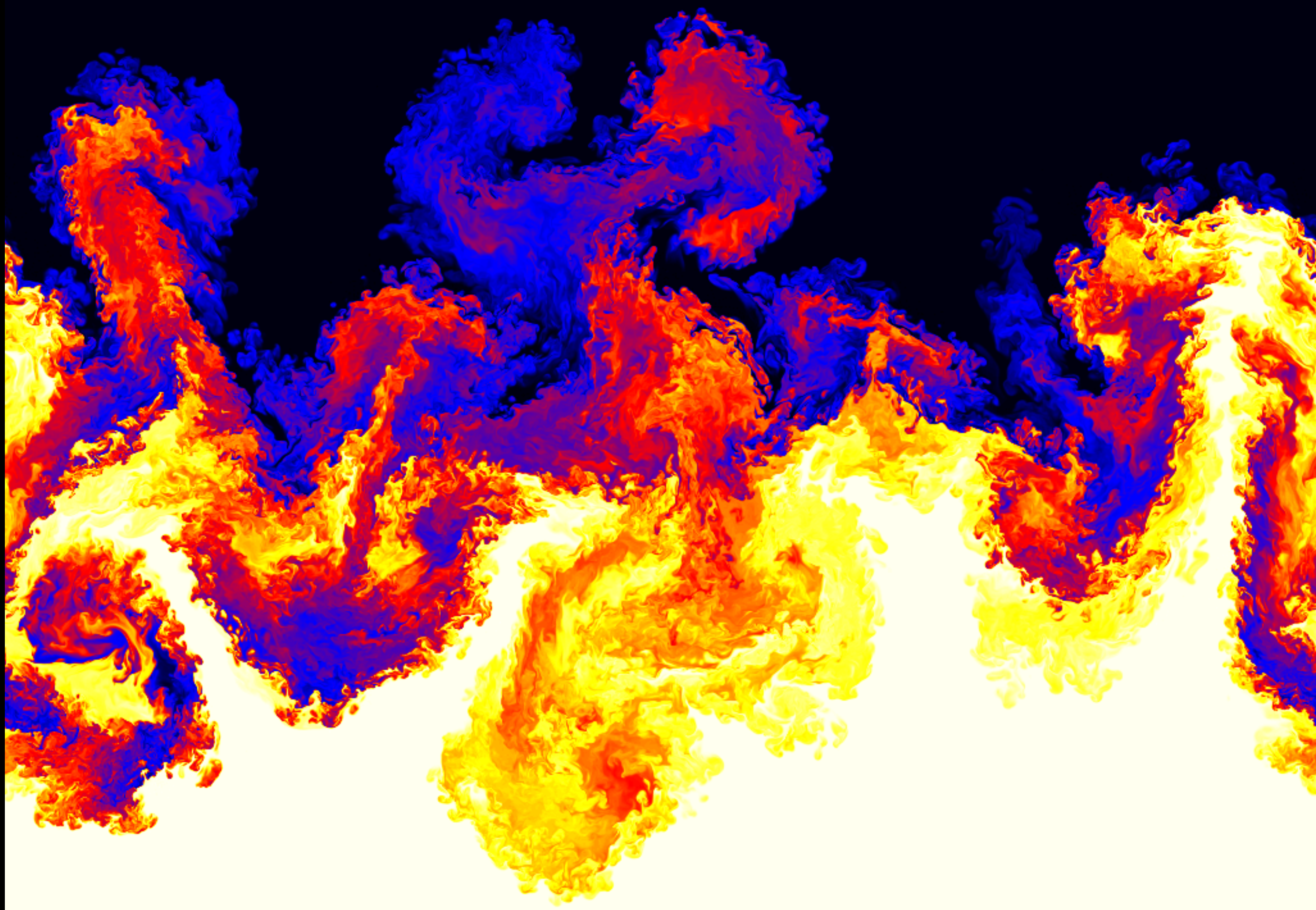
Turbulent transport of active scalar: Rayleigh-Taylor turbulence



Guido Boffetta (Torino)
Filippo De Lillo (Torino) Andrea Mazzino (Genova)
Stefano Musacchio (Nice) Lara Vozella (Genova)

Rayleigh-Taylor turbulence: a toy model for thermal convection
(no boundaries, no large scale circulation, ...)

Outline

* statistics of small scales: passive temperature
  Rayleigh-Tayor turbulence as adiabatically evolving NS turbulence + passive scalar.
  Phenomenology: Kolmogorov scaling + intermittency

* time evolution of large scales: active temperature
  Nonlinear diffusion model for large scale mixing
  Global heat transport realizes the “ultimate state” of thermal convection

* heat transfer enhancement in viscoelastic thermal convection

A (The) problem in thermal convection

"Given the (dimensionless) temperature difference and the fluid characteristic, find the turbulent flow properties"

Within the Oberbeck-Boussinesq approximation, for which transport coefficients do not depend on T and the (incompressible) equation of motion are

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

$$\rho(T) = \rho(T_0)\left[1 - \beta(T - T_0)\right]$$

"The problem" simplifies:

given

$$Ra = \frac{\beta g \theta_0 L^3}{\kappa\nu}$$

$$Pr = \frac{\nu}{\kappa}$$

find

$$Re = \frac{v_{rms} L}{\nu}$$

$$Nu = \frac{< v_z T > L}{\kappa\theta_0}$$

Mixing efficiency: turbulent heat transport

In terms of dimensionless numbers, the parameter space of turbulent convection is given by Ra and Pr, while the turbulent state is quantified by Re and Nu.

$$Ra = \frac{\beta g \theta_0 L^3}{\kappa \nu} \qquad Pr = \frac{\nu}{\kappa}$$

$$Re = \frac{v_{rms} L}{\nu} \qquad Nu = \frac{< v_z T > L}{\kappa \theta_0}$$

The problem is to find the relations

Re(Ra,Pr) and Nu(Ra,Pr)

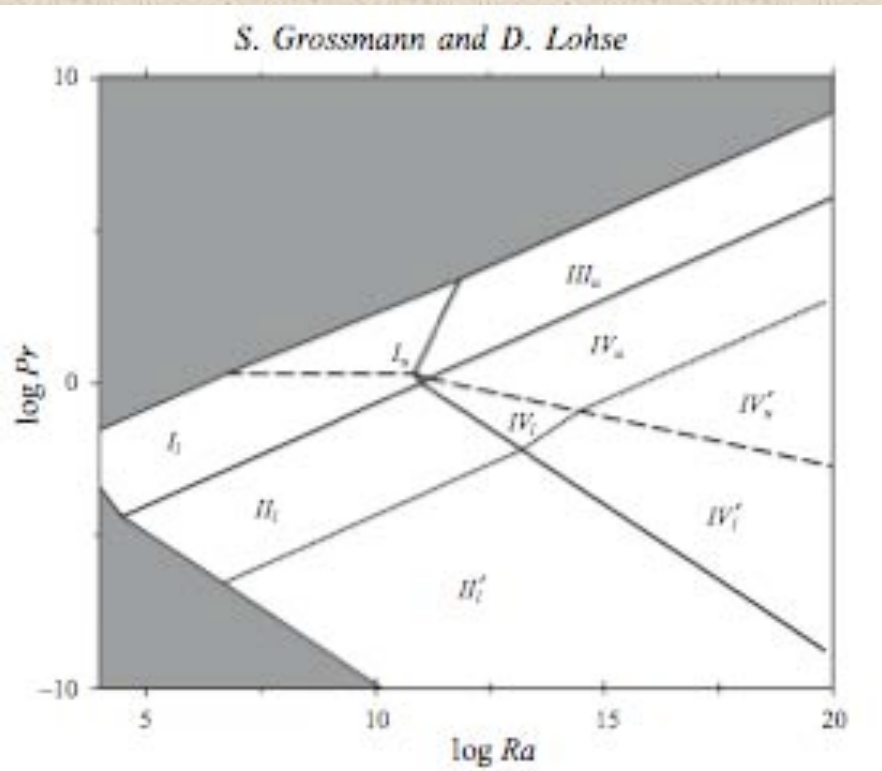
Turbulent heat transport is expected to reach an asymptotic regime at large value of thermal forcing with simple power laws $Re \approx Ra^{\alpha} Pr^{\sigma}$ $\qquad Nu \approx Ra^{\gamma} Pr^{\delta}$

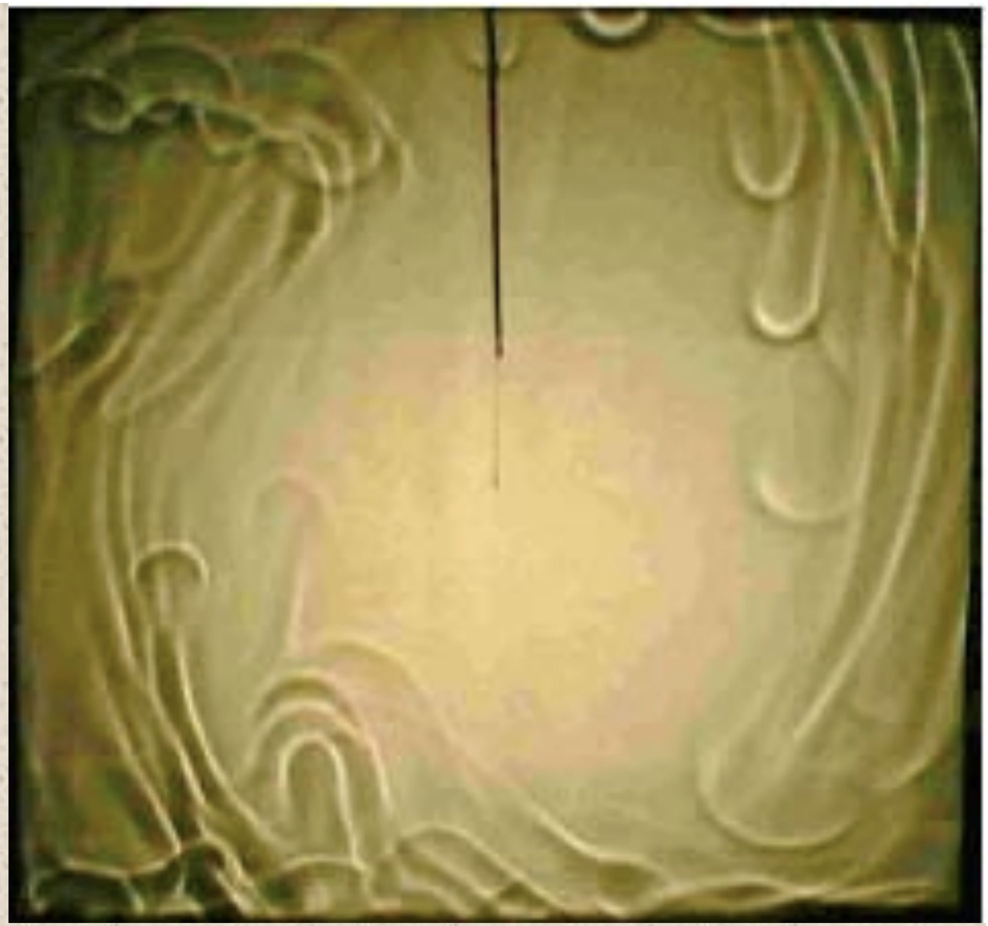
For very large Ra, Kraichnan predicted [1962] an ultimate regime in which

$$Nu \approx Ra^{1/2}$$

This ultimate state of thermal convection is expected to appear when the contribution of boundary layers becomes irrelevant

In Rayleigh-Benard convection
the situation is complicated by
the presence of boundary layers
and large scale circulations
[Grossmann and Lohse, JFM **407** (2000)]





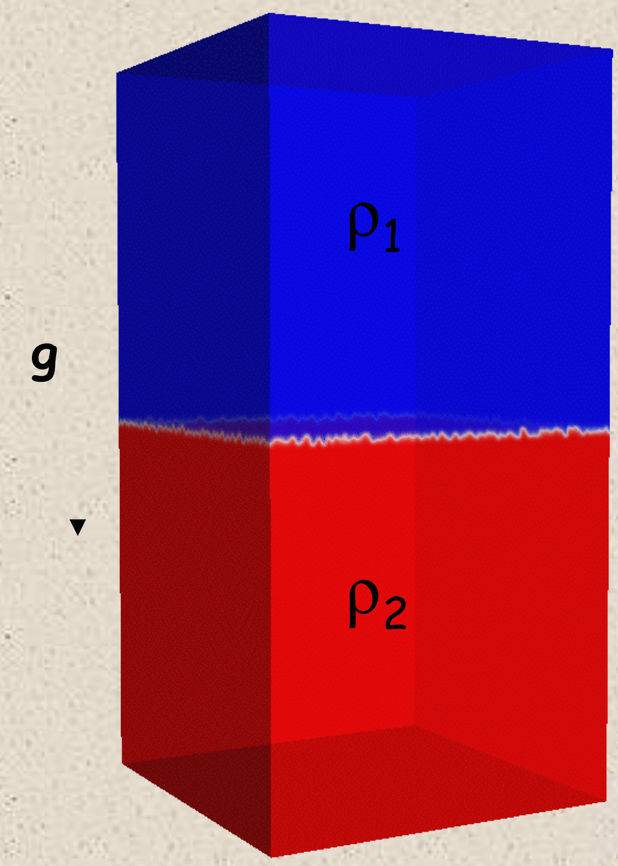
Ra=6.8×10^{8}

no universal behavior

Rayleigh-Taylor instability

Instability on the interface of two fluids of different densities with relative acceleration.

