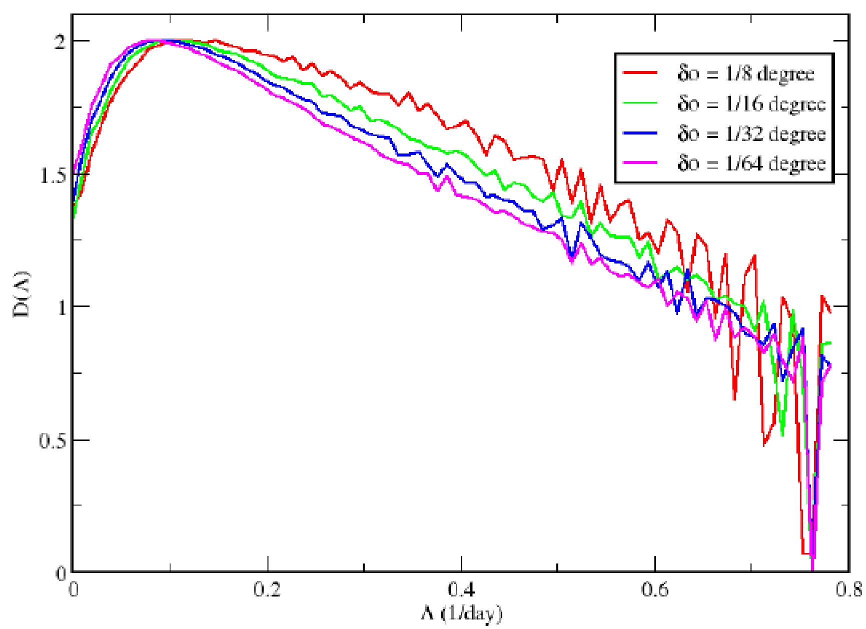
Δ_0 = velocity resolution

Increasing the spatial resolution we improve the identification of the mesoscale and submesoscale structures, and the large scales structures remain.

Histograms of FSLEs at differents scales



Fractal dimension of FSLEs at differents scales



$$P(\delta_0, \Lambda) = P(\delta_0, \Lambda_c)\, \delta_0^{d-D(\Lambda)}$$

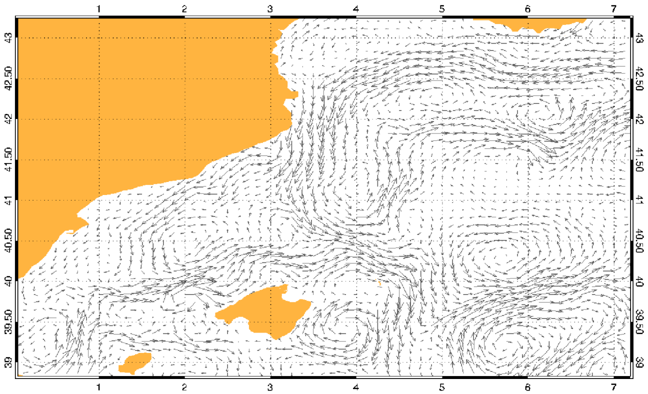
$$D(\Lambda) = d - \frac{\log \frac{P(\delta_0,\Lambda)}{P(\delta_0,\Lambda_c)}}{\log \delta_0}$$

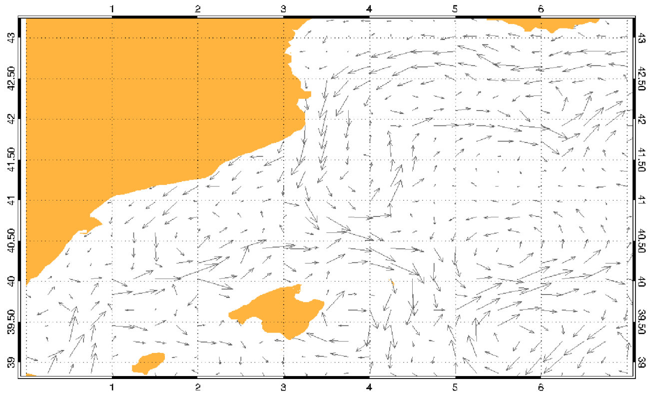
FSLE display a multifractal character.

