Transport of particles in fluid flows

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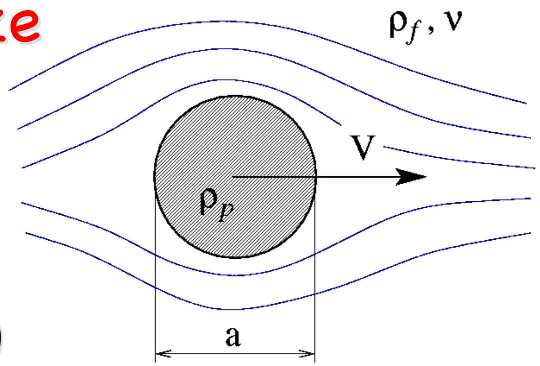
Two kinds of particles

Tracers= same as fluid elements

- same density of the fluid $\rho_p = \rho_f$
- point-like
- same velocity of the underlying fluid velocity

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{v}(t) = \boldsymbol{u}(\boldsymbol{X}(t), t)$$

Inertial particles= mass impurities of finite size



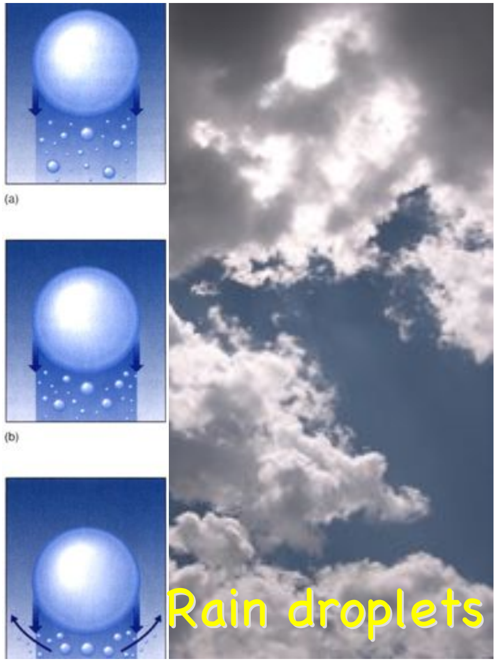
- density different from that of the fluid $\rho_p \neq \rho_f$
- finite size
- friction (Stokes) and other forces should be included
- shape may be important (we assume spherical shape)
- velocity mismatch with that of the fluid

Simplified dynamics under some assumptions

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$

$$\frac{d\boldsymbol{V}}{dt} = \boldsymbol{F}(\boldsymbol{V}, \boldsymbol{u}(\boldsymbol{X}(t), t), a, \nu, \ldots)$$

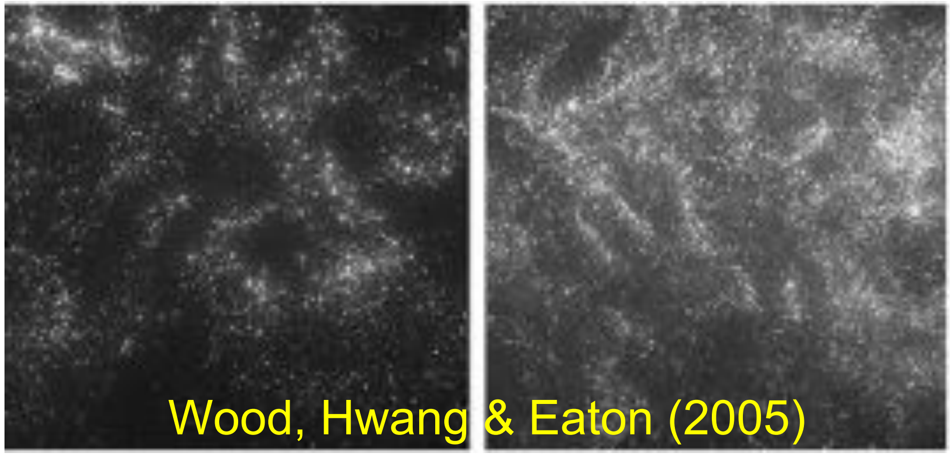
Inertial particles



Finite-size & mass impurities in fluid flows

Goal

Dynamical and statistical properties of particles evolving in turbulence
focus on clustering observed in experiments



Clustering important for

- particle interaction rates by enhancing contact probability
  (collisions, chemical reactions, etc...)
- the fluctuations in the concentration of a pollutant (Bec's talk)

Phenomenology of Turbulence

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = \nu \Delta \boldsymbol{u} - \tfrac{1}{\rho_f} \nabla p + \boldsymbol{f} \qquad \nabla \cdot \boldsymbol{u} = 0$$

$$Re = \frac{LU}{\nu} = \frac{\text{inertial t.}}{\text{dissipative t.}} \gg 1$$

Basic properties

- K41 energy cascade from large (~L) scale to the small dissipative scales (~η = Kolmogorov length scale)

- inertial range $\eta \ll r \ll L$ $\quad \delta_r u = u(x+r) - u(x) \sim r^{1/3}$

  Many characteristic time scales $\quad \tau_r = \dfrac{r}{\delta_r u} \sim r^{2/3}$

- dissipative range $r < \eta$ $\quad \delta_r u = u(x+r) - u(x) \sim r$

Fast evolving scale: characteristic time ---> $\tau_f = \tau_\eta = \dfrac{L}{U} Re^{-1/2}$

Simplified particle dynamics

Assumptions:

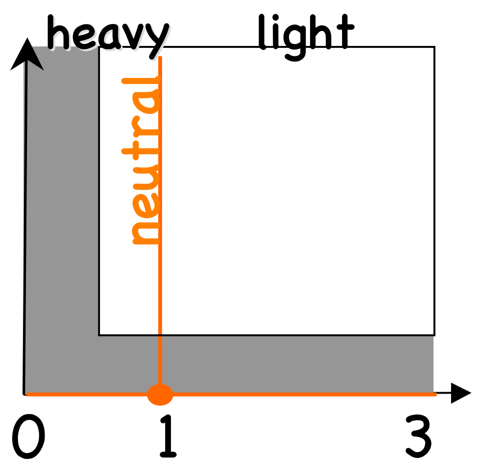
Small particles $a \ll \eta$
Small local Re $a|u-V|/\nu \ll 1$
No feedback on the fluid (passive particles)
No collisions (dilute suspensions)

Stokes number

$$St = \frac{\tau_p}{\tau_f}$$

Stokes time $\tau_p = \frac{a^2}{3\nu\beta}$

Fast fluid time scale τ_f



Density contrast

$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

$0 \le \beta < 1$ heavy
$\beta = 1$ neutral
$1 < \beta \le 3$ light

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$
$$\frac{d\boldsymbol{V}}{dt} = \beta\frac{D\boldsymbol{u}(\boldsymbol{X})}{Dt} + \frac{\boldsymbol{u}(\boldsymbol{X},t) - \boldsymbol{V}}{St}$$

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{V}$$
$$\frac{d\boldsymbol{V}}{dt} = \frac{\boldsymbol{u}(\boldsymbol{X}(t),t) - \boldsymbol{V}}{St}$$

Minimal interesting model

Very heavy particle $\beta=0$
(e.g. water droplets in air $\beta=10^{-3}$)

Inertial Particles as dynamical systems

Particle in d-dimensional space

$$\begin{aligned}\dot{\boldsymbol{X}} &= \boldsymbol{V} \\ \dot{\boldsymbol{V}} &= \beta D_t \boldsymbol{u}(\boldsymbol{X}) + \frac{\boldsymbol{u}(\boldsymbol{X},t) - \boldsymbol{V}}{St}\end{aligned} \qquad \boldsymbol{X},\boldsymbol{V} \in \mathrm{R}^d$$

$\boldsymbol{u}(\boldsymbol{x},t)$

Differentiable at small scales (r<η)

Well defined dissipative dynamical system in 2d-dimensional phase-space

$$\dot{\boldsymbol{Z}} = \boldsymbol{F}(\boldsymbol{Z},t) \qquad \boldsymbol{F} = (\boldsymbol{V}, \beta D_t \boldsymbol{u}(\boldsymbol{x},t) + \tfrac{\boldsymbol{u}-\boldsymbol{V}}{St}) \qquad \boldsymbol{Z} = (\boldsymbol{X},\boldsymbol{V}) \in \mathrm{R}^{2d}$$

$\mathbb{L}_{ij} = \partial_j F_i$ Jacobian (stability matrix)

$\sigma_{ij} = \partial_j u_i$ Strain matrix

$$\mathbb{L} = \begin{pmatrix} \mathbb{O} & \mathbb{I} \\ \beta D_t \sigma + \frac{\sigma}{St} & -\frac{\mathbb{I}}{St} \end{pmatrix}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{F} = Tr(\mathbb{L}) = -\frac{d}{St} < 0$$

constant phase-space contraction rate, i.e. phase-space
Volumes contract exponentially with rate -d/St
Motions evolve onto an attractor in phase space