Rayleigh (1883): unstable stratification in gravitational field
Taylor (1950): generalization to all acceleration mechanisms



$\rho_1 > \rho_2$

Linear stability analysis for incompressible, inviscid, miscible fluids predicts a growth rate

$$\lambda = \sqrt{Agk}$$

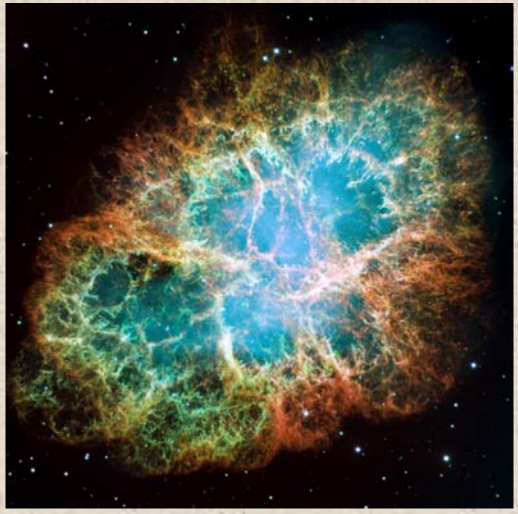
High wavenumbers are stabilized by viscosity, surface tension,...

Atwood number

$$A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

Examples of RT instability

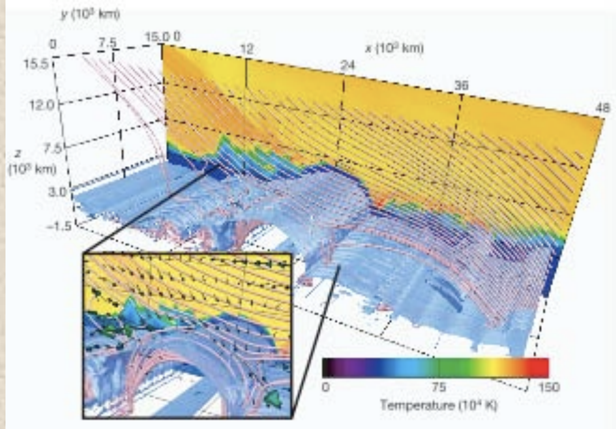
Many applications in natural phenomena and technological problems:



supernova explosion
acceleration mechanism for thermonuclear flame front

atmospheric physics
mammatus clouds

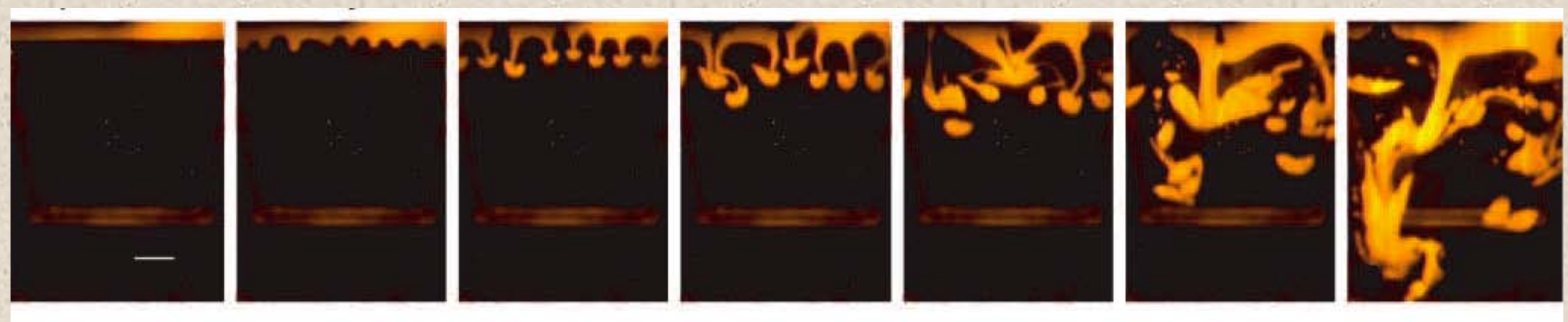
solar corona heating



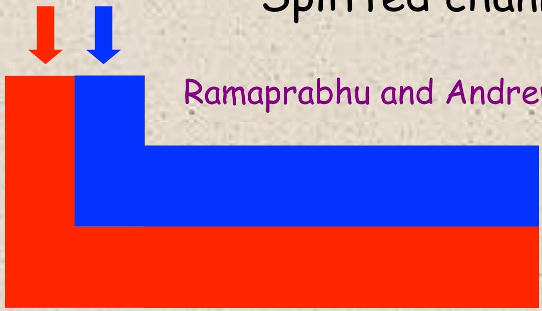
laser ignition of DT pellets

Rayleigh-Taylor experiments

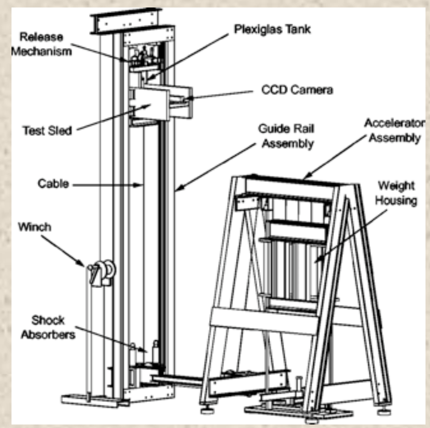
Heavy paramagnetic fluid levitated over a light diamagnetic solution Kadau et al, PNAS **104** (2007)



Splitted channel

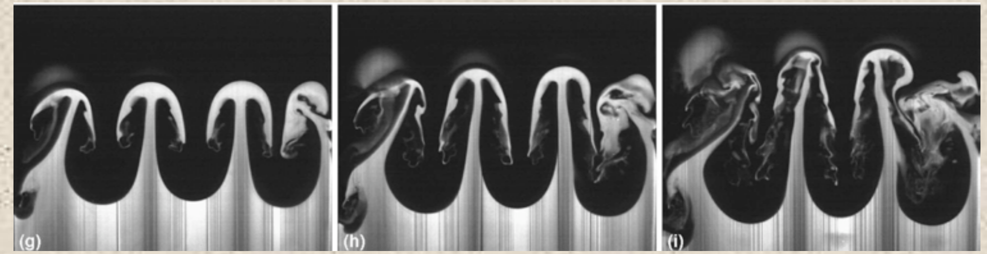


Ramaprabhu and Andrews, JFM **502** (2004)



Accelerated sled

Waddell, Niederhaus, Jacobs, POF **13** (2001)



Difficulties in preparing initial condition

Equation of motion and setup

Single fluid at two temperatures (densities)

Temperature jump: $\theta_0 = T_2 - T_1$

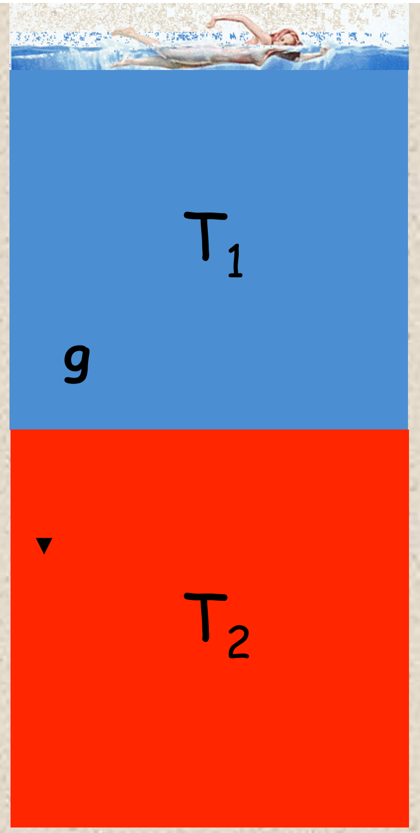
Atwood: $A \equiv \dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \simeq \dfrac{1}{2}\beta\theta_0$ (β: thermal expansion coef.)

For small A the Boussinesq approximation for an incompressible fluid holds:

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} - \beta \mathbf{g} T \\ \partial_t T + \mathbf{u} \cdot \nabla T = \kappa \Delta T \end{cases}$$

Time dependent turbulence with initial condition:

$$\begin{cases} \mathbf{u}(\mathbf{x}, 0) = 0 \\ T(\mathbf{x}, 0) = -(1/2)\theta_0 \, \mathrm{sgn}(z) \end{cases}$$

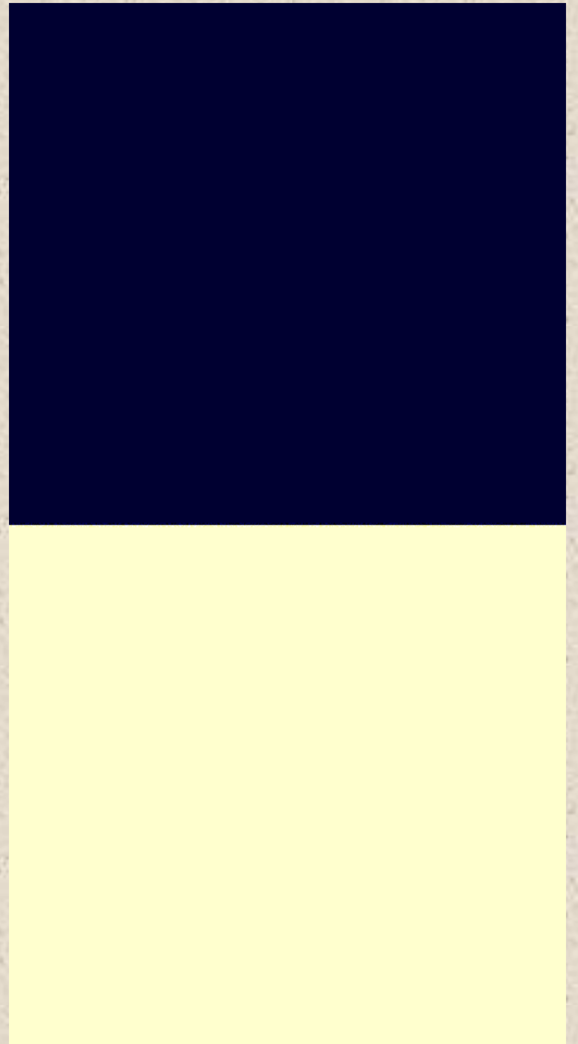


Phenomenology of RT turbulence

Energy balance:
turbulent kinetic energy $E=(1/2)\langle u^2\rangle$ produced from potential energy $P=-\beta g\langle zT\rangle$

$$\varepsilon = \nu\left\langle (\nabla u)^2 \right\rangle$$

$$\frac{dE}{dt} = \beta g \left\langle wT \right\rangle - \varepsilon = -\frac{dP}{dt} - \varepsilon$$



Dimensional balance: $\frac{du^2_{rms}}{dt} \simeq \beta g \theta_0 u_{rms}$ therefore