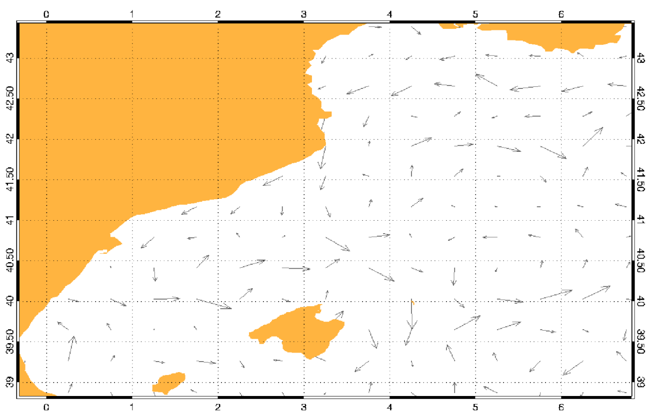
What happens if the spatial resolution of the velocity data is decreased?

Can we recover the structures?

Velocity field at different spatial resolutions





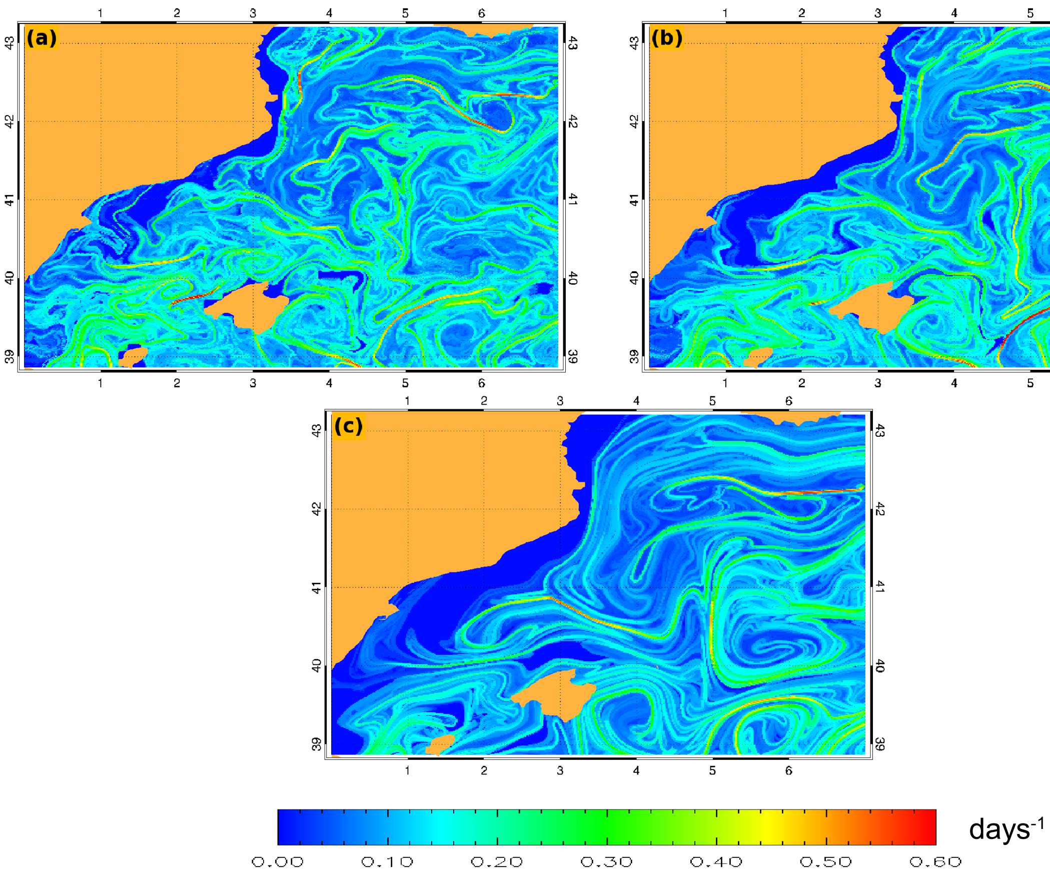


a) $\Delta_0 = 1/8^0$

b) $\Delta_0 = 1/4^0$

c) $\Delta_0 = 1/2^0$

FSLE at different spatial resolution of the velocity data



a) $\Delta_0 = 1/8^0$ $\delta_0 = 1/64^0$

b) $\Delta_0 = 1/4^0$ $\delta_0 = 1/64^0$

c) $\Delta_0 = 1/2^0$ $\delta_0 = 1/64^0$

δ_0 = spatial scale of FSLE

Δ_0 = velocity resolution

The large main structures remain even when the velocity field resolution is decreased from 10km to 50 km, and the small ones change.

Conclusions

- - Increasing the spatial resolution of FSLEs we improve the identification
- of surface mesoscale structures.
- - The main surface mesoscales structures in the ocean remain when the spatial resolution of the velocity data decreases.
- The spatial distribution of FSLE displays a multifractal character:
- The FSLE are rather robust.
- Mesoscale structures are maintained when the eddy diffusion is included

References

- d'Ovidio et al, GRL (2004)
- Rossi et al, GRL (2008)
- Rossi et al, Non. Proc. Geoph. (2009)
- Tew Kai et al, PNAS (2009)
- d'Ovidio et al, Deep Sea Research I (2009)
- Hernandez-Carrasco et al, submitted (2010).