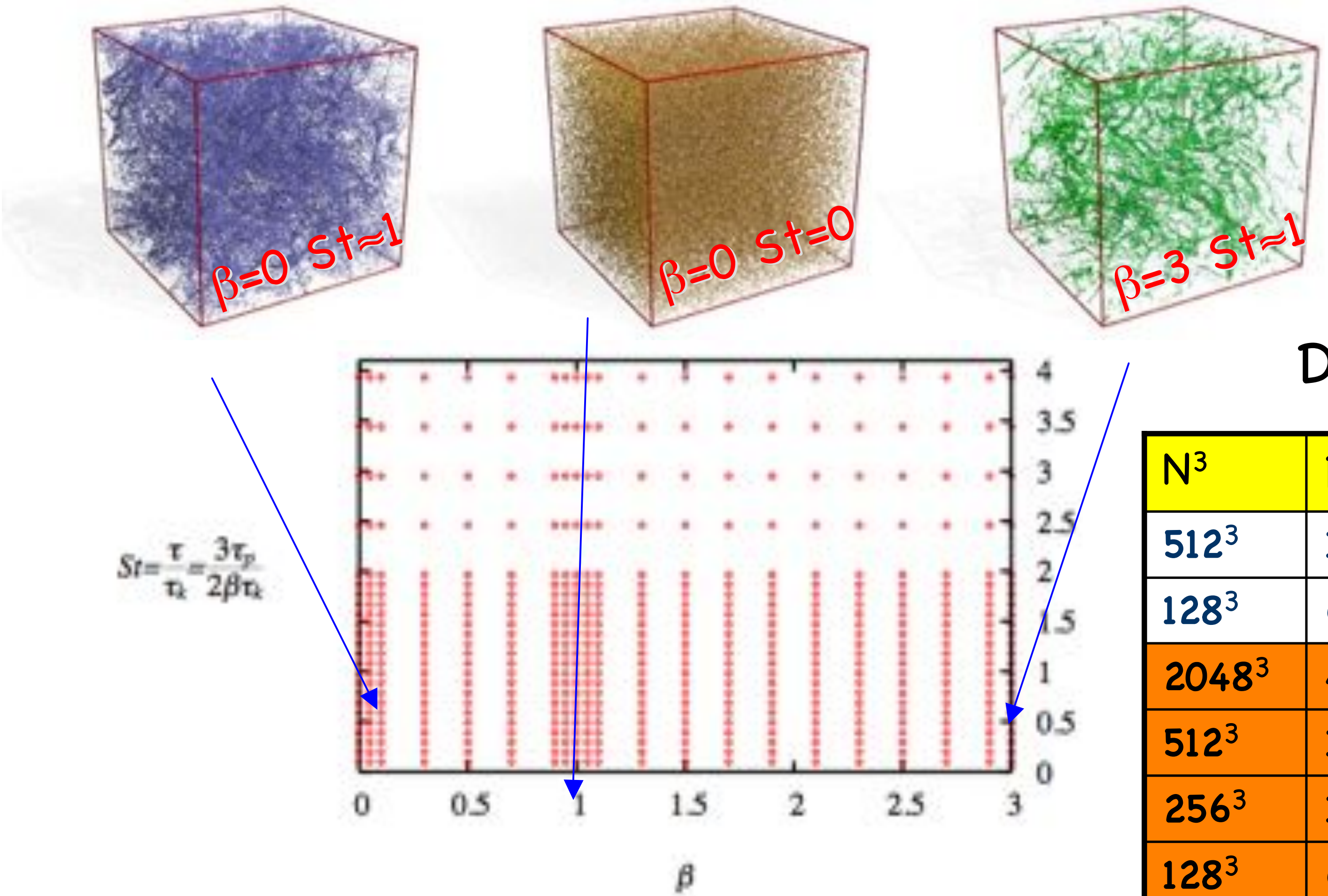
Particles in turbulence

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = \nu \Delta \boldsymbol{u} - \tfrac{1}{\rho_f} \boldsymbol{\nabla} p + \boldsymbol{f}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

$$\dot{\boldsymbol{X}} = \boldsymbol{V}$$

$$\dot{\boldsymbol{V}} = \beta D_t \boldsymbol{u}(\boldsymbol{X}) + \frac{\boldsymbol{u}(\boldsymbol{X}, t) - \boldsymbol{V}}{St}$$



DNS summary

N^3	Re_λ	β	St
512^3	185	0->3	0.16->4
128^3	65	0->3	0.16->4
2048^3	400	0	0.16->70
512^3	185	0	0.16->3.5
256^3	105	0	0.16->3.5
128^3	65	0	0.16->3.5

Mechanisms at work

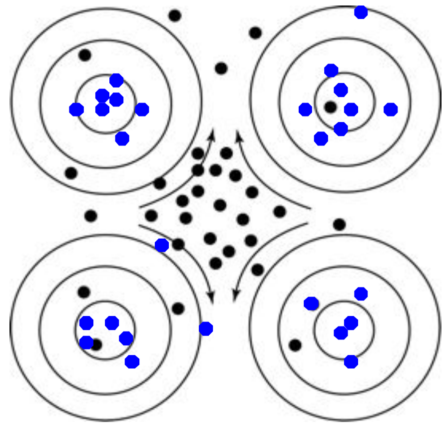
- Ejection/injection of heavy/light particles from/in vortices preferential concentration

$$\frac{D\boldsymbol{u}(\boldsymbol{X},t)}{Dt} \approx \frac{d\boldsymbol{V}}{dt} = \beta\frac{D\boldsymbol{u}(\boldsymbol{X},t)}{Dt} + \frac{u(\boldsymbol{X},t)-\boldsymbol{V}}{\tau}$$

$$\tau \ll 1 \quad \rightarrow \quad \boldsymbol{V} \approx \boldsymbol{u} + (\beta-1)\tau\frac{D\boldsymbol{u}}{Dt}$$

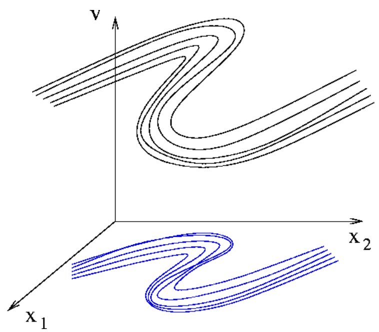
$$\boldsymbol{\nabla}\cdot\boldsymbol{V} \approx (\beta-1)\tau\boldsymbol{\nabla}\cdot(\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}) = \tau(\beta-1)(S^2-\Omega^2)$$

(Maxey 1987; Balkovsky, Falkovich, Fouxon 2001)



- $\beta<1$ heavy
- $\beta>1$ light

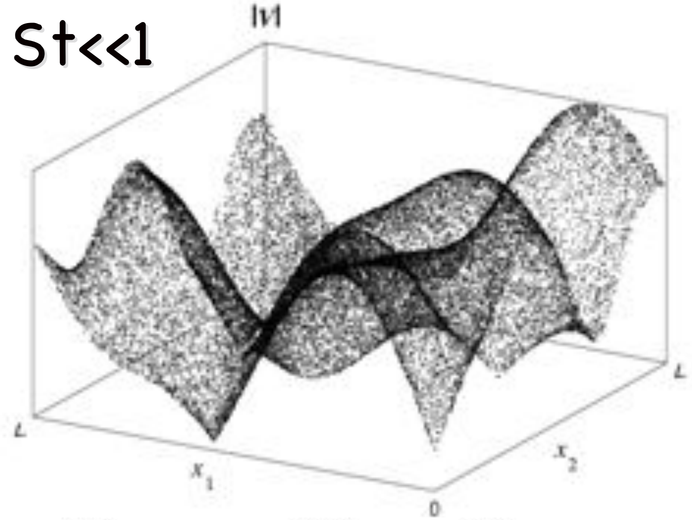
- Dissipative dynamics in phase-space: volumes contraction & particles may arrive very close with very different velocities