Large scale velocity fluctuations grow as $u_{rms}(t) \approx Agt$

Turbulent mizing layer of width h(t) grows as $h(t) \approx Agt^2$

Kinetic energy in pumped in the system at a rate $\varepsilon \simeq \frac{u^2}{\tau} \simeq \frac{u^3}{h} \simeq (Ag)^2 t$

-> time evolving turbulence

Fermi's model (Fermi and von Neumann, '50)

UNITED STATES ATOMIC ENERGY COMMISSION

TAYLOR INSTABILITY OF INCOMPRESSIBLE LIQUIDS

By
Enrico Fermi
John von Neumann

Potential U and kinetic K energy variation for generating two rectangular plumes

$$U = \frac{\rho_2 - \rho_1}{2} L^2 g a^2 \qquad K = \frac{\rho_2 + \rho_1}{2} L^2 \dot{a}^2 a$$

Lagrange equations $$\frac{d}{dt}\frac{\partial K}{\partial \dot{a}} - \frac{\partial K}{\partial a} = -r\frac{\partial U}{\partial a}$$

(r=fraction of potential energy converted into large scale energy)

gives $$\ddot{a}a + \frac{1}{2}\dot{a}^2 = \frac{1}{4} rAga$$

and therefore, for the width of the mixing layer h(t)=2a(t)

$$h(t) = h_0 + \sqrt{(r/2)Agh_0 t + (r/8)Agt^2}$$

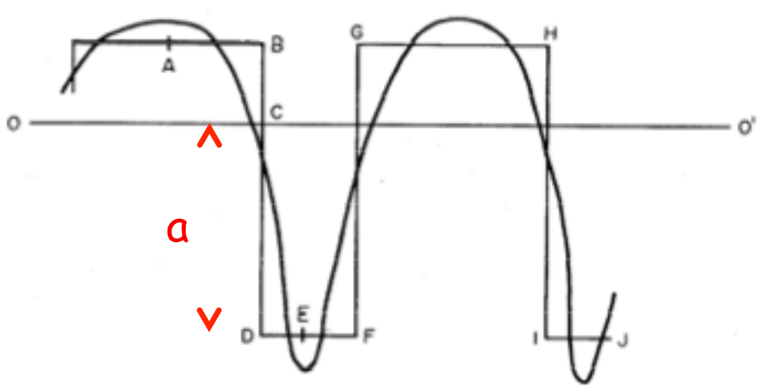


Fig. 1 square plume

Small scale theory of RT turbulence

M. Chertkov, PRL **91** (2003)

Buoyancy is negligible at small scales

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

$$\beta g \delta_r T \ll \frac{\delta_r u^2}{r}$$ (small Richardson number)

temperature fluctuations passively transported by a scale-independent energy flux

$$\varepsilon(t) \approx (Ag)^2 t$$

small scale fluctuations follow Kolmogorov-Obukhov scaling

$$\delta_r u(t) \simeq u_L(t)\left(\frac{r}{h(t)}\right)^{1/3} \simeq (\beta g\theta_0)^{2/3} t^{1/3} r^{1/3}$$

$$\delta_r T(t) \simeq \theta_0 \left(\frac{r}{h(t)}\right)^{1/3} \simeq \frac{\theta_0}{(\beta g \theta_0)^{1/3}} t^{-2/3} r^{1/3}$$

consistency:

$$Ri = \frac{\beta g \delta_r T(t)}{\delta_r u^2(t)/r} \simeq \left(\frac{r}{L(t)}\right)^{2/3} \to 0$$

Inconsistent in 2D where the energy flows to large scale (buoyancy dominated)

RT turbulence in 2D

M. Chertkov, PRL **91** (2003)

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

Buoyancy balances inertia at all scales

$$\beta g \delta_r T \approx \frac{\delta_r u^2}{r} \qquad \text{(Ri=O(1))}$$

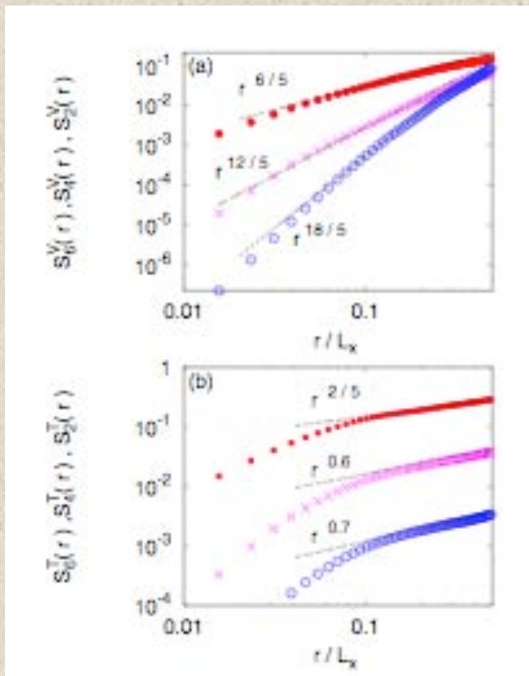
temperature fluctuations cascade to small scales at a rate

$$\varepsilon_T(t) \simeq \frac{\delta_r u \delta_r T^2}{r} \simeq \frac{\delta_r u^5}{r^3(\beta g)^2} \simeq \frac{u_L^5}{h^3(\beta g)^2}$$

small scale fluctuations follow Bolgiano scaling

$$\delta_r u(t) \simeq u_L(t)\left(\frac{r}{h(t)}\right)^{3/5} \simeq (\beta g \theta_0)^{2/5} t^{-1/5} r^{3/5}$$

$$\delta_r T(t) \simeq \theta_0 \left(\frac{r}{h(t)}\right)^{1/5} \simeq \frac{\theta_0}{(\beta g \theta_0)^{1/5}} t^{-2/5} r^{1/5}$$



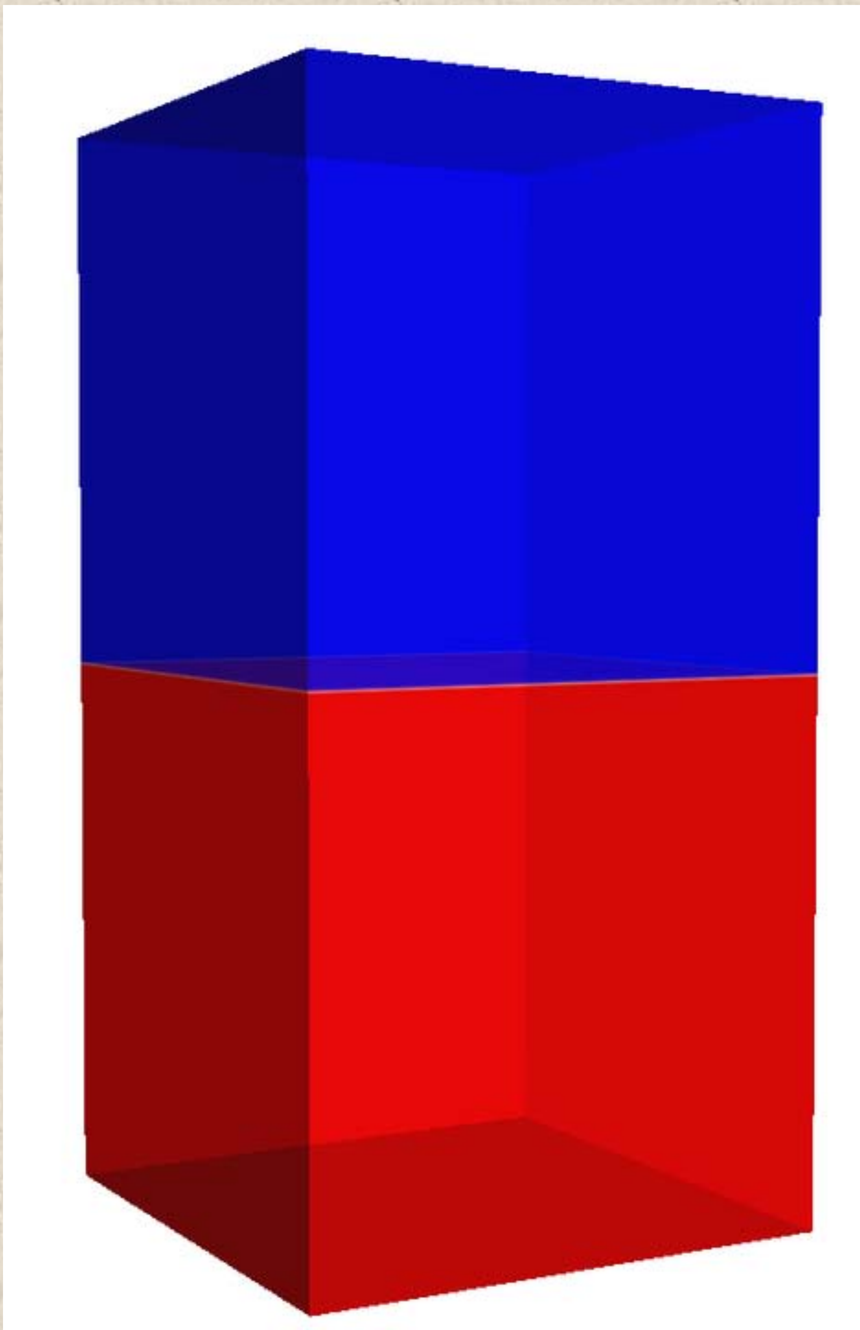
A.Celani, A.Mazzino, L.Vozella, PRL **96** (2006)

Numerical simulations

Numerical integration of Boussinesq equation

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

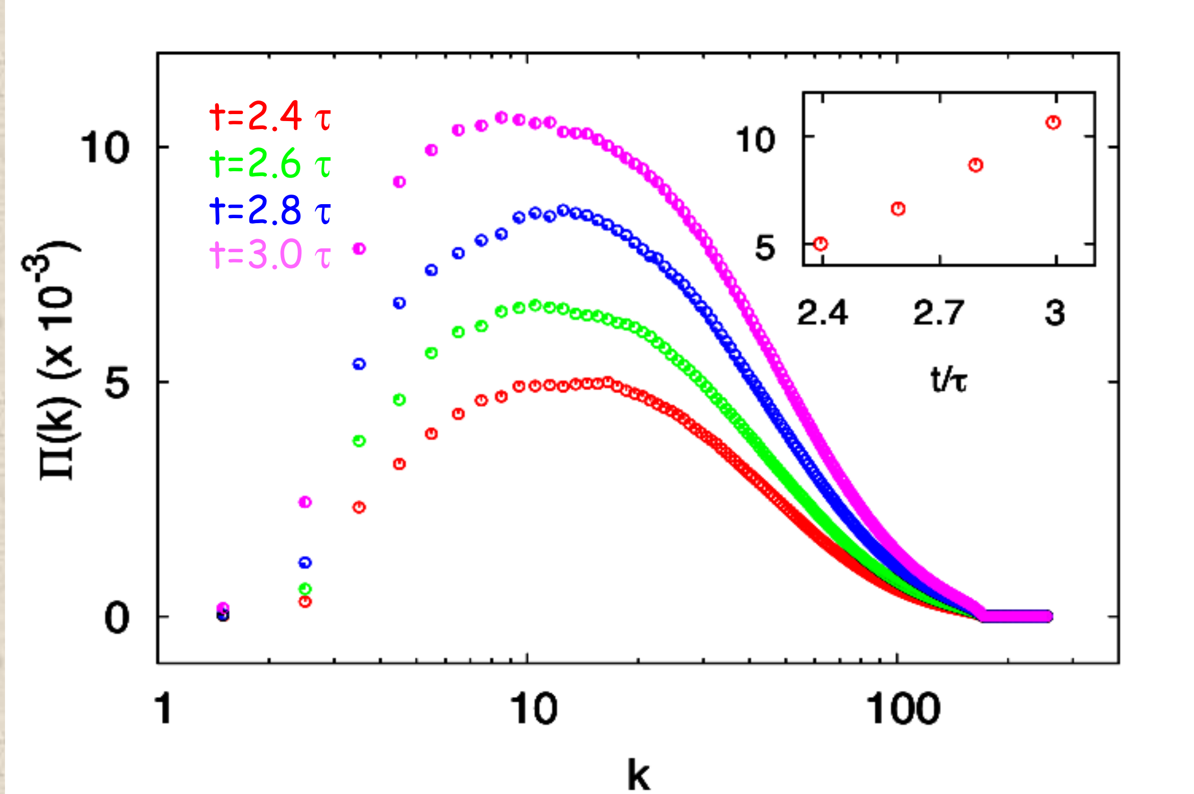
	N_x	N_y	N_z	$\nu=\kappa$
A	256	256	1024	6×10^{-4}
B	512	512	2048	3×10^{-4}
C	1024	1024	2048	1×10^{-4}