- Avalaible at: http://www.ifisc.uib-csic.es/publications

ADITIONAL MATERIAL

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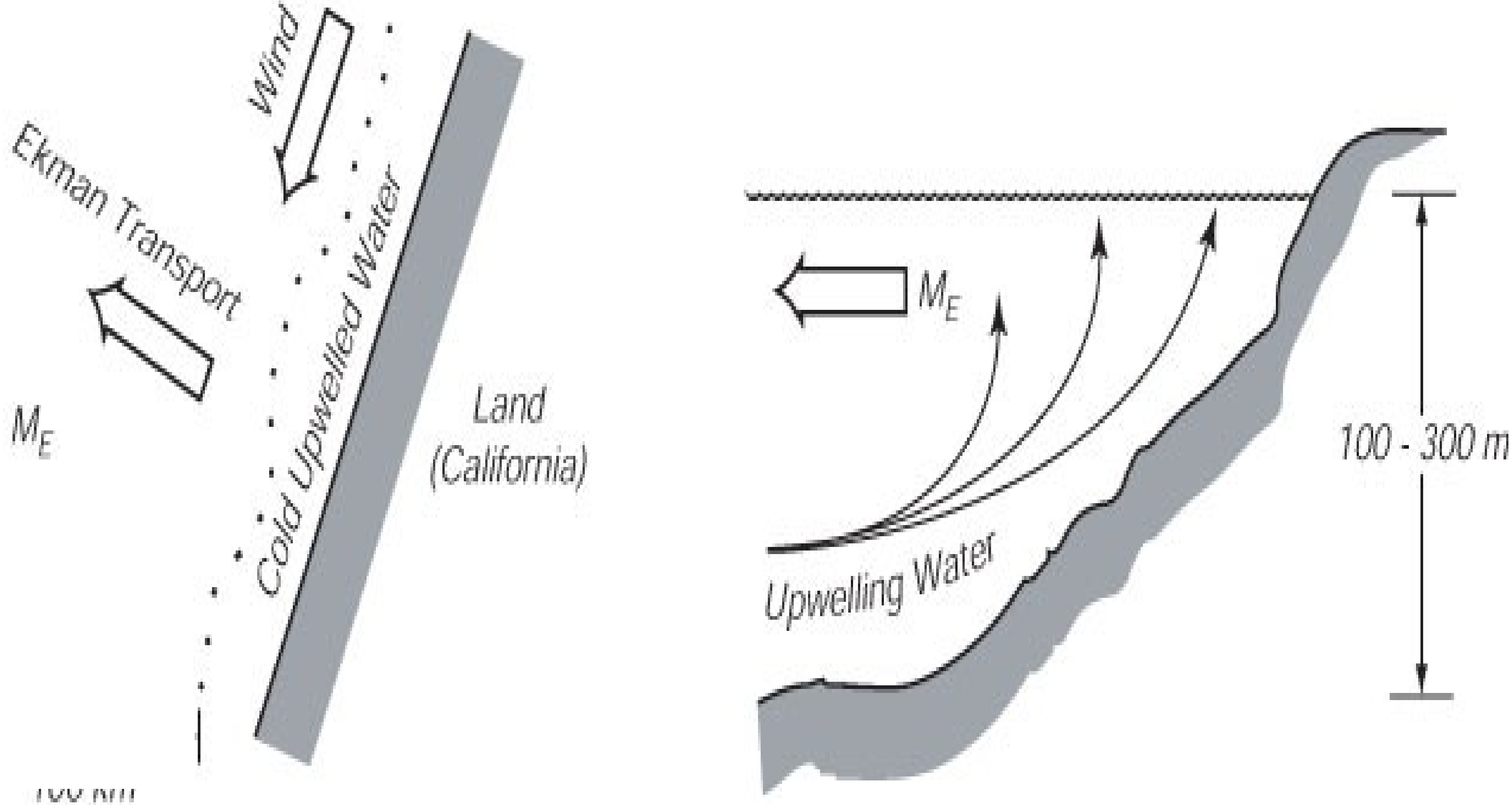
Upwelling dynamics: Ekman transport