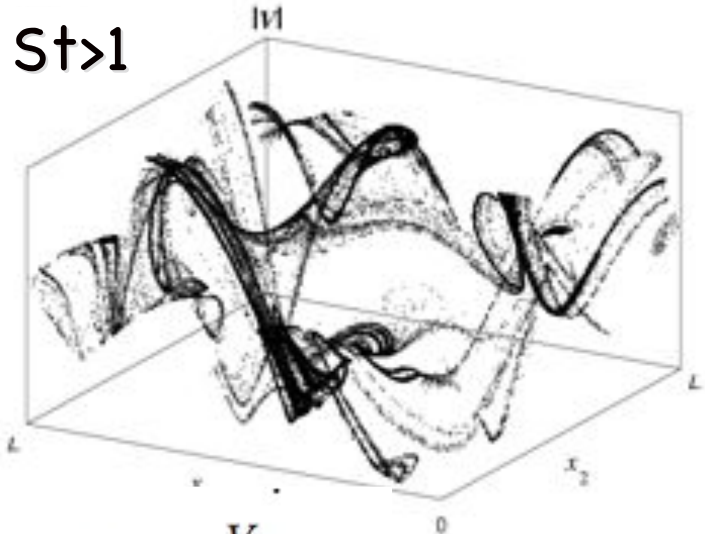
- Finite response time to fluid fluctuations (filter of fast time scales)

Phase space dynamics

St<<1

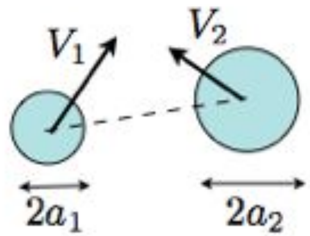


St>1



$$\boldsymbol{R} = \boldsymbol{X}_1 - \boldsymbol{X}_2; \qquad R = |\boldsymbol{R}|$$

$$\delta_R V_{\|} = (\boldsymbol{V}_1 - \boldsymbol{V}2) \cdot \frac{\boldsymbol{R}}{R}$$



Collision rate

$$k(r) \sim p_2(r) \langle |\delta_R V_{\|}| R = r \rangle$$

$r = a_1 + a_2$

Enhanced encounters by clustering

Enhanced relative velocity

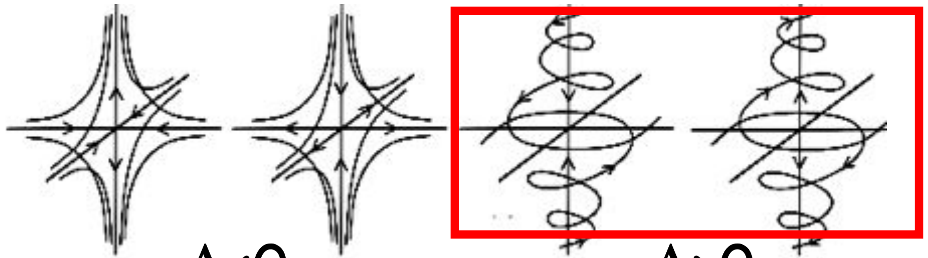
Correlation with the flow

Preferential concentration

Heavy particles like strain regions

Light particles like rotating regions

Strain rotation



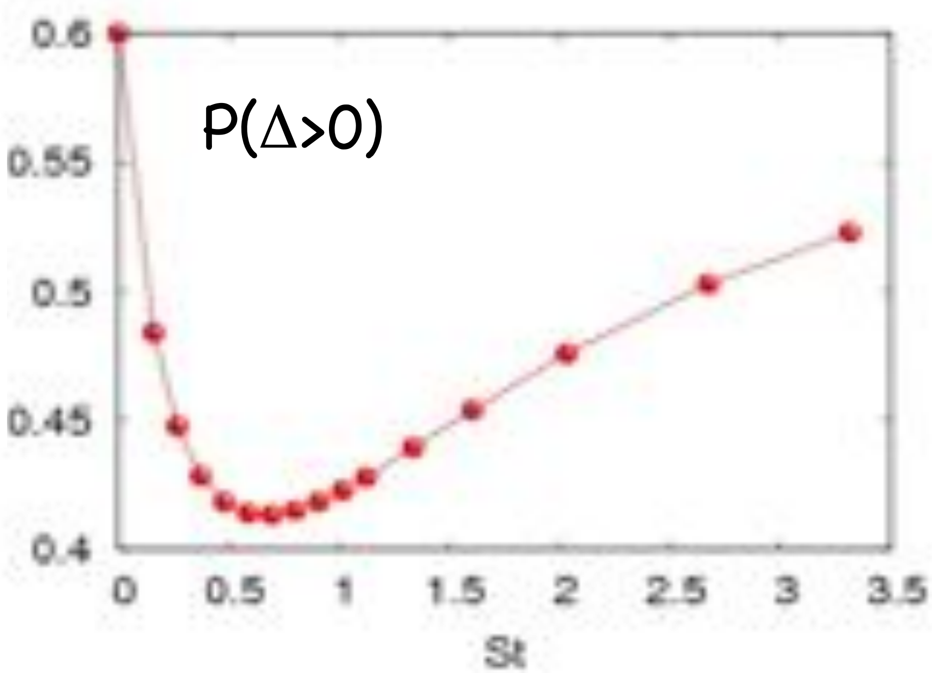
Δ<0 Δ>0

$$\Delta = \left(\frac{\det[\hat{\sigma}]}{2}\right)^2 - \left(\frac{\mathrm{Tr}[\hat{\sigma}^2]}{6}\right)^3$$

$$\hat{\sigma}_{ij} = \partial_i u_j$$

$\Delta \leq 0$ $3\ \mathcal{R}$ eigen

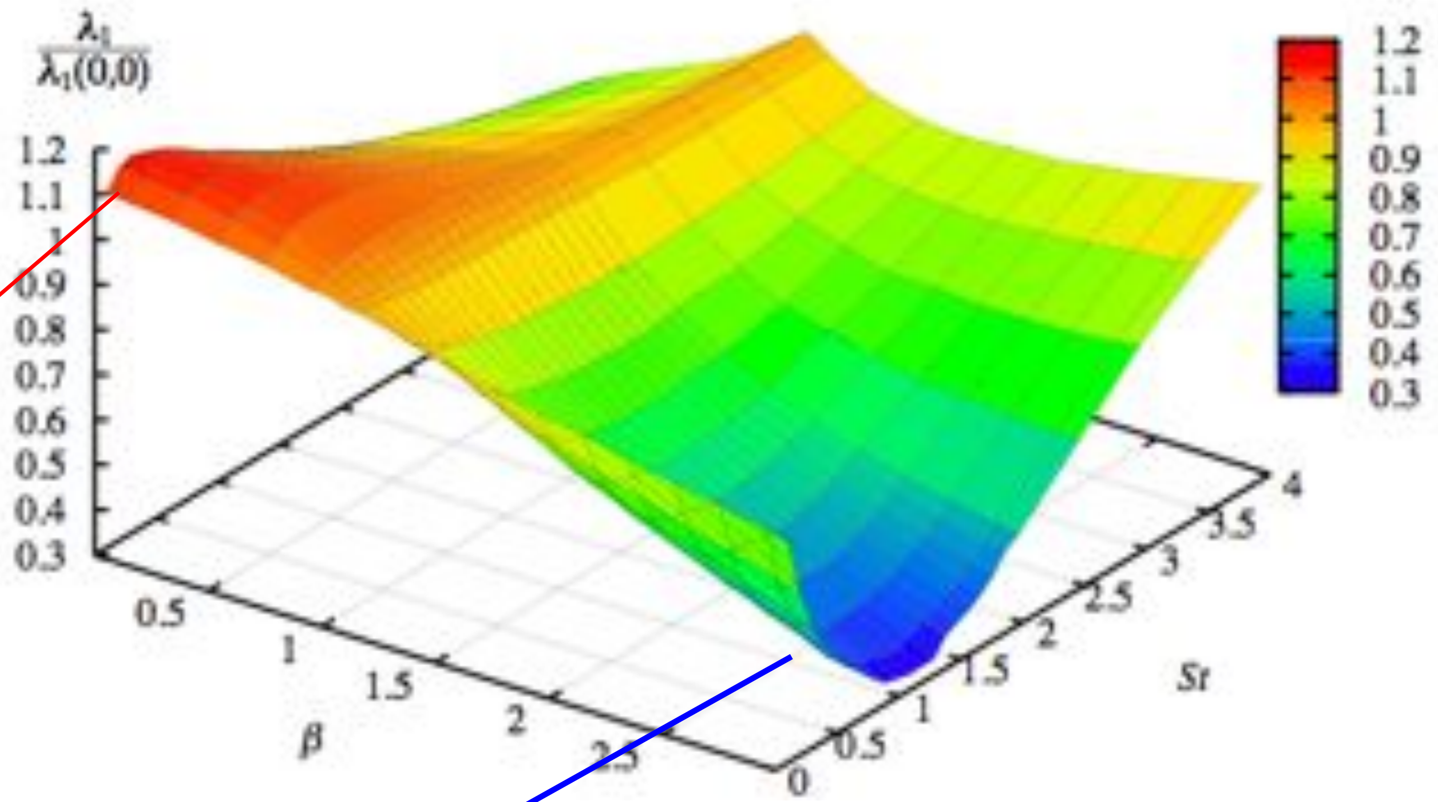
$\Delta > 0$ $1\ \mathcal{R} + 2\ \mathcal{C}$ eigen.