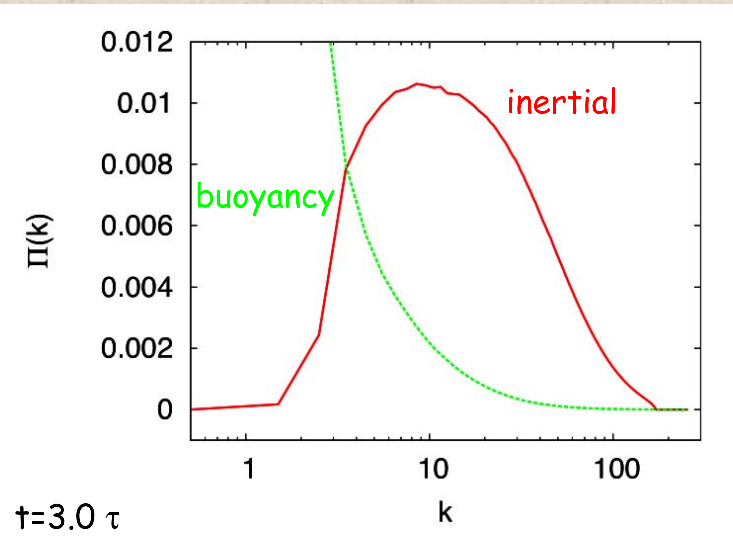
Energy flux

Inertial range of scale-independent kinetic energy flux $\Pi(k,t)$

$$\varepsilon = \Pi(k,t) \approx \frac{u_L^3}{h} \simeq t$$



RT as an adiabatically evolving turbulence (small scales adapt to large scale variations)



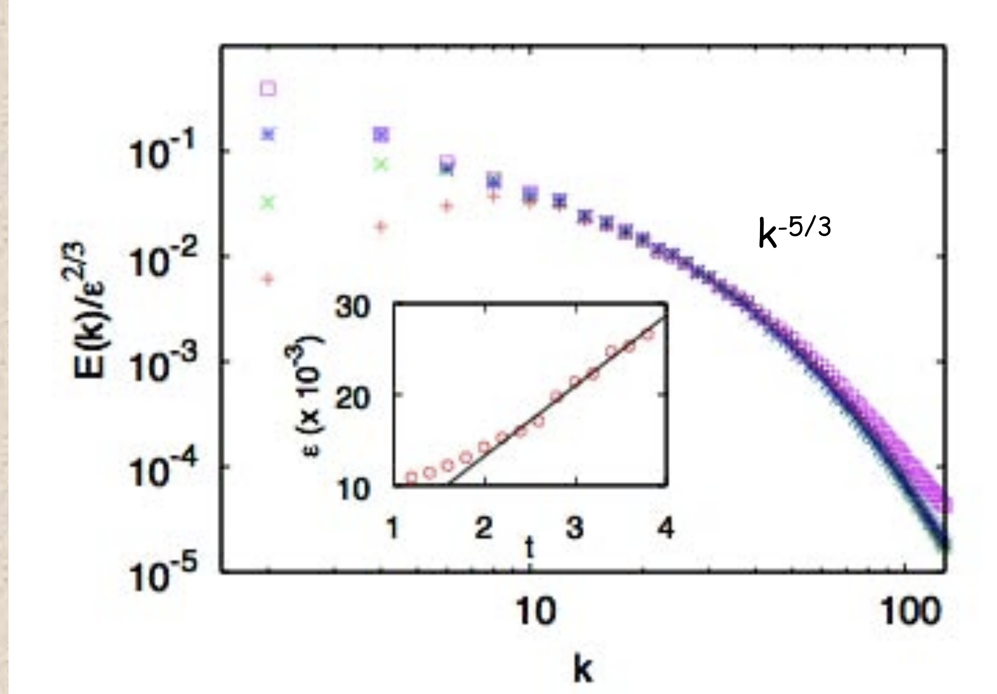
Buoyancy contribution to the energy flux becomes negligible at small scales

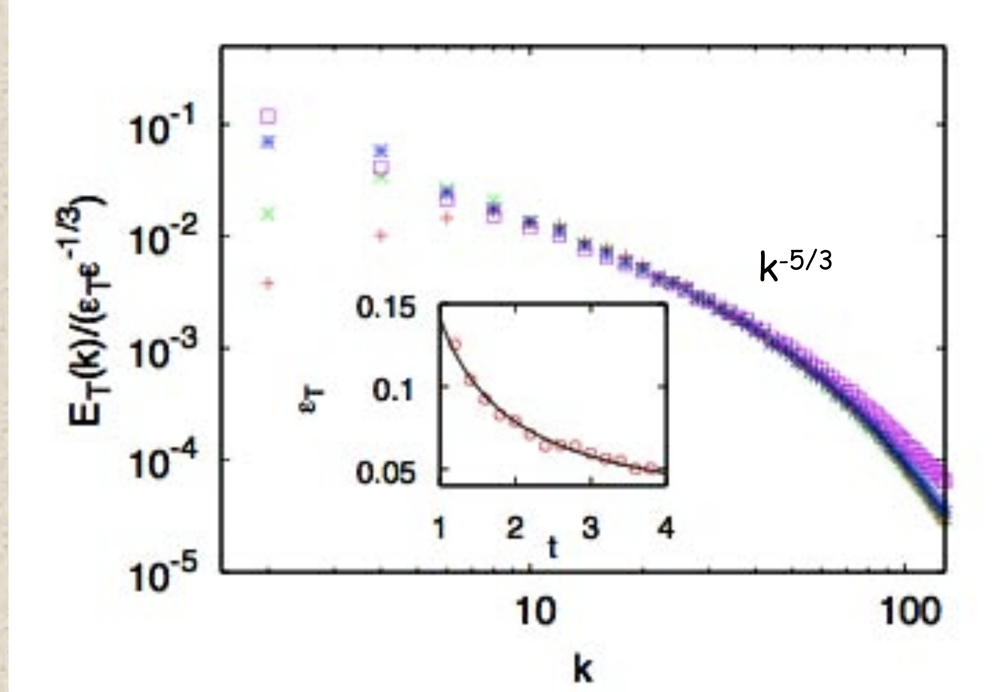
$$Ri = \frac{\beta g \delta_r T(t)}{\delta_r u^2(t) / r} \simeq \left(\frac{r}{L(t)}\right)^{2/3} \to 0$$

Spectra collapse (Kolmogorov scaling)

Collapse of kinetic energy and temperature variance spectra at t/τ=1.0, 1.4, 1.8, 3.8

Insets: time evolution of kinetic energy dissipation $\varepsilon \approx t$ and temperature variance dissipation $\varepsilon_T \approx t^{-1}$





Spatial-temporal scaling in agreement with dimensional theory

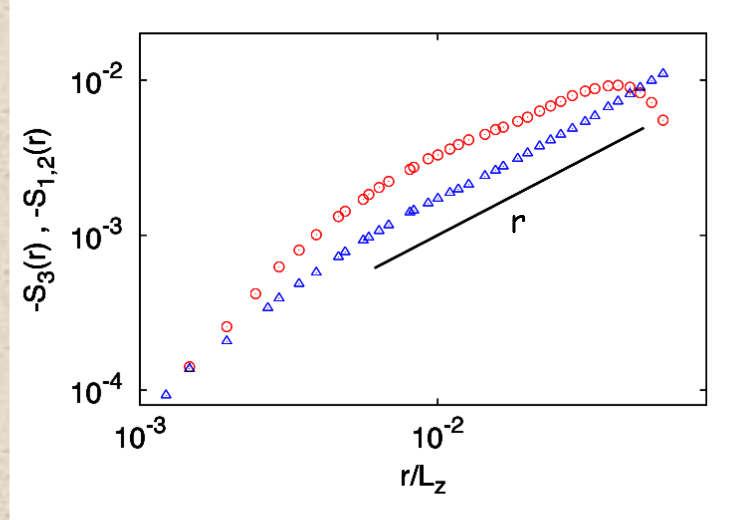
$$E(k,t) \simeq t^{2/3} k^{-5/3}$$

$$E_T(k,t) \simeq t^{-4/3} k^{-5/3}$$

Structure functions and small scale intermittency

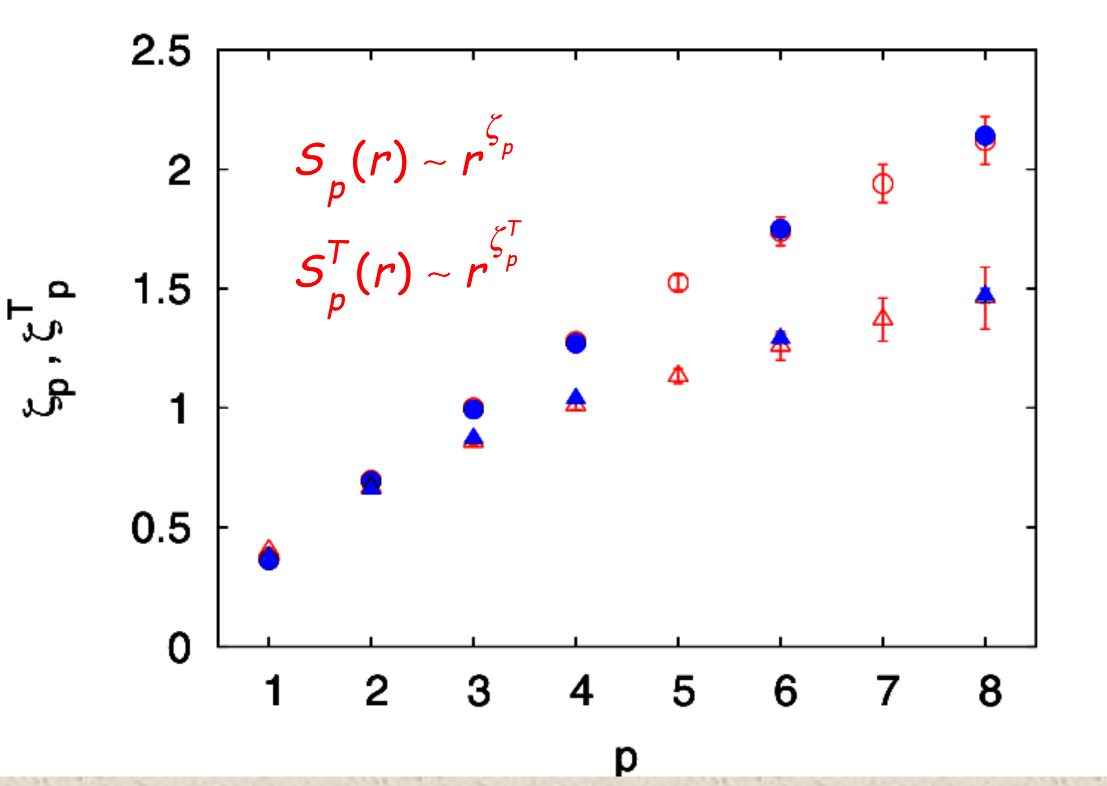
$$S_p(r,t) \equiv \left\langle \left(\delta_r v_{\parallel}(t)\right)^p \right\rangle \simeq v_{rms}(t)^p \left(\frac{r}{h(t)}\right)^{\zeta_p}$$

$$S_p^T(r,t) \equiv \left\langle \left(\delta_r T(t)\right)^p \right\rangle \simeq \theta_0^p \left(\frac{r}{h(t)}\right)^{\zeta_p^T}$$



$$S_3(r) = \left\langle \delta_r u_{\parallel}^3 \right\rangle$$

$$S_{1,2}(r) = \left\langle \delta_r u_{\parallel} \delta_r T^2 \right\rangle$$



RT turbulence

NS turbulence + passive scalar (R_λ=427, Watanabe & Gotoh, New J Phys. 2004)