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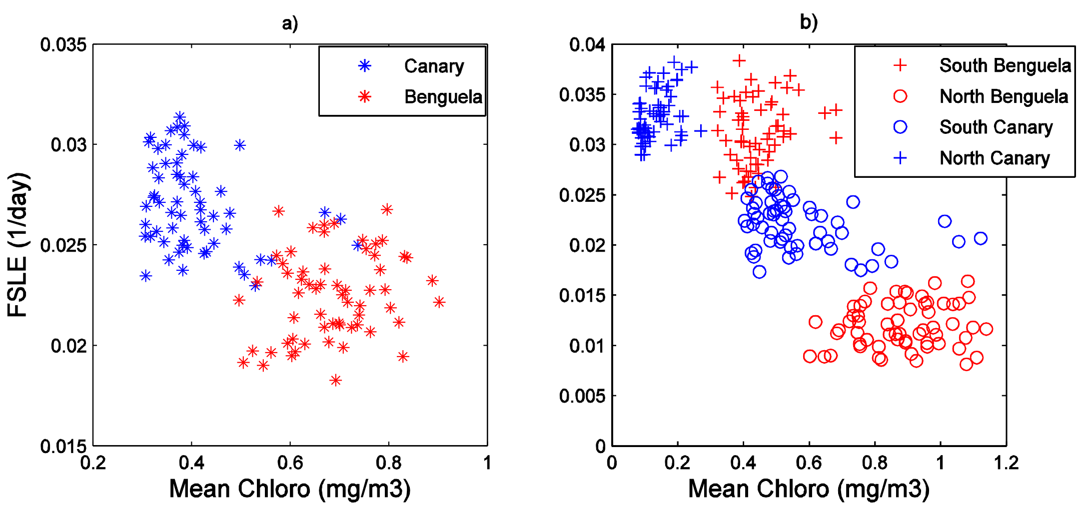
Spatial averaged westward Ekman transport versus averaged chlorophyll per subsystem



Negative (Blue)→ Westward transport offshore

Positive (red) → Eastward

Positive correlation: Westward offshore transporte→Higher chloro content

Behaviour

- They are unable to dive or rest on the water surface (permeable plumage).

- Special foraging strategy : They feed primarily in multi-species flocks in association with subsurface predators, especially tuna but also large fishes or dolphins that bring preys (small flying fishes, squids, crustaceans,..) to the marine surface.

Energy efficient flight : they use thermals to soar before gliding over long Distances.

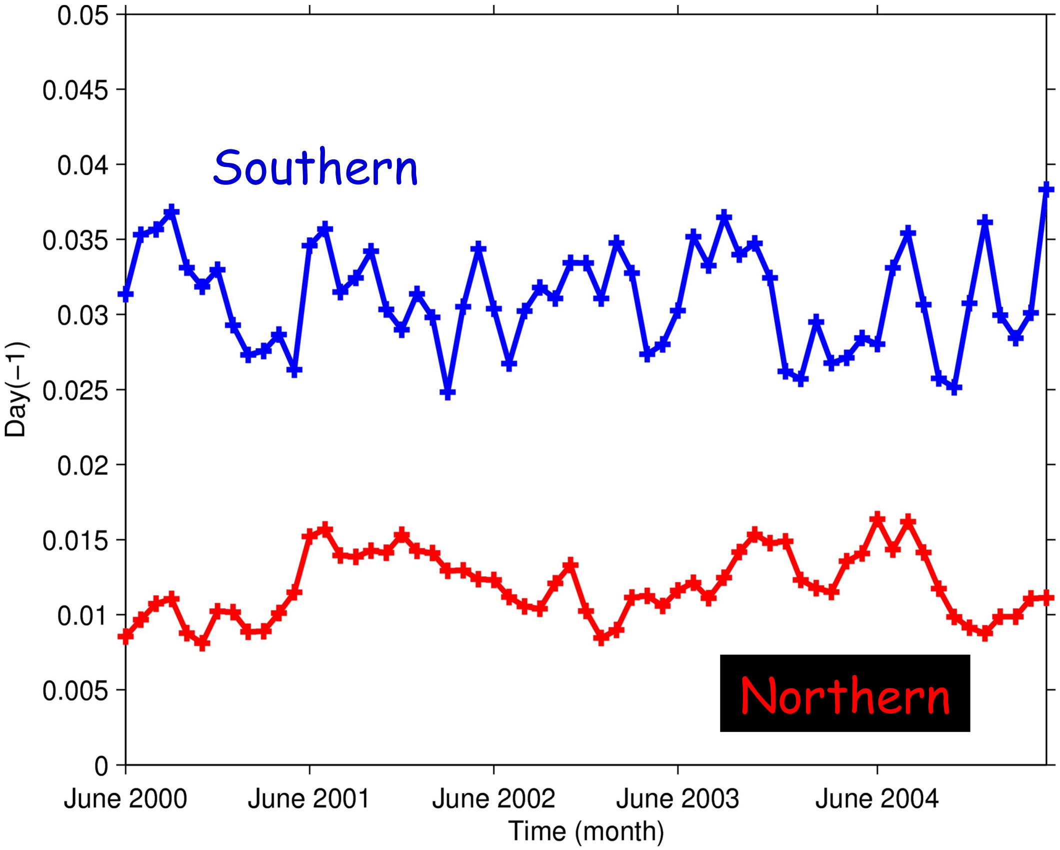
Can fly days and nights over weeks without touching ground.