Bec et al (2006)

Lyapunov Exponents

Another signature of the uneven distribution of particles



Heavy

St<<1

$\lambda_1(St) > \lambda_1(St=0)$

stay longer in strain-regions

Light

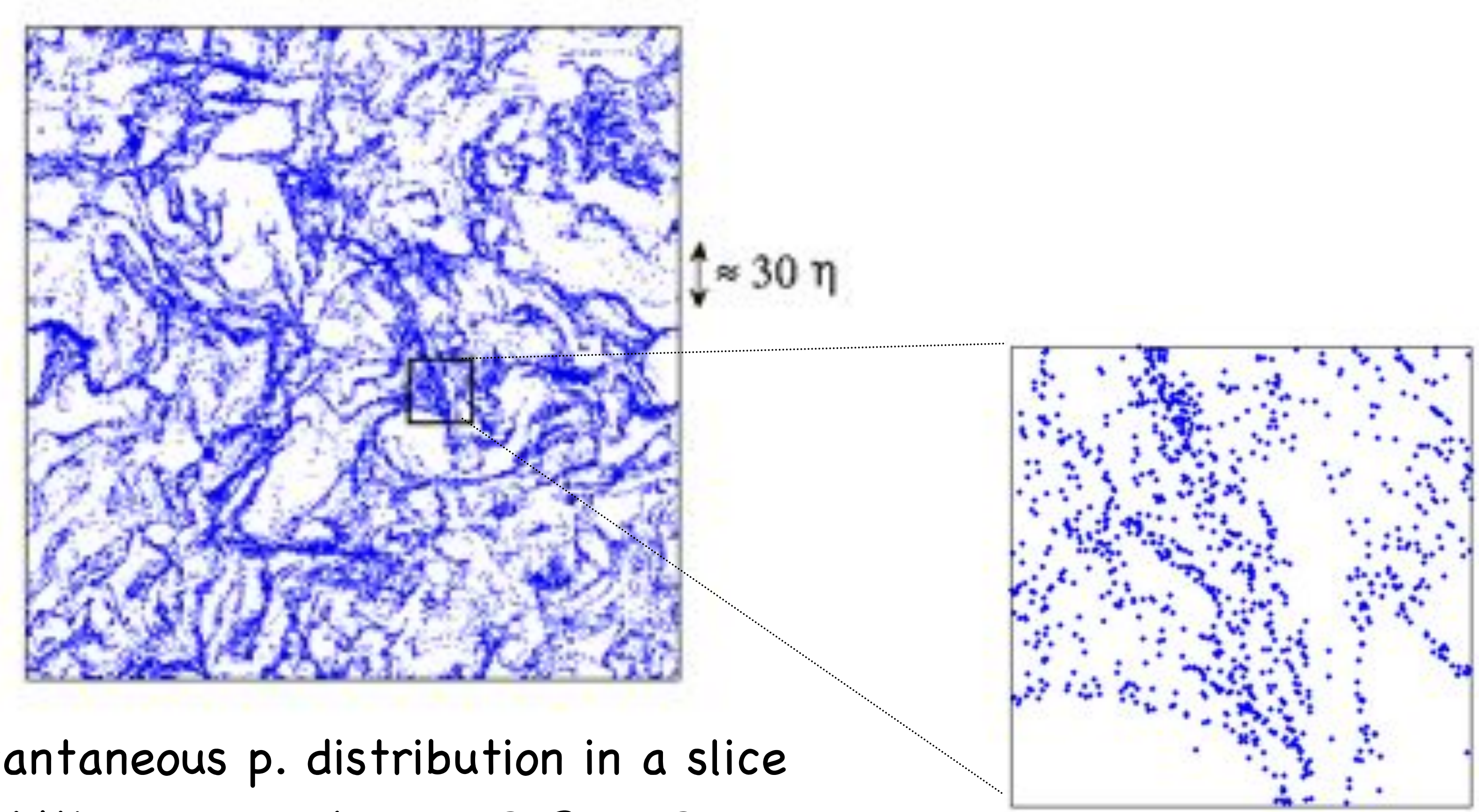
$\lambda_1(St) < \lambda_1(St=0)$

staying away from strain-regions

Calzavarini, MC, Lohse & Toschi 2008

Two kinds of clustering

Particle clustering is observed both
in the dissipative and in the inertial range

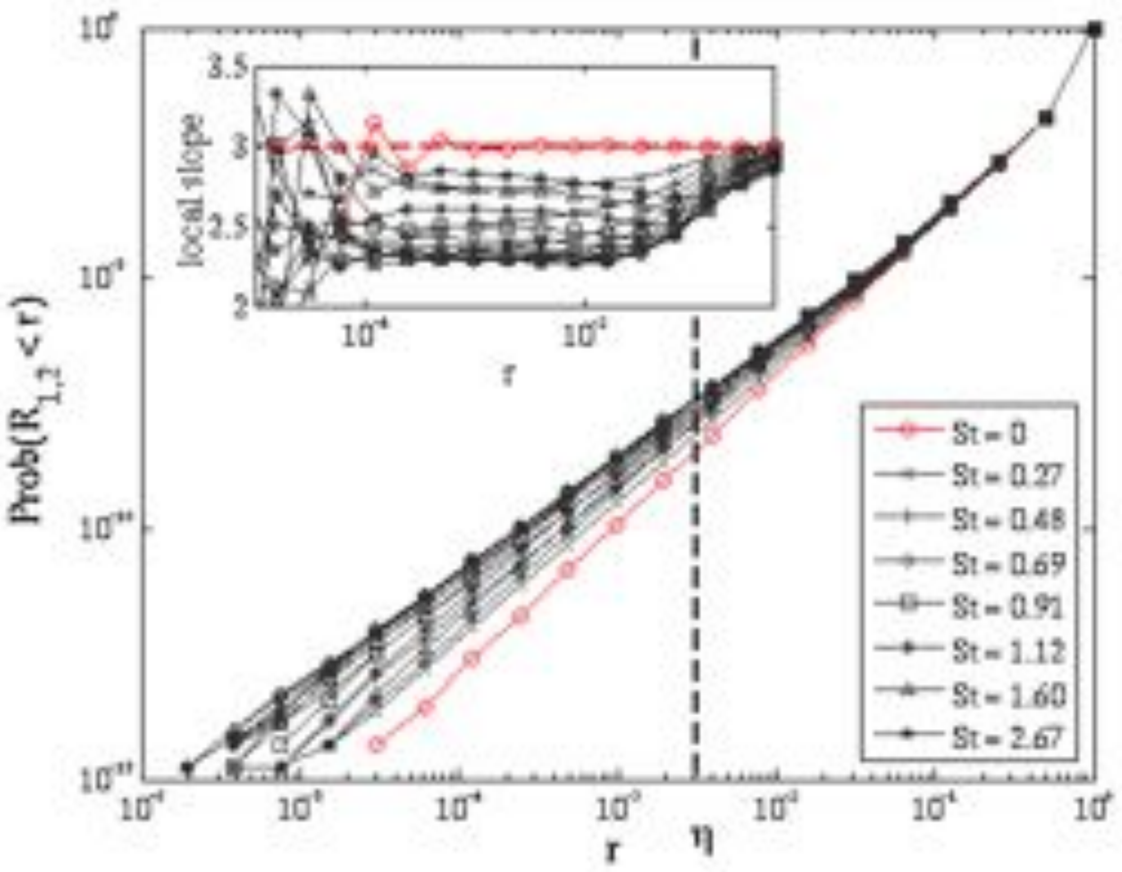


Instantaneous p. distribution in a slice
of width $\approx 2.5\eta$. St_η = 0.58 R_λ = 185

Bec, Biferale, MC, Lanotte, Musacchio & Toschi (2007)

Clustering at r<η

- Smooth flow -> fractal distribution
- Everything must be a function of St_η & Re_λ only