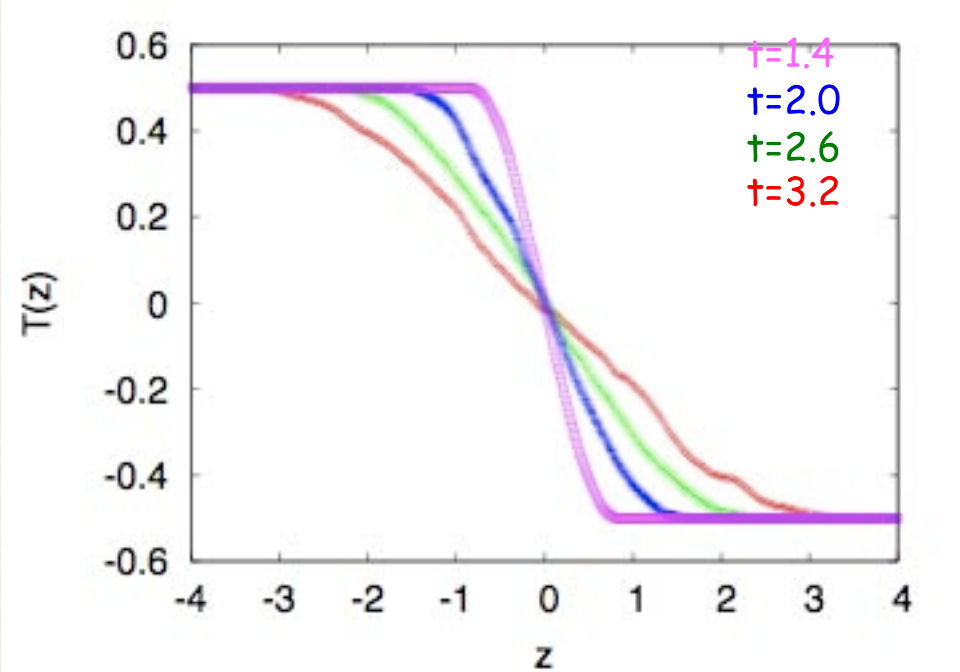
Same scaling exponents of NS turbulence:
universality with respect to the forcing mechanism

Large scale statistics:

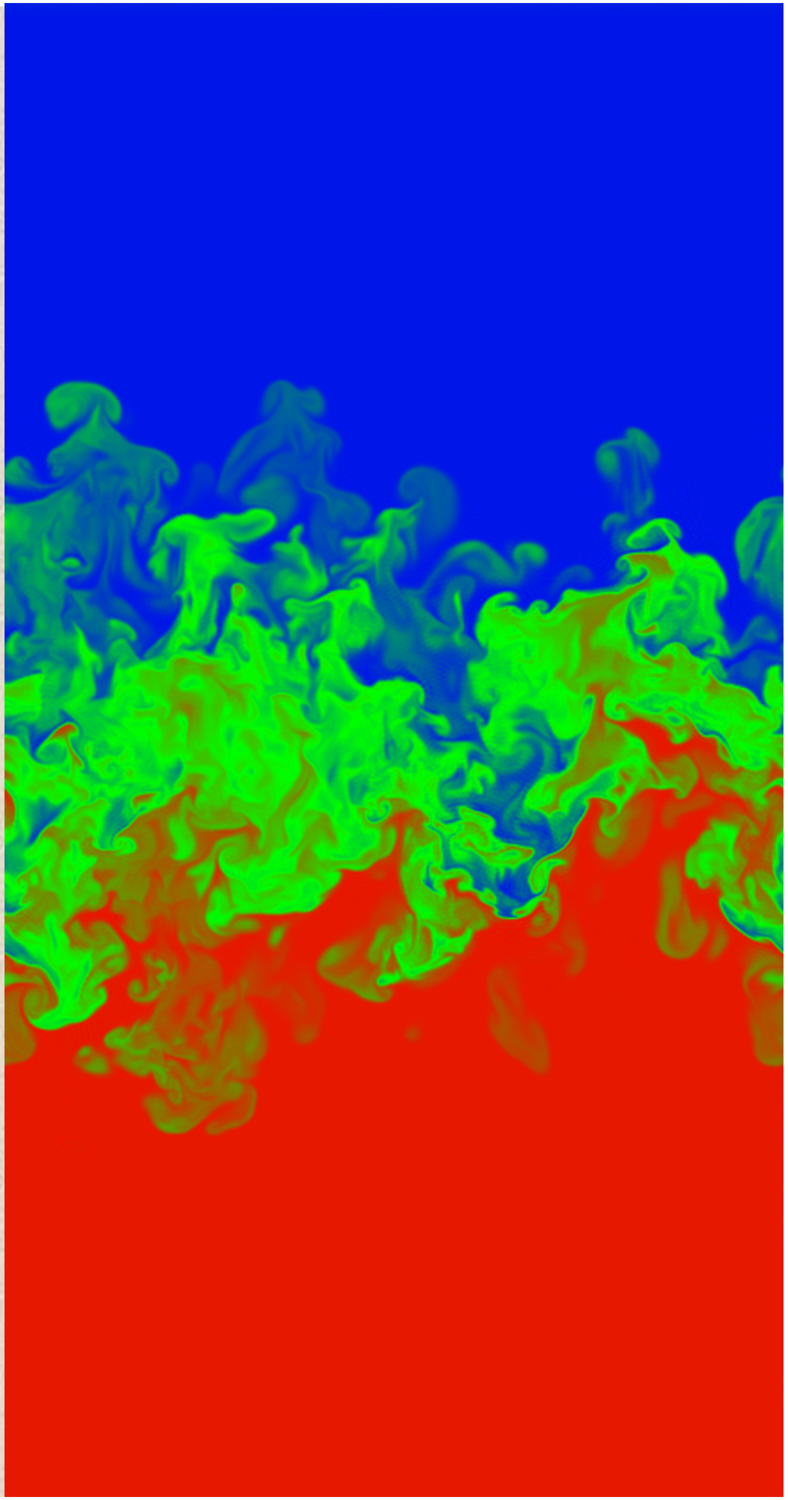
a nonlinear diffusion model for
spatial-temporal evolution of the mixing layer

G.Boffetta, F.De Lillo, S.Musacchio PRL **104** (2010)

Evolution of the mixing layer



mean temperature profile



Mean normalized temperature profile

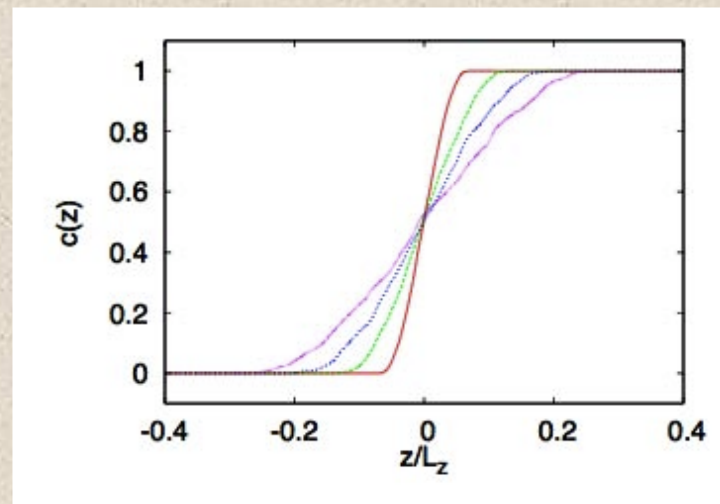
$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta \mathbf{g} T \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \end{cases}$$

$$\bar{c}(z,t) \equiv \frac{1}{L_x L_y}\int c(x,y,z,t)\,dx\,dy$$

$c=(T_{max}-T)/(T_{max}-T_{min})$, $0 \le c \le 1$

evolves with

$$\partial_t \bar{c} + \partial_z \overline{(wc)} = \kappa \partial_z^2 \bar{c}$$



Eddy diffusivity model
(Prandtl mixing layer)

$$\partial_t \bar{c} = \partial_z \left(K(z,t) \partial_z \bar{c} \right)$$

with $K(z,t) \simeq h^2 \partial_z w$ and $w \simeq \beta g T t$ one obtains ($h \approx Agt^2$)

$$\partial_t \bar{c} = a(Ag)^3 t^5 \partial_z \left(\partial_z \bar{c} \right)^2$$

$$\varphi = 3\partial_z \bar{c} / \left(a(Ag)^3 \right)$$

$$t' = t^6$$

Nonlinear diffusion equation
(concentration dependent diffusivity)
[R.E. Pattle, Quart. J Mech. Appl. Math. XXI, 1959]

$$\partial_{t'} \varphi = \partial_z \left(\varphi^n \partial_z \varphi \right)$$

Boussinesq equation

Self-similar solution

$$\bar{c}(z,t) = \frac{1}{4}\frac{z}{z_1}\left[3 - \left(\frac{z}{z_1}\right)^2\right] + \frac{1}{2} \qquad |z| \le z_1$$

$\bar{c}(z,t) = 0 \qquad z<-z_1$

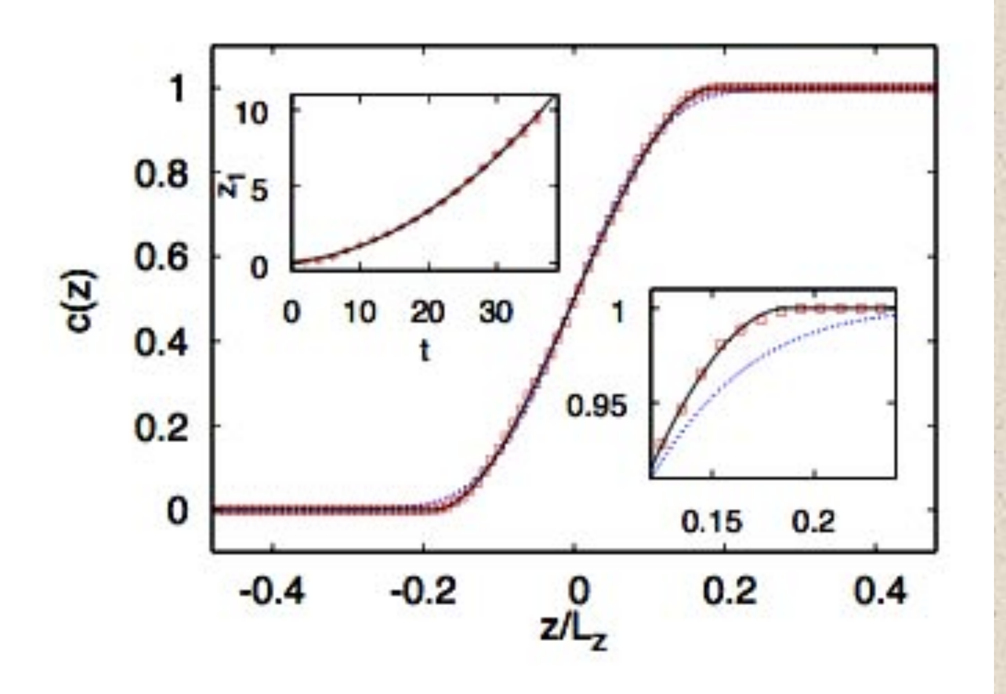
$\bar{c}(z,t) = 1 \qquad z>z_1$

mixing layer half width

$z_1 = \gamma A g t^2$

$\gamma = (3a/2)^{1/3}$

very good fit of the numerical profile with one free parameter γ



Mixing layer growth law: $h(t) = \alpha A g t^2$

with $\alpha=(3/4)\gamma\approx0.025$

close to values obtained with different methods.