Travelling time at high altitudes to locate patches of prey and come close to surface to feed,

Reduced flight speed : good indicator of foraging activity,

Feeding occurs only during daytime (peaks in the morning and evening)

Backward FSLE spatially averaged in the Benguela, per subsystem



- **Stronger mixing activity in the southern part.**

- **High seasonal variability in the southern system whereas the northern one is quite stable.**

How to explain this inverse relationship (*more turbulent less biomass*)?

- In open ocean, eddies tend to enhance biological productivity (particularly in low nutrient environments). This seems not to apply to upwelling regions (rich nutrient).

- Most straightforward conclusion: Horizontal turbulent mixing of nutrients in surface waters of the most productive subsystems is second order for biomass enhancement as compared to the vertical mechanisms.

- Areas with high FSLE are correlated with intense vertical movements (both up and down). Areas with low FSLE are dominated by upwards vertical velocities. *GROWING EVIDENCE OF THE PRESENCE OF STRONG VERTICAL DYNAMICS AT ANY PLACE WHERE SUBMESOSCALE OCCURS (Mahadevan et al 2006,2007).*

- Areas where Ekman drift dominates over mesoscale activity there is not a large dispersion of particles.

- Mixing modifies the 3D flow → weakening of the Ekman transport induced upwelling.

- Other factors: river inputs, dust, topography, etc..

Other factors influencing production …

Data: surface current and chlorophyll

Surface velocity data (u,v) computed at each grid point ($\frac{1}{4}$°) composed of:

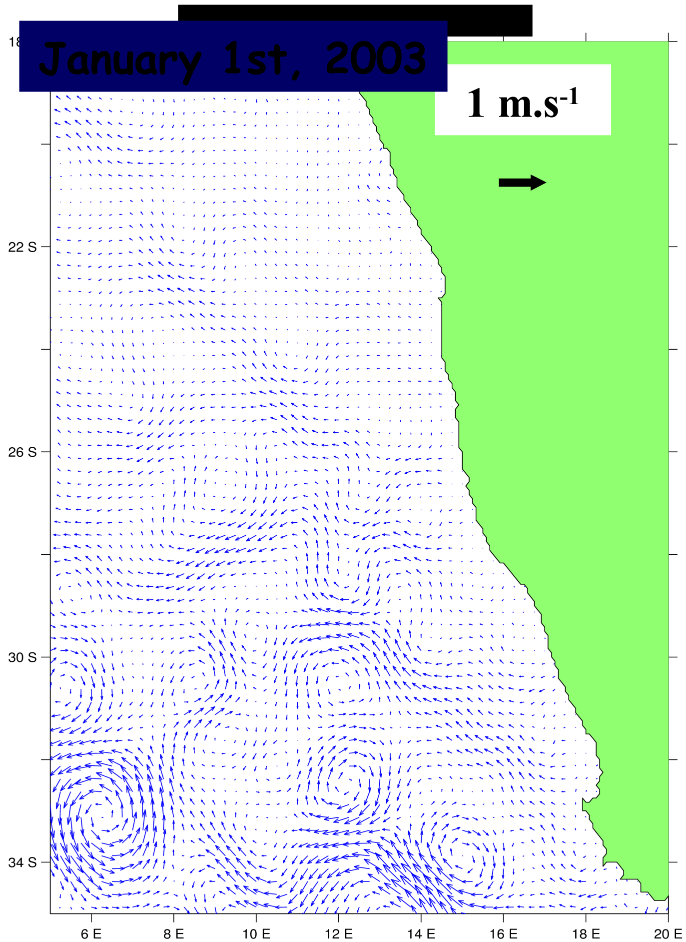
Geostrophic currents computed from a composite SSH field,

Ekman currents from QuikSCAT wind stress fields.