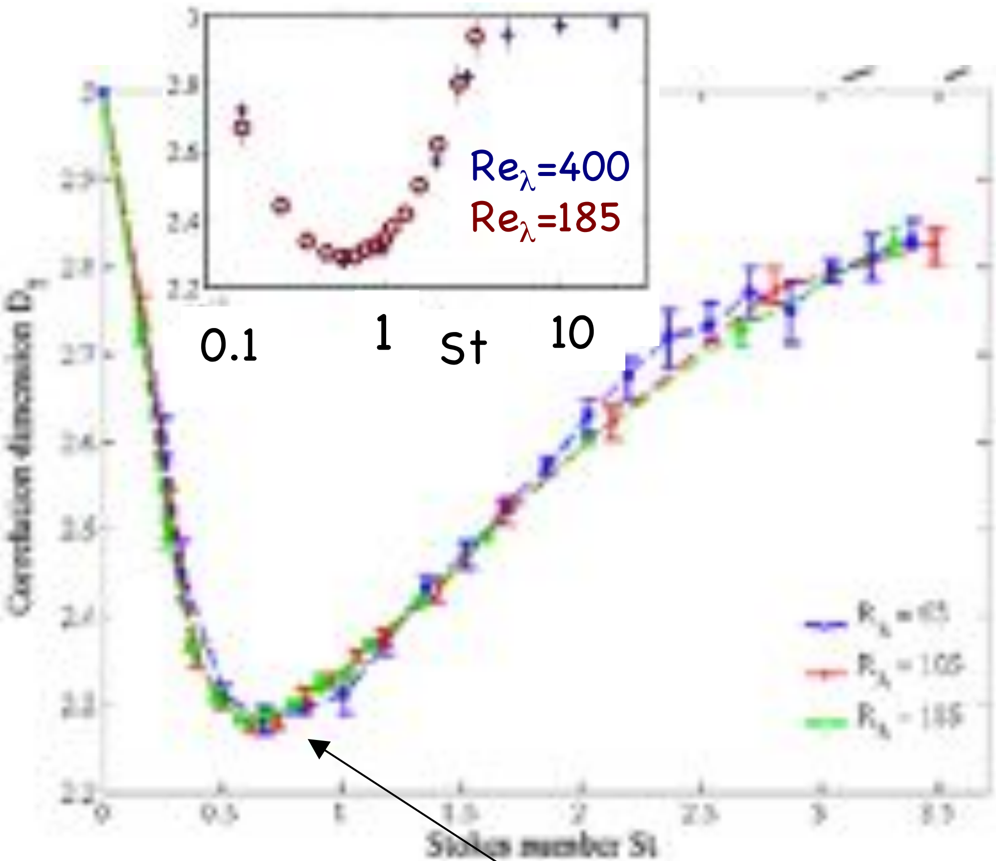


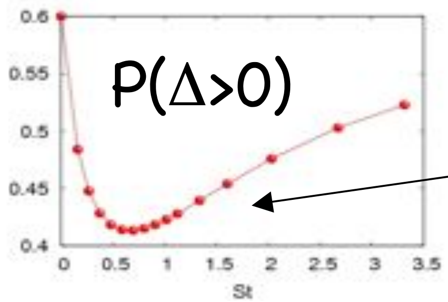
correlation dimension

$$P_2^<(r) \sim r^{\mathcal{D}_2}$$

Here β=0
Heavy particles

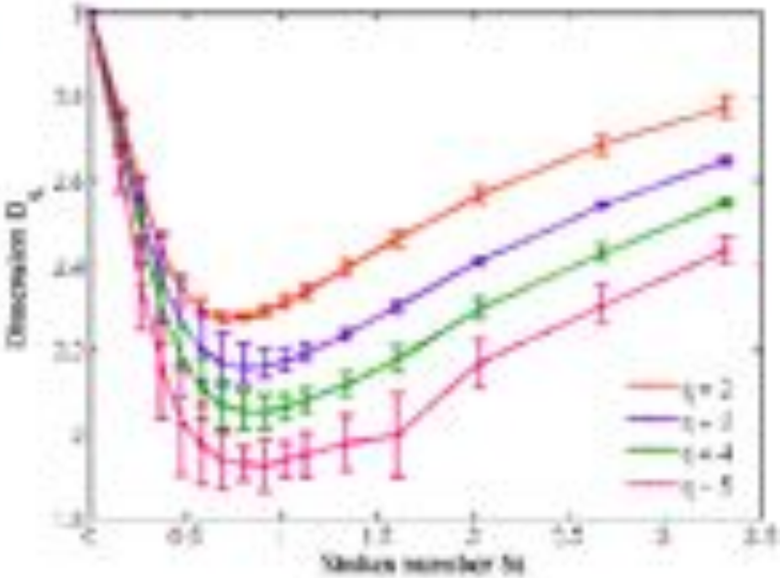
Correlation dimension





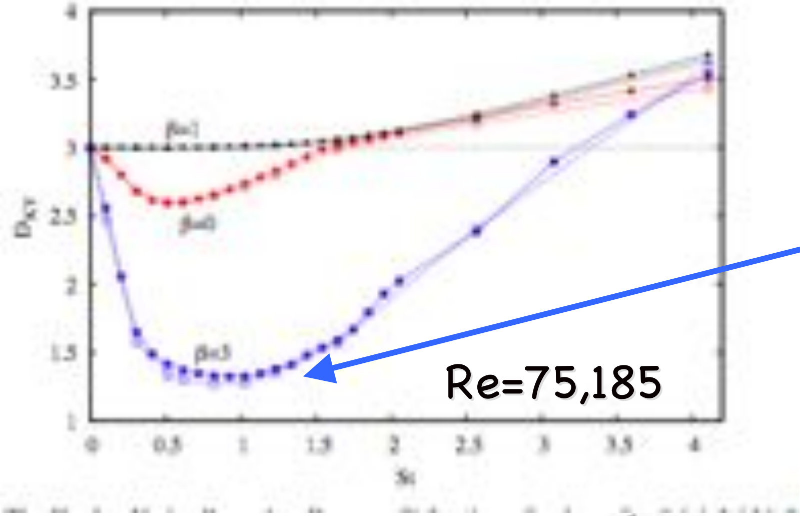
Clustering & Preferential concentration are correlated

- Maximum of clustering for $St_\eta \approx 1$
- D_2 almost independent of Re_λ,
- $D_q \neq D_2$ multifractality



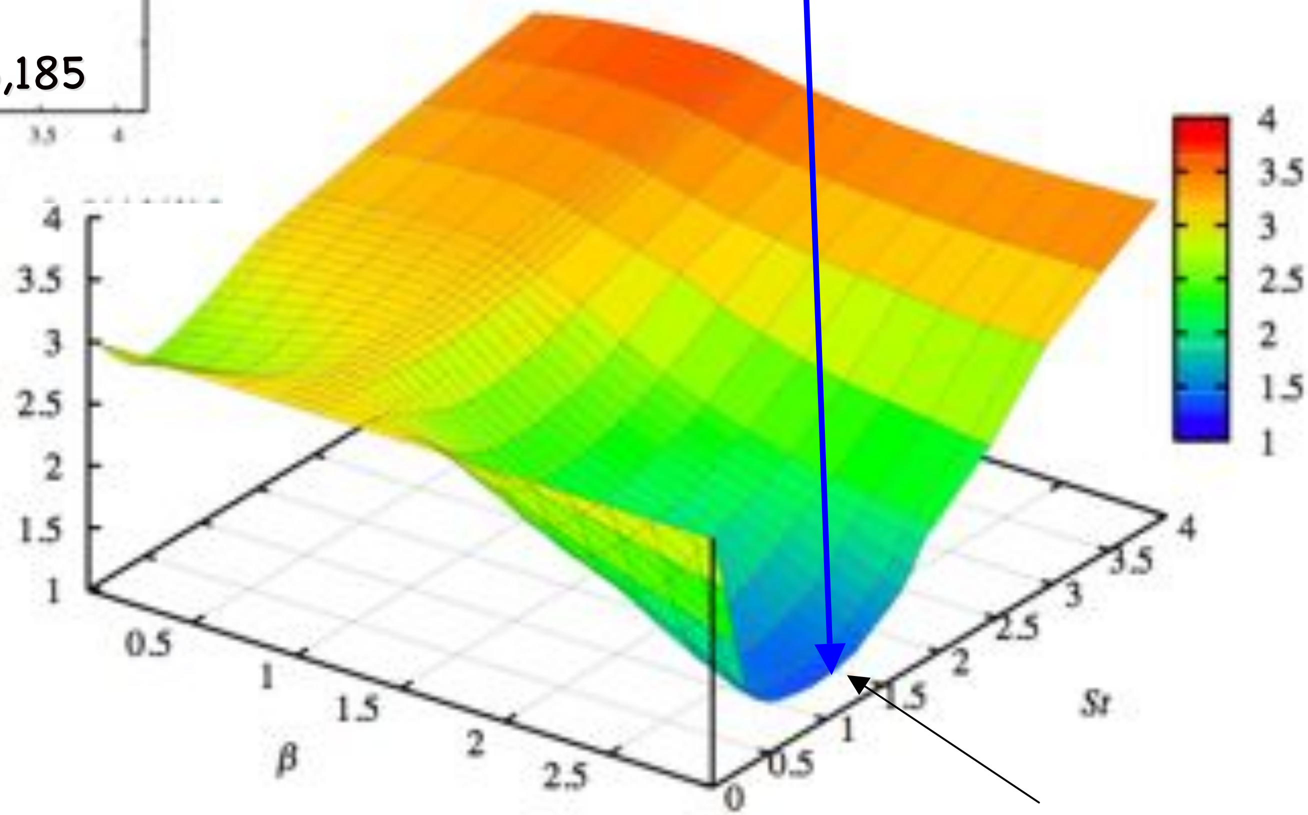
Heavy particles (β=0)