Linear diffusion model does not work

Assuming constant K(t) diffusion model gives

$$\bar{c}(z,t) = \frac{1}{2}\left[1 + erf\left(\frac{z}{z_1}\right)\right]$$

simulation

nonlinear diffusion model

linear diffusion model

From geometrical to dynamical quantities

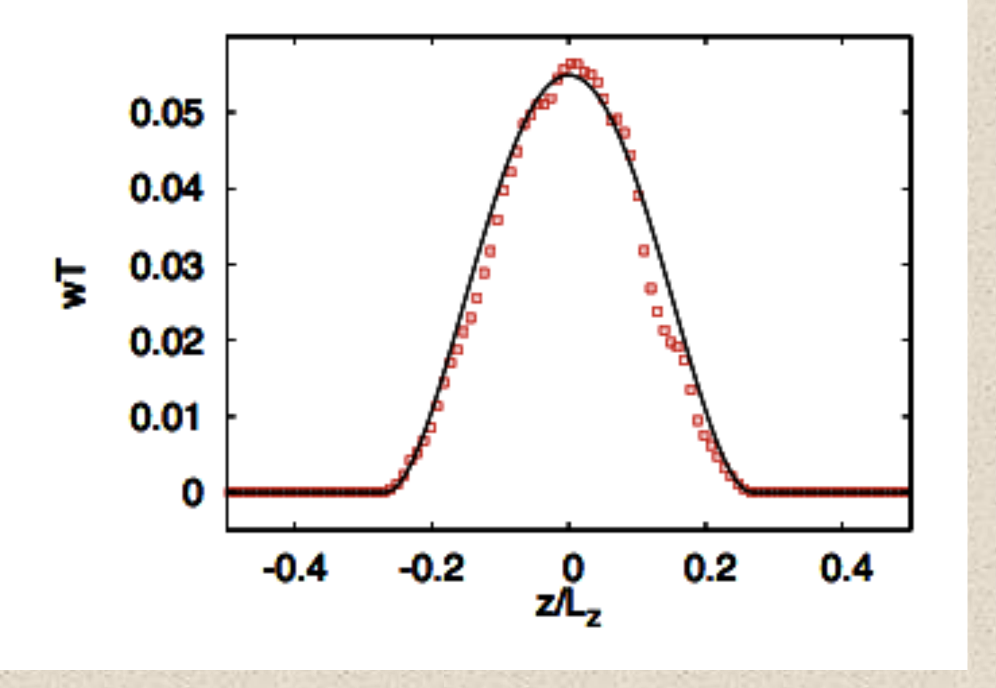
From the equation for temperature profile $\partial_t \bar{c} + \partial_z \overline{(wc)} = \kappa \partial_z^2 \bar{c}$

and the diffusion model $\partial_t \bar{c} = a(Ag)^3 t^5 \partial_z \left(\partial_z \bar{c}\right)^2$

one has an expression for the turbulent heat flux $\overline{wT} = a(Ag)^3 t^5 \left(\partial_z \bar{T}\right)^2 / \theta_0$

which fits well the numerical data

$$\overline{wT} = \frac{3}{8} \gamma \theta_0 A g t \left[\left(\frac{z}{z1} \right)^2 - 1 \right]^2$$



Mixing efficiency in RT turbulence: Kraichnan's ultimate state

From the mean temperature and heat flux profiles

$$\bar{T}(z,t) = \frac{\theta_0}{4}\frac{z}{z_1}\left[\left(\frac{z}{z_1}\right)^2 - 3\right] \qquad \overline{wT} = \frac{3}{8}\gamma\theta_0 Agt\left[\left(\frac{z}{z1}\right)^2 - 1\right]^2$$

$$Nu = \langle wT \rangle h / (\kappa\theta_0)$$

$$Ra = Agh^3 / (\nu\kappa)$$

$$Pr = \nu / \kappa$$

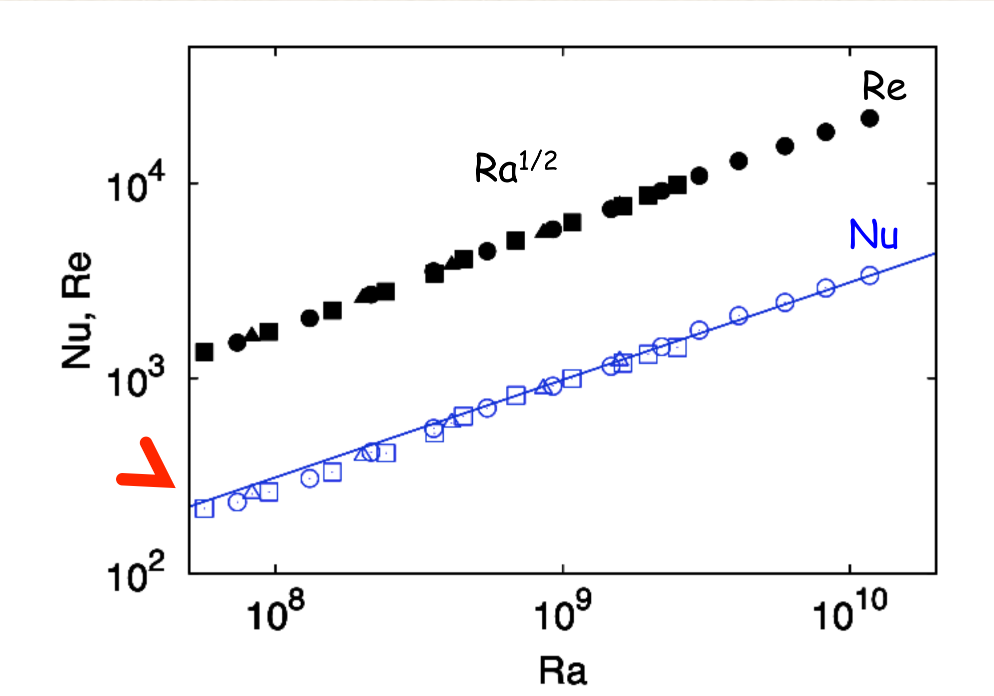
integrating over ther mixing layer width h(t) one obtains the prediction

$$Nu = \left(\frac{\gamma}{50}\right)^{1/2} Pr^{1/2} Ra^{1/2}$$

Dimensionally, also

$$Re \approx Pr^{-1/2} Ra^{1/2}$$

"Ultimate state" of thermal convection
Kraichnan (1962) (without free parameters)

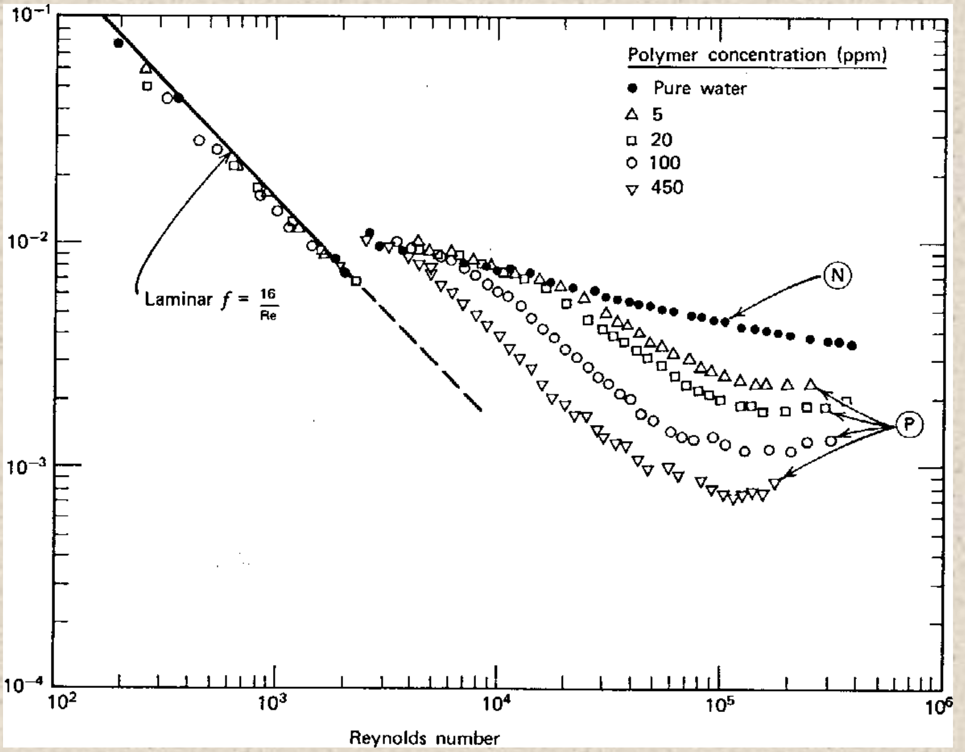


□:256x256x1024
O: 512x512x2048
Δ: 1024x1024x1024

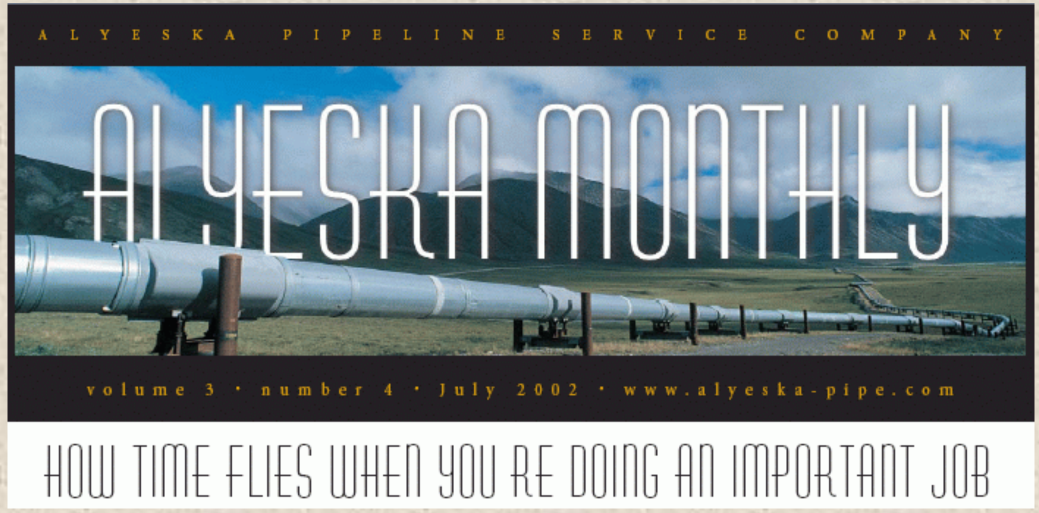
Effects of polymers on thermal convection: viscoelastic Rayleigh-Taylor turbulence

G.Boffetta, A.Mazzino, S.Musacchio, L. Vozella, **Phys. Rev. Lett. 104**, 184501 (2010) + JFM **643** (2010)

The addition of small amount of long-chain polymers can have dramatic effects on flowing fluids.



Drag reduction:
turbulent drag can be reduced up to about 80% (Toms 1949)



Oil-soluble polymers are added in Trans-Alaskan Pipeline to reduce the number of pumping stations

Is there any effect of polymers on turbulent convection ?