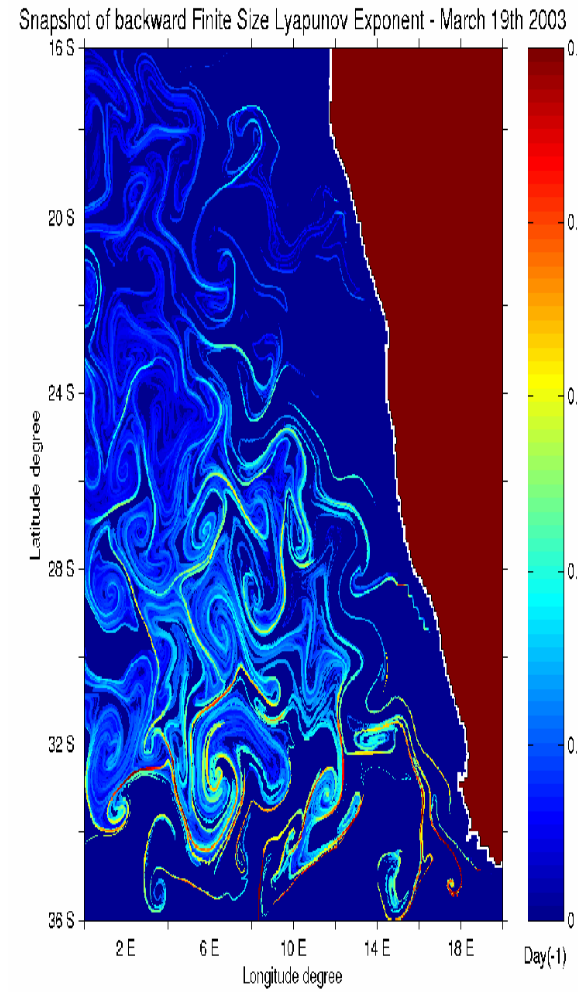
Chlorophyll a surface concentration from monthly SeaWiFS product.

Period covered: June 2000 – June 2005.

Velocity



FSLE



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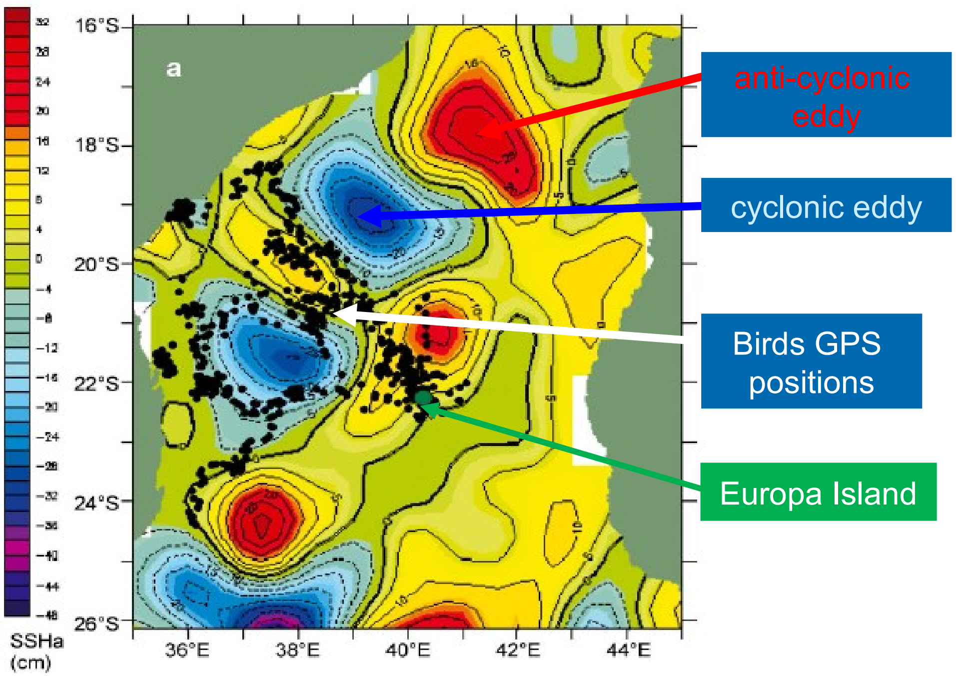
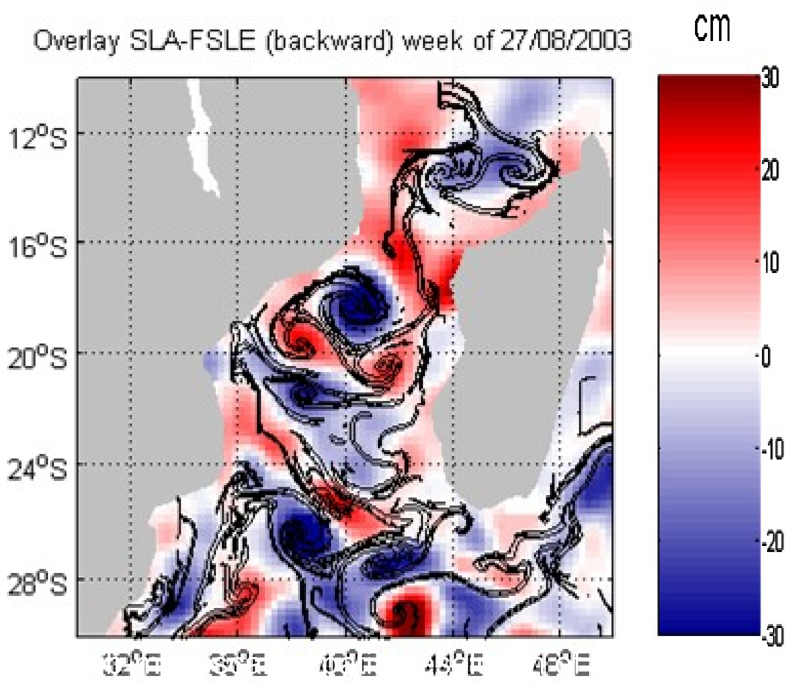
SSH (cm): Eulerian view