Kaplan-Yorke dimension

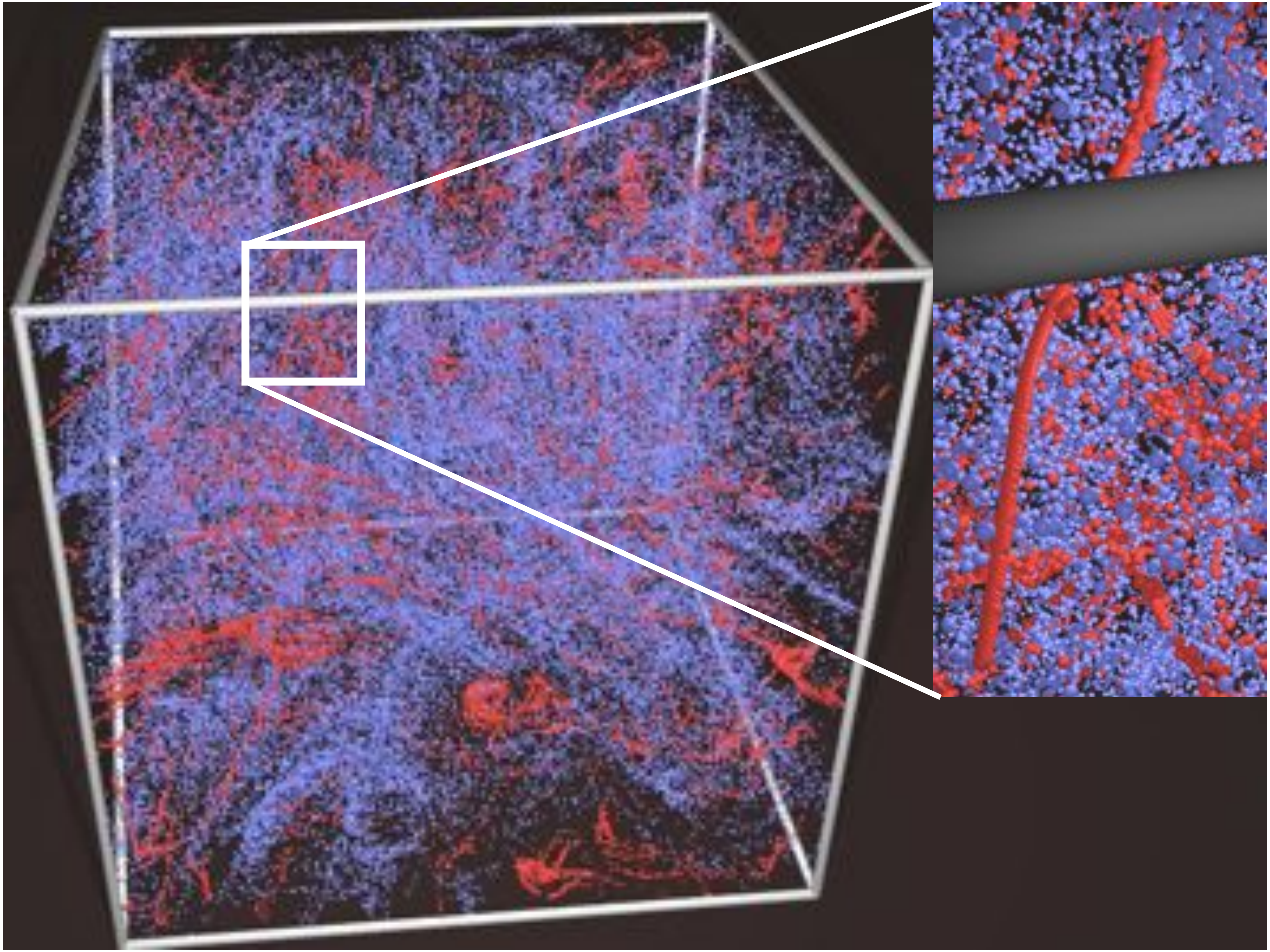


Light particles stronger clustering
$D_2 \approx 1$ signature of vortex filaments

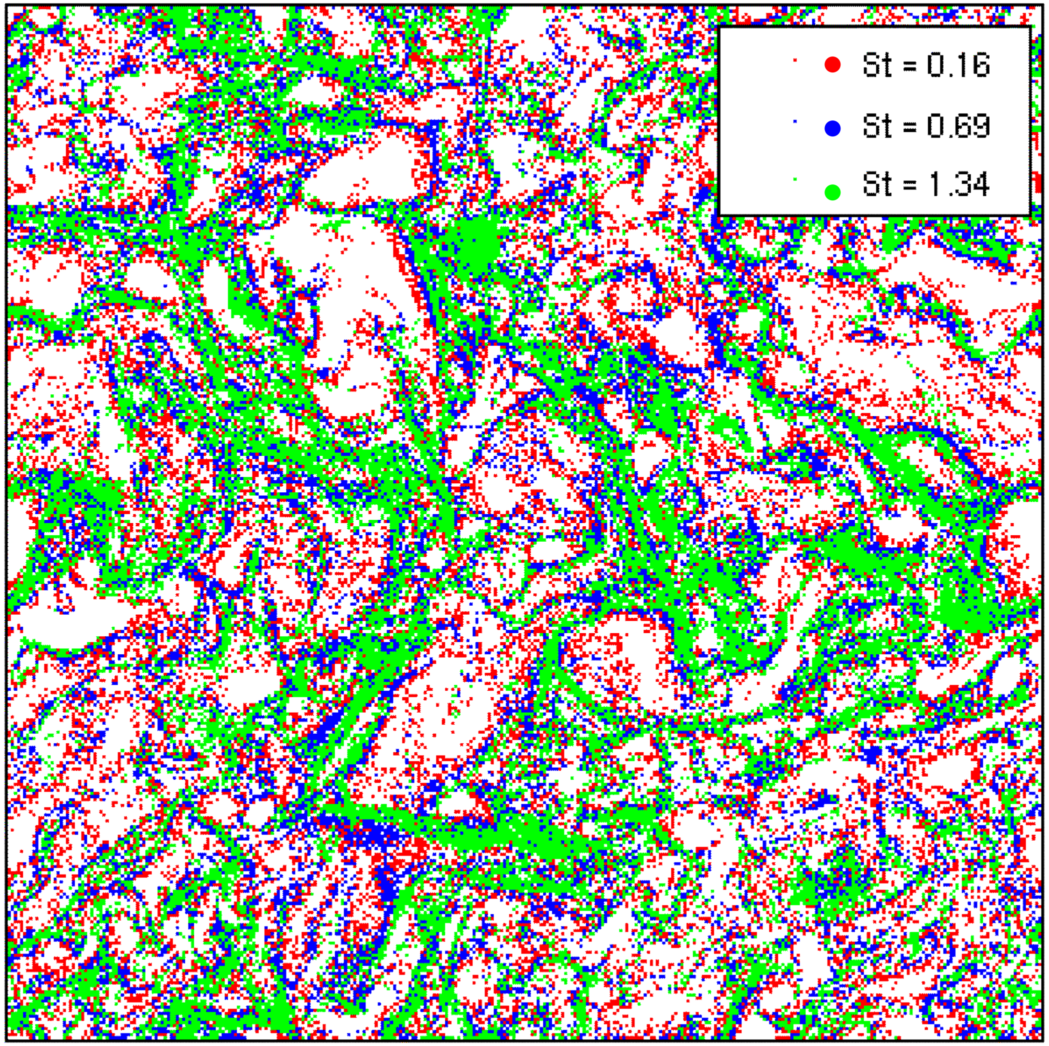
$$d_\lambda = \mathrm{K} + \frac{\sum_{i=1,\mathrm{K}}^{+} \lambda_i}{|\lambda_{\mathrm{K}+1}|}$$



Light particles: neglecting collisions might be a problem!