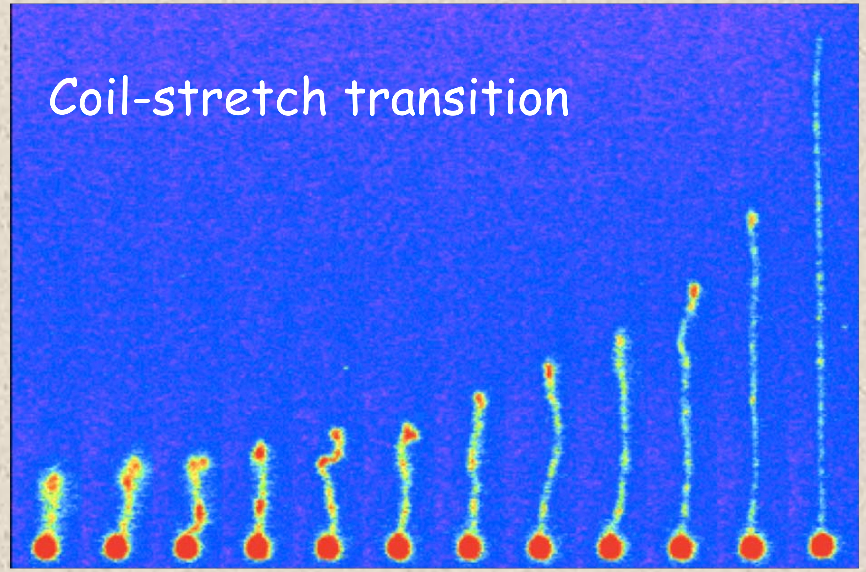
Viscoelastic RT turbulence

Fluid model:
Boussinesq+Oldroyd-B

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} - \beta\mathbf{g}T + (2\nu\eta/\tau_p)\nabla\cdot\sigma \\ \partial_t T + \mathbf{u}\cdot\nabla T = \kappa\Delta T \\ \partial_t \sigma + \mathbf{u}\cdot\nabla\sigma = (\nabla\mathbf{u})^T\cdot\sigma + \sigma\cdot(\nabla\mathbf{u}) - (2/\tau_p)(\sigma - \mathbf{1}) \end{cases}$$

Polymer conformation tensor
$\sigma_{ij}=\langle R_i R_j\rangle/R_0^2$

$$E = P + K + \Sigma = -\beta g\langle zT\rangle + \frac{1}{2}\langle u^2\rangle + \frac{\nu\eta}{\tau_p}\langle tr(\sigma) - 3\rangle$$



Scales in RT turbulence

r

$L \sim t^2$

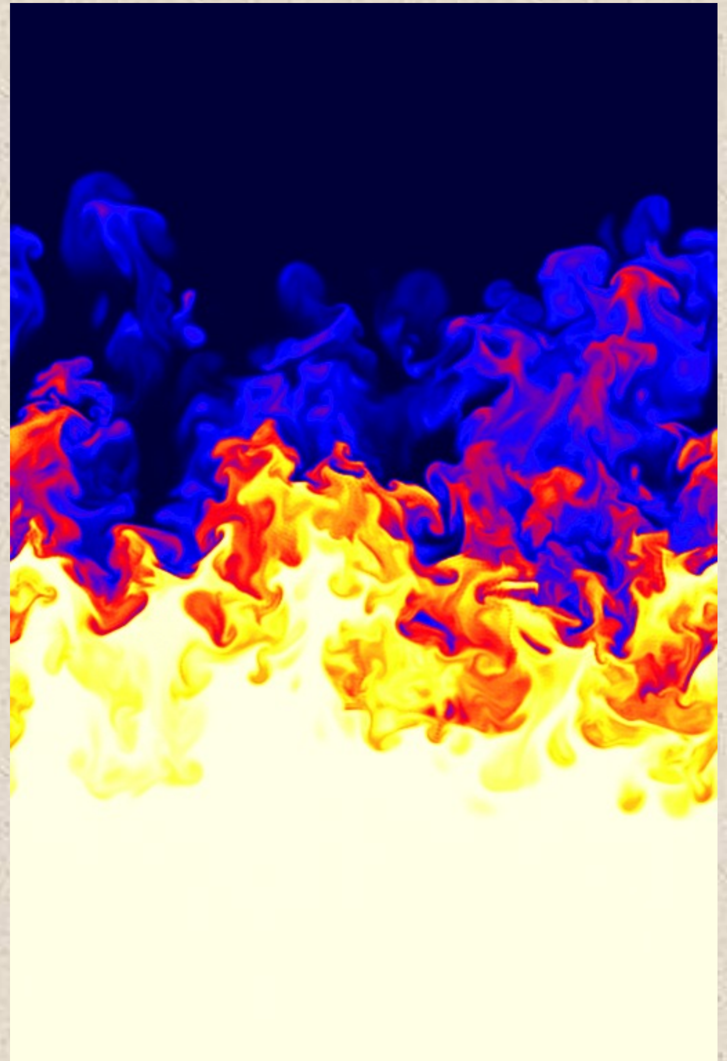
$r_L \sim t^{1/2}$ Lumley scale
$\tau_p/\tau(r_L)=1$

$\eta \sim t^{-1/4}$

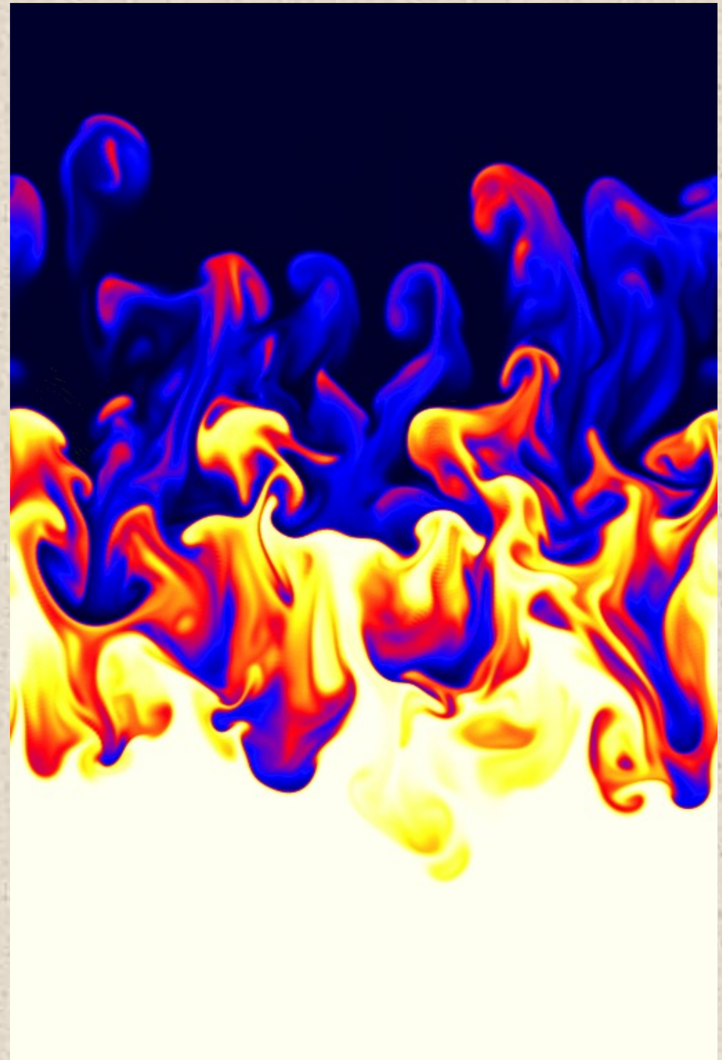
t

polymers become stretched
(and active) when $\nabla u > 1/\tau_p$
i.e. when $Wi = \frac{\tau_p}{\tau_\eta} > 1$

Weissenber number grows as $Wi \approx t^{1/2}$



Newtonian



viscoelastic

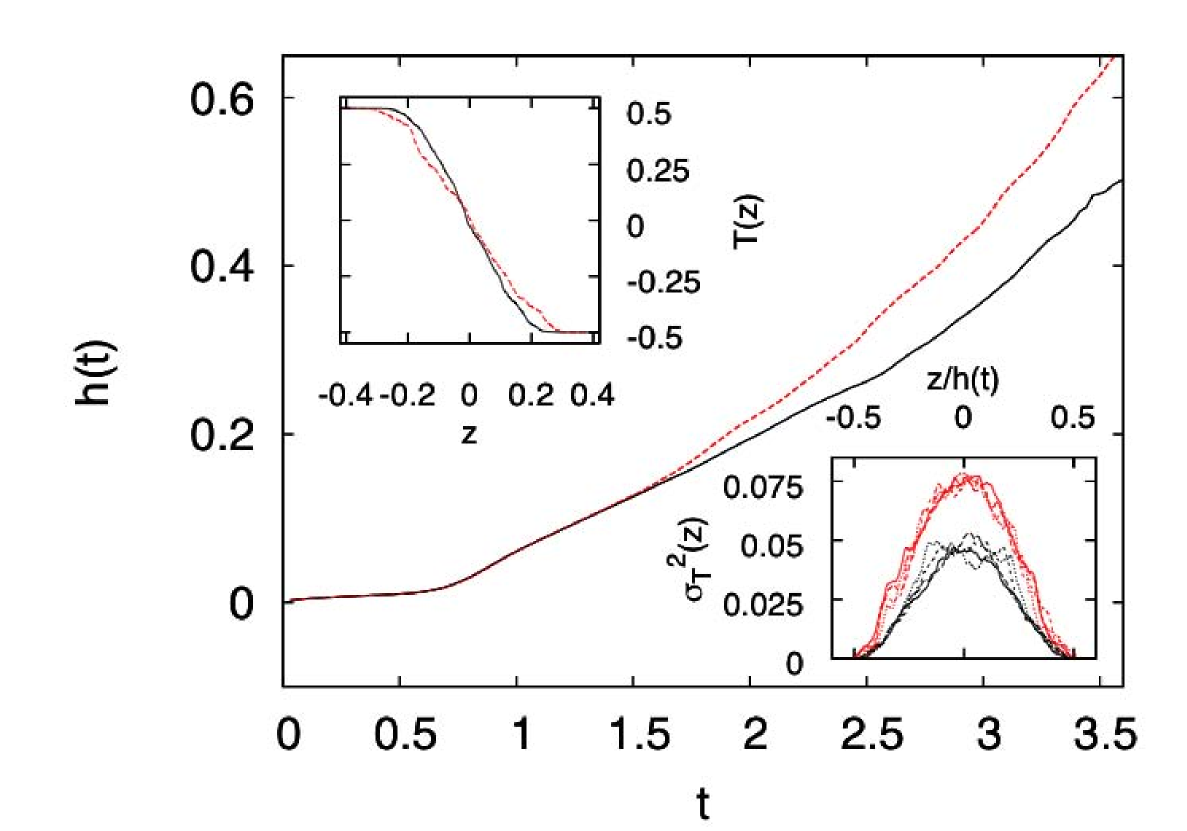
Two effects induced by polymers:

- faster thermal plumes: acceleration of mixing layer growth ("drag" reduction)
- small scale turbulence reduced: heat transfer enhancement

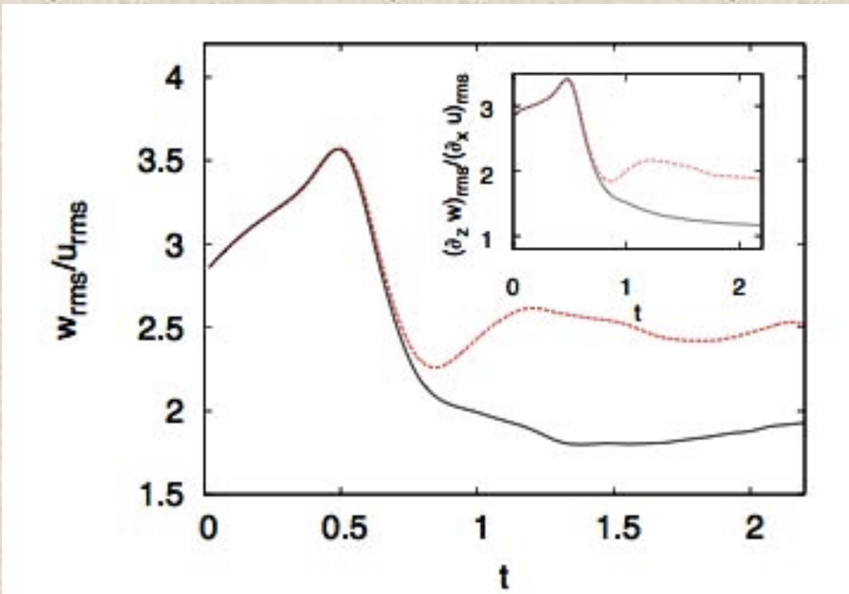
Acceleration of the growth of mixing layer

When polymers are stretched mixing layer grows faster than in the Newtonian case (30% larger at final time).