Lagrangian FSLEs versus SSH



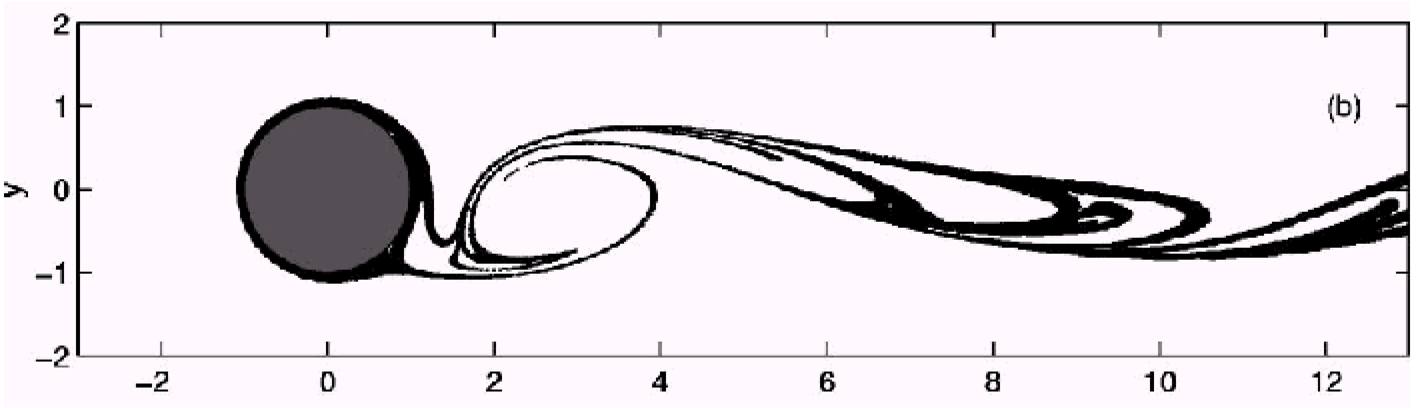
Weimerskirch et al,2004



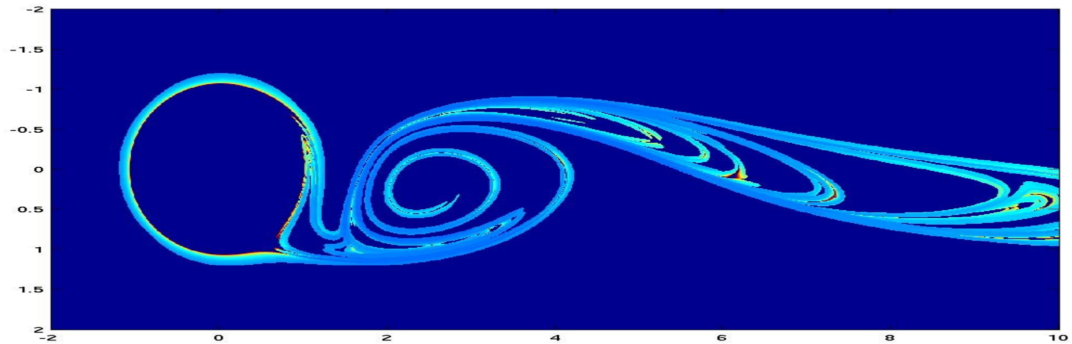
The Lagrangian FSLE gives access to submesoscale structures

Lagrangian Coherent Structures: **|FSLE| > 0.1 day^{-1}**

The spatial dependence of the FSLE allows the detection of stable and unstable manifolds of hyperbolic objects