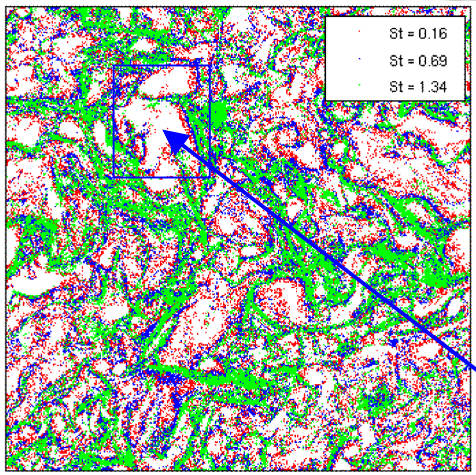


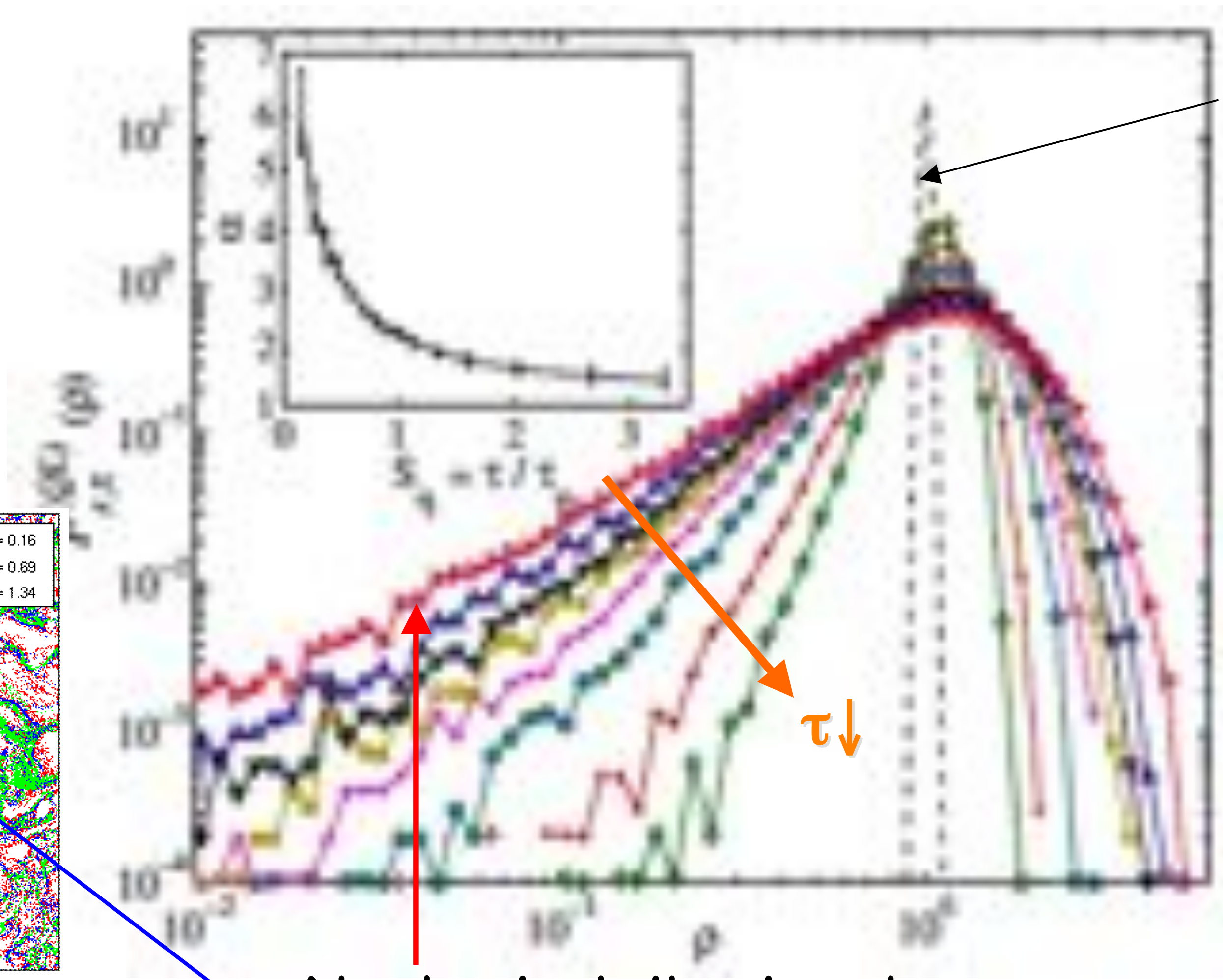
Clustering at inertial scales



- Voids & structures from η to L
- Distribution of particles over scales?
- What is the dependence on St_η? Or what is the proper parameter?

Coarse grained density





Algebraic tails signature of voids $p(\rho) \propto \rho^{\alpha(\tau,r)}$

What is the relevant time scale of inertial range clustering

For St->0 we have that

$$\boldsymbol{V} \approx \boldsymbol{u} - \tau D_t \boldsymbol{u} = u - \tau(\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u})$$

$$\boldsymbol{\nabla} \cdot V = -\tau \boldsymbol{\nabla} \cdot (\boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u}) = \boldsymbol{\tau} \nabla^2 p$$ Effective compressibility

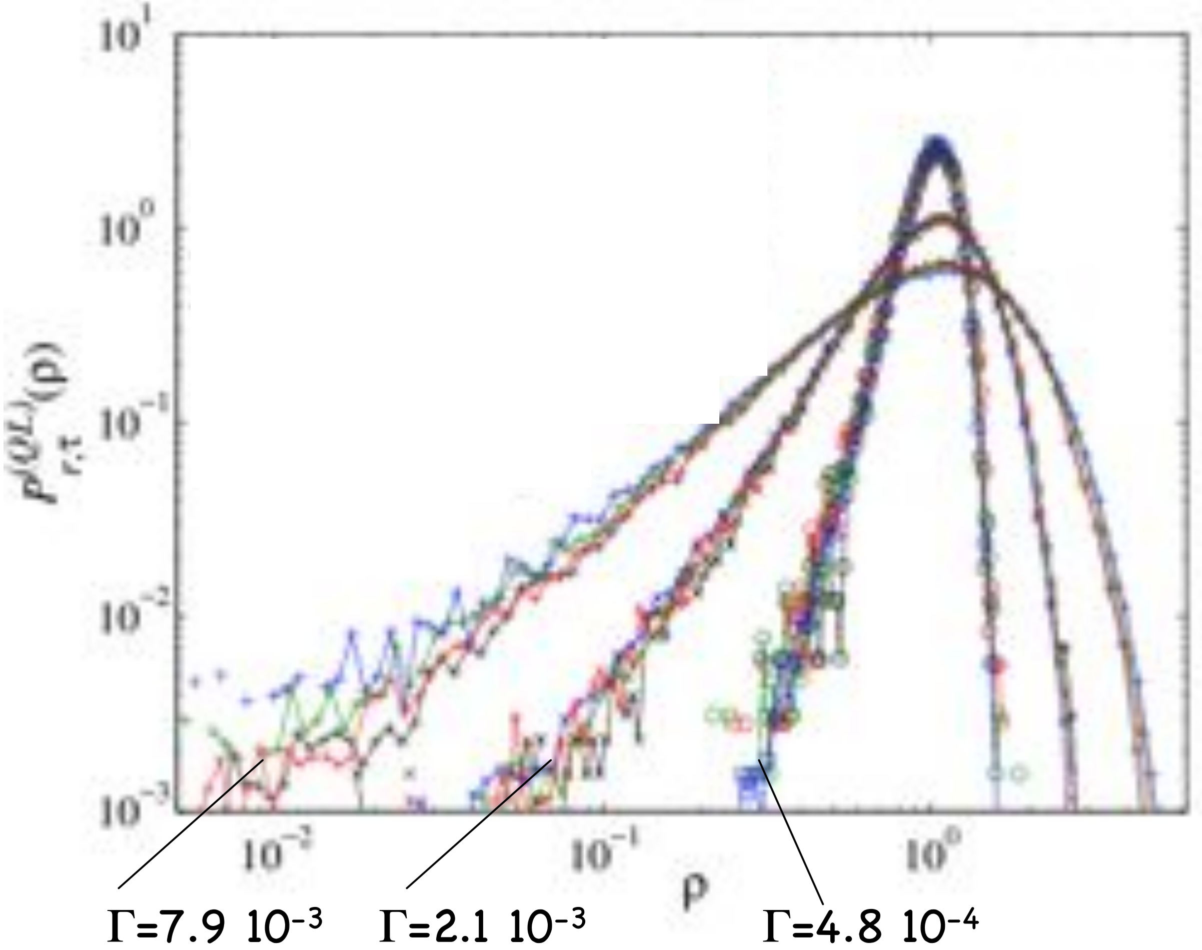
We can estimate the phase-space contraction rate for
A particle blob of size r when the Stokes time is τ

$$\frac{1}{\mathcal{T}_{r,\tau}} = \frac{1}{r^3} \int_{[0:r]^3} d^3x \, \boldsymbol{\nabla} \cdot \boldsymbol{V} \sim -\frac{\tau \delta_r a}{r} \sim \frac{\tau \delta_r \nabla p}{r}$$