Consistent with the speed-up of RT instability (linear analysis)
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella, JFM **643** (2010)

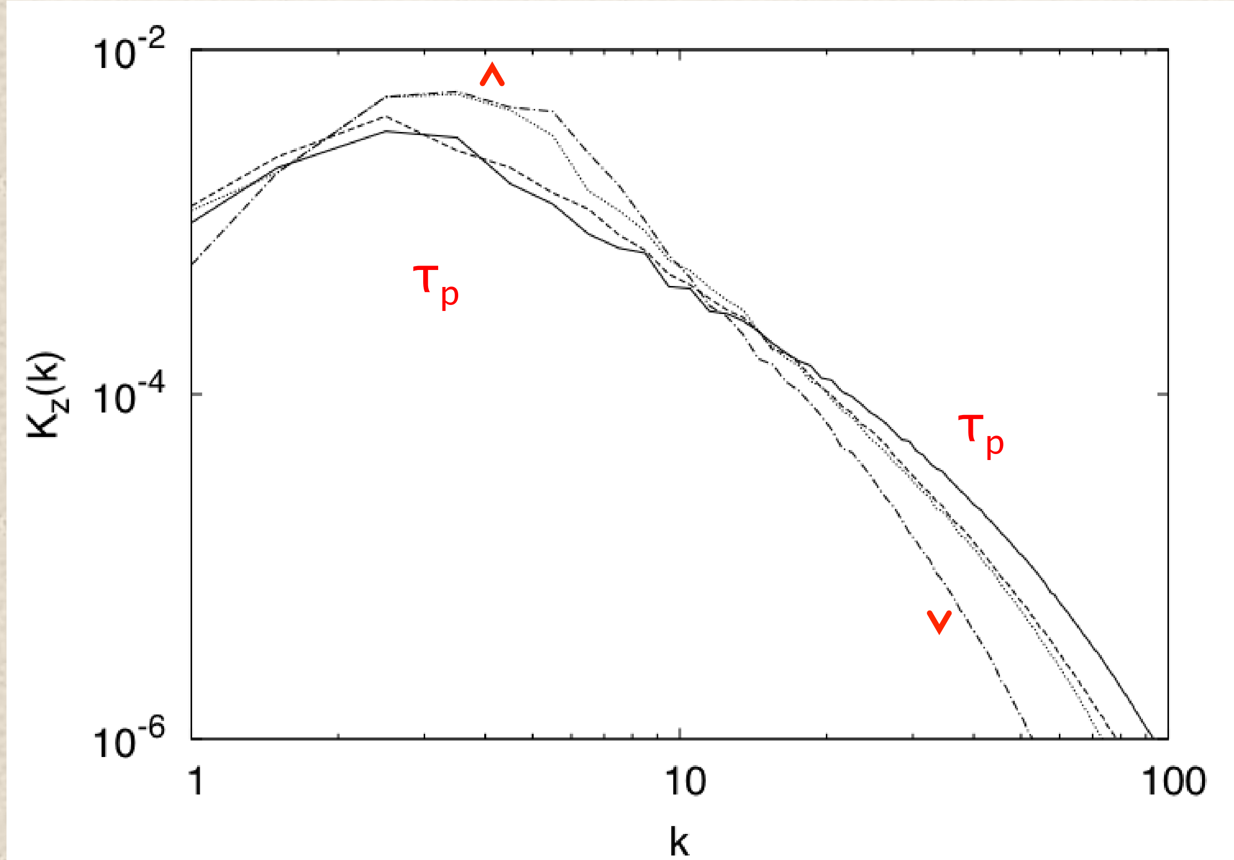


Kinetic energy stronger and more anisotropic



Newtonian
viscoelastic

Energy spectra



more energy at large scales (mixing layer accelerates)

less energy at small scales (reduced mixing)

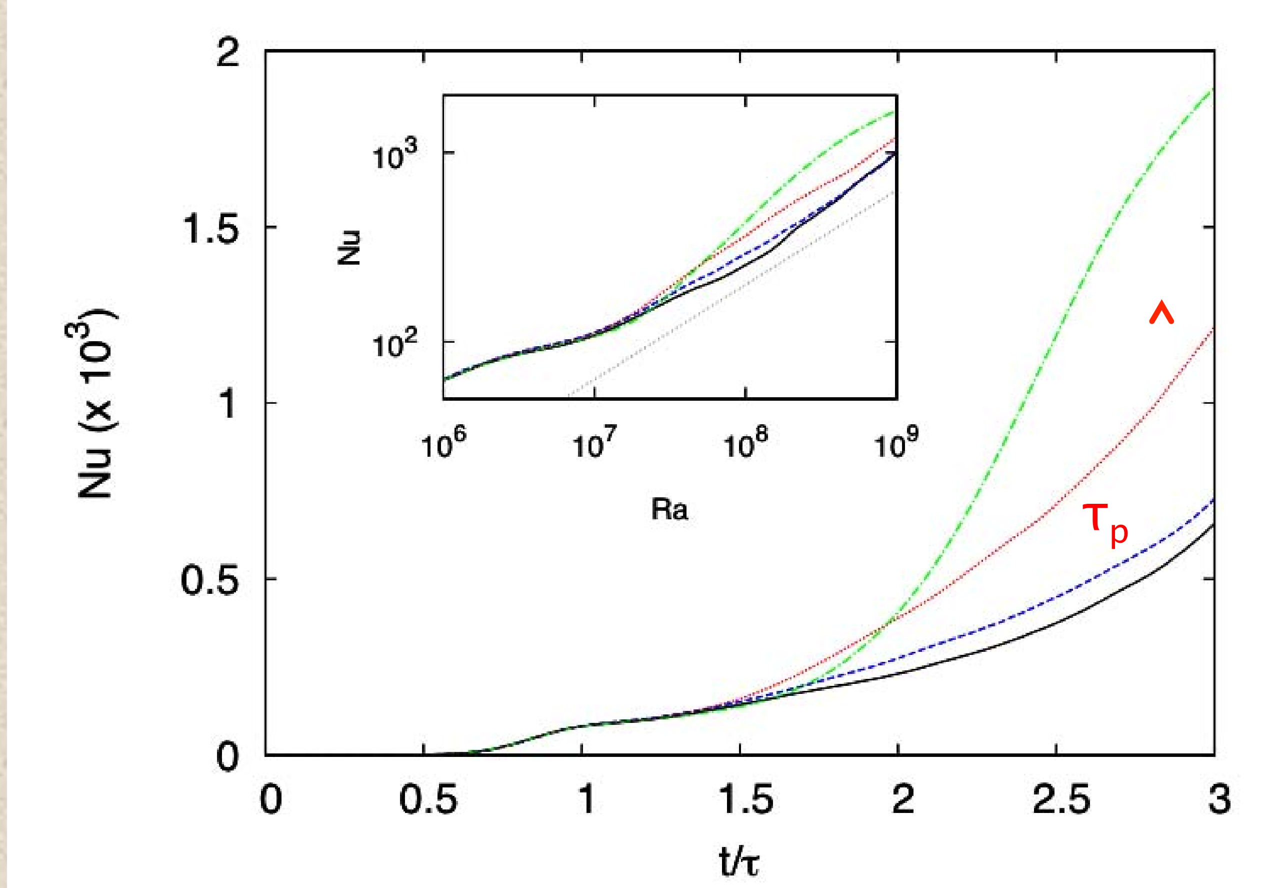
Heat transfer enhancement

Nu is larger in presence of polymers.

Nu increases more than Ra therefore

$$Nu = C\,Pr^{1/2}\,Ra^{1/2}$$

$$C = \left(\frac{\gamma}{50}\right)^{1/2}$$



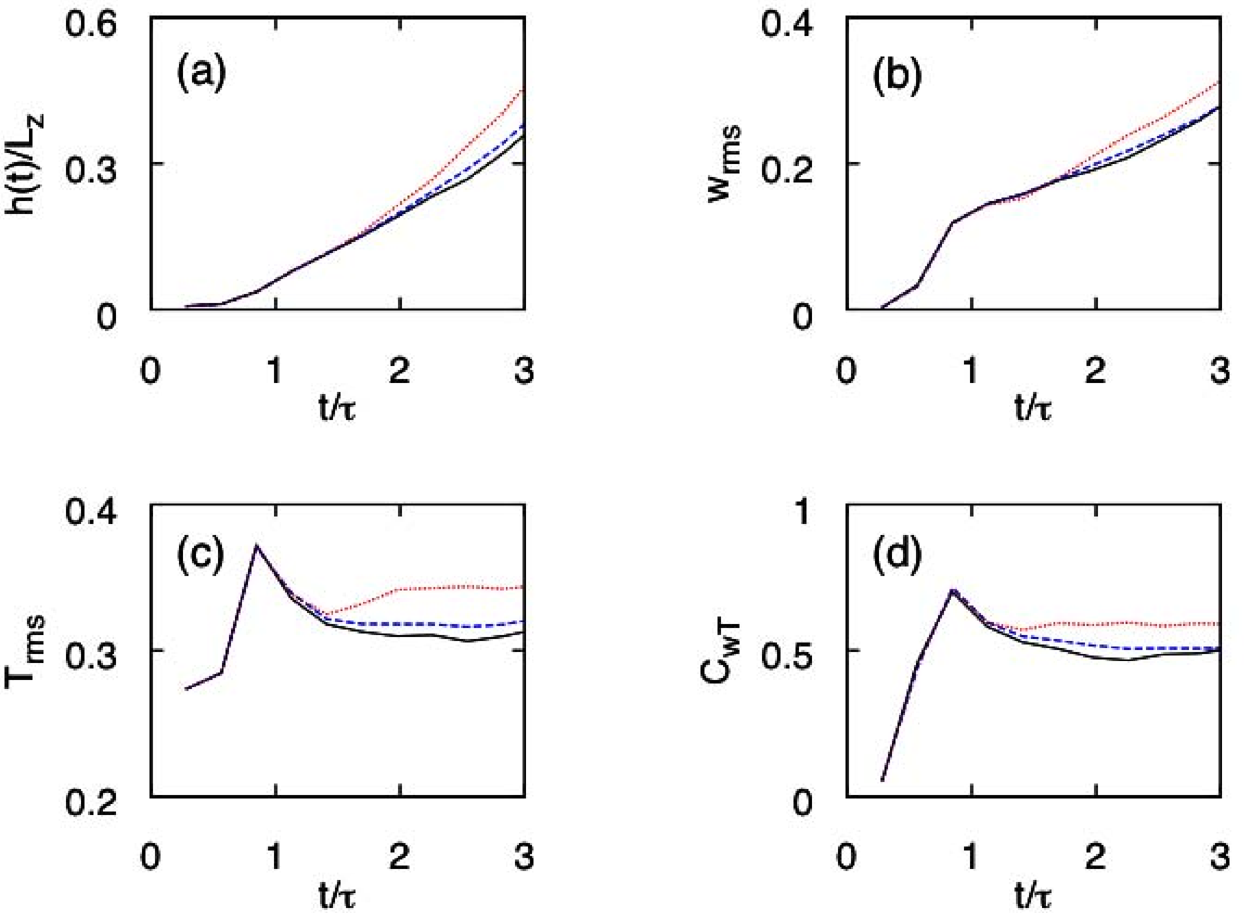
$C_N = 0.022 \pm 0.002$

$C_{VE} = 0.029 \pm 0.002$ (30% more)

$\tau_p = 0$
$\tau_p = 1$
$\tau_p = 2$
$\tau_p = 10$

The origin of heat transfer enhancement ?

$$Nu = \frac{1}{k\theta_0} h w_{rms} T_{rms} C(w,T) \qquad C(w,T) = \frac{\langle wT \rangle}{w_{rms} T_{rms}}$$



faster thermal plumes (h and w), reduced turbulent mixing (T) and stronger correlation between T and w (more coherent plumes)

$\tau_p=0$
$\tau_p=1$
$\tau_p=2$

Three of these effects are not specific of RT turbulence: what about other systems?
Similar result for mean gradient and shell model simulations R.Benzi et al, PRL **104** (2010)
but not (yet) in RB experiments: G.Ahlers, A.Nikolaenko PRL **104** (2010)

Conclusions

Rayleigh-Taylor turbulence as a simple model of thermal convection:

* at small scale temperature becomes passive,

  Navier-Stokes + passive scalar phenomenology

* large scale well described by a (nonlinear) eddy diffusivity model

  ultimate state of thermal convection

* viscoelastic RT turbulence

  acceleration of the mixing layer growth (drag reduction)

  heat transfer enhancement

G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella PRE **79**, 065301 (2009)
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella JFM **643**, 127 (2010)
G.Boffetta, F.De Lillo, S.Musacchio PRL **104**, 034505 (2010)
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella **Phys. Fluids 22** 035109 (2010)
G.Boffetta, A.Mazzino, S.Musacchio, L.Vozella PRL **104**, 184501 (2010)

http://www.to.infn.it/~boffetta