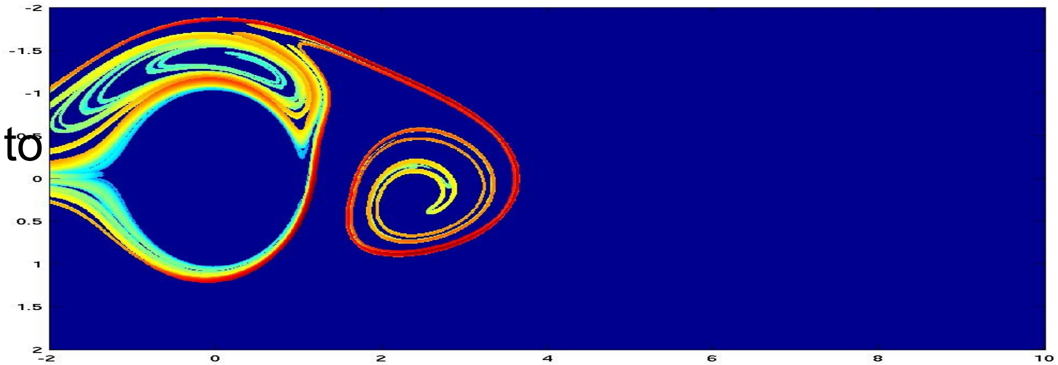


MAXIMA of FSLE: Lagrangian Coherent Structures (LCSs)



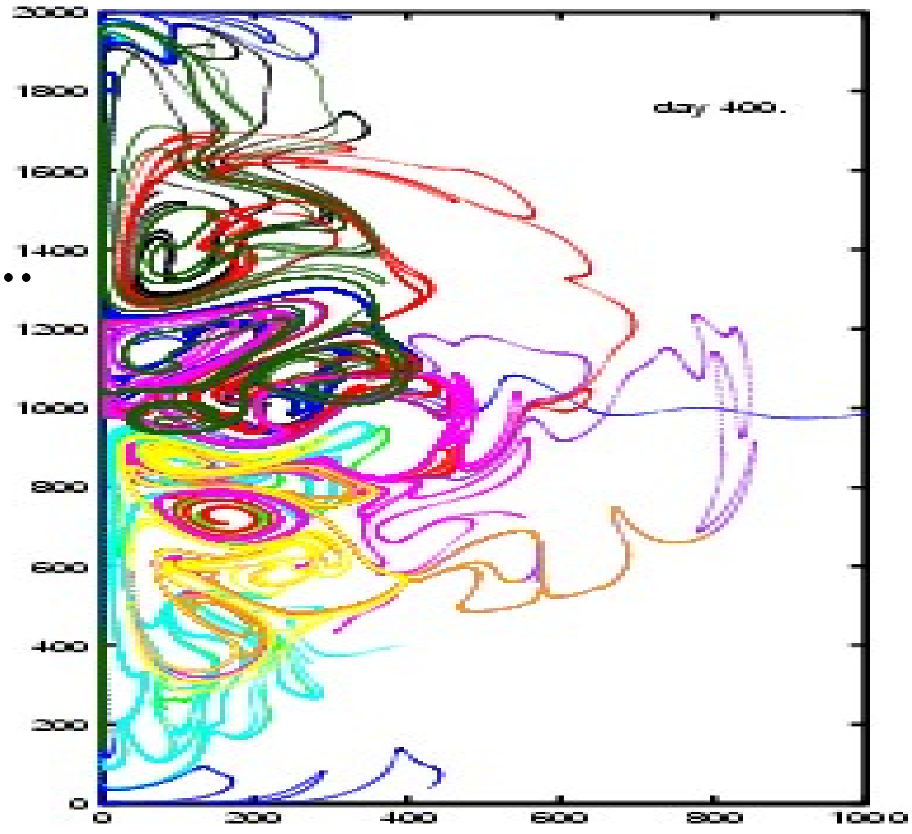
FSLE values from **time-backwards** trajectories

The lines organizing the flow seem to be the manifolds associated to strongest local Lyapunov exponents (backards and forward)



FSLE values from **time-forward** trajectories

But unsteady flows …



Set of invariant manifolds for a model of wind-driven Double gyre ocean circulation from Mancho, Small and Wiggins, 2005

Is there any particular subset of hyperbolic points and manifolds organizing the dynamics (the equivalent to the fixed points in autonomous systems) ? How to select them among this mess ?

Mesoscale eddies are fundamental structures in the habitat of marine communities:

i) Enrichment: primary production occurs.
ii) Concentration: areas where food accumulates.
iii) Retention: food is trapped.
iv) Transport: eddies transport them to oligotrophic zones.
v) They are one of the main responsibles of the formation of sub-mesoscale filamental structures.

Still a better understanding of the influence of submesoscale structures in marine ecosystems