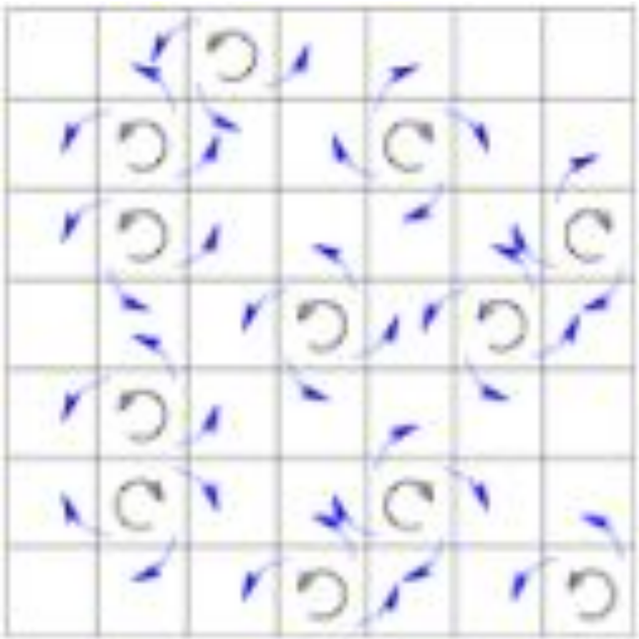
It relates to pressure

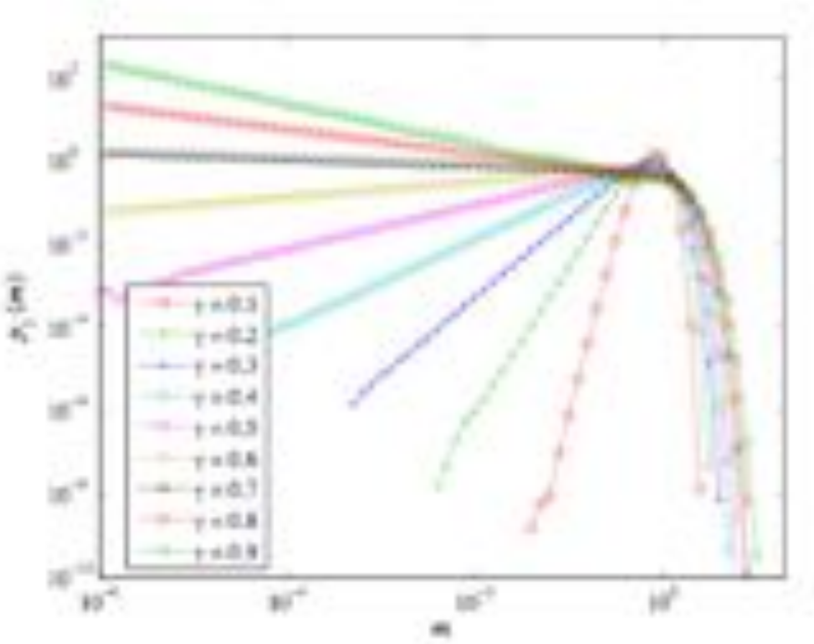
Nondimensional contraction rate

$$\Gamma = \frac{\tau_\eta}{\mathcal{T}_{r,\tau}} \sim Re^{1/4}\, S_\eta \left(\frac{r}{\eta}\right)^{-5/3} \sim Re^{-1}\, S_\eta \left(\frac{r}{L}\right)^{-5/3}$$



Inertial particles & Statistical Physics





Bec & Chétrite (2007)

Developing statistical models for mass transport
which retains phenomenological ingredients

Inertial particles & Statistical Physics

$$\dot{\boldsymbol{X}} = \boldsymbol{V}$$
$$m\dot{\boldsymbol{V}} = -\gamma \boldsymbol{V} + \boldsymbol{\eta}(t)$$

$$\langle \eta_i(t)\eta_j(t)\rangle = 2\gamma T \delta_{ij}\delta(t-t')$$

Brownian motion (Langevin)

$$\dot{\boldsymbol{X}} = \boldsymbol{V}$$
$$\dot{\boldsymbol{V}} = -\frac{\boldsymbol{V}}{St} + \frac{\boldsymbol{u}(\boldsymbol{X},t)}{St}$$

$$\langle u_i(\boldsymbol{x},t)u_j(\boldsymbol{y},t')\rangle = C_{ij}(\boldsymbol{x}-\boldsymbol{y};t-t')$$

Inertial (heavy) particles
Brownian motion in a
Disordered media with
Nontrivial spatio-temporal
Correlation (turbulence)

Simplified model: retaining only spatial correlations
mimicking turbulent ones (Kraichnan model for the velocity)

$$\langle u_i(\boldsymbol{x},t)u_j(\boldsymbol{y},t')\rangle = D_{ij}(\boldsymbol{x}-\boldsymbol{y})\delta(t-t')$$

Inertial particles & Statistical Physics

$$\dot{\boldsymbol{X}} = \boldsymbol{V}$$
$$\dot{\boldsymbol{V}} = -\frac{\boldsymbol{V}}{St} + \frac{\boldsymbol{u}(\boldsymbol{X},t)}{St}$$

$\Longleftrightarrow$

$$\ddot{\boldsymbol{X}} + \frac{1}{St}\dot{\boldsymbol{X}} = \frac{\boldsymbol{u}(\boldsymbol{X},t)}{St}$$

e.g. separation between 2 particles $R=X_1-X_2$

$$\ddot{\boldsymbol{R}} + \frac{1}{St}\dot{\boldsymbol{R}} = \frac{\delta_R \boldsymbol{u}}{St}$$

For smooth velocities

$$\delta_R \boldsymbol{u} = \hat{\sigma}(t)\boldsymbol{R}$$
$$\hat{\sigma}_{ij} = \partial_j u_i$$

$$\ddot{\boldsymbol{R}} + \frac{1}{St}\dot{\boldsymbol{R}} = \frac{1}{St}\hat{\sigma}\boldsymbol{R}$$

d=1

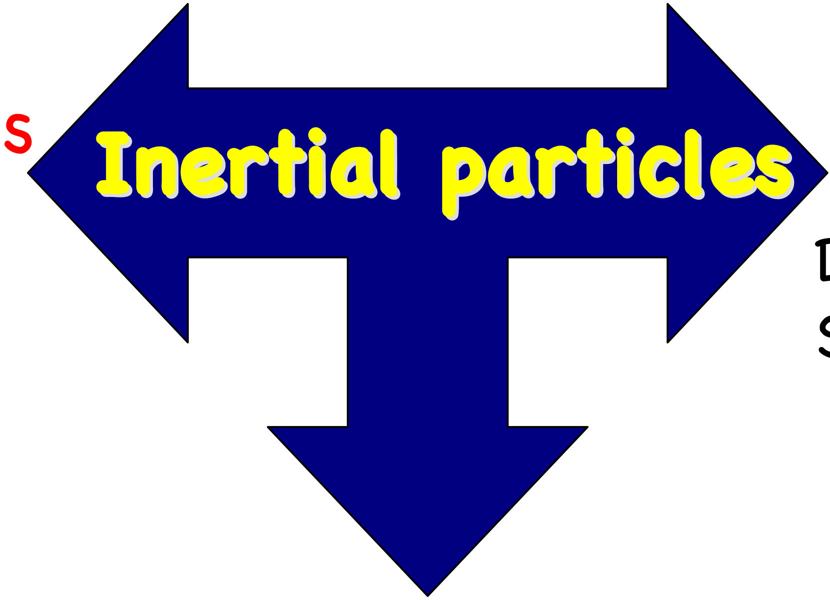
$$R = \psi e^{-t/(2St)}$$

$$-\psi'' + \frac{1}{St}\sigma\psi = -\frac{1}{4St^2}\psi$$

Equivalent to Anderson localization:
time->space Localization length->Lyapunov exponent

Derevyanko, Falkovich, Turitsyn, Turitsyn JoT (2007) (1d)
Horvai arXiv:nlin/0511023v1 [nlin.CD] (2d)

Fluid dynamics



Statistical Physics

Disordered systems

Stochastic processes

Dynamical Systems

Chaos/Fractals

Thanks

- J. Bec
- L. Biferale
- G. Boffetta
- E. Calzavarini
- A. Celani
- R. Hillerbrandt
- D. Lohse
- S. Musacchio
- F. Toschi
- K. Turitsyn

Reading list

Inertial Particles in Turbulence

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