Inertial particle clustering and random walks in random environments

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in collaboration with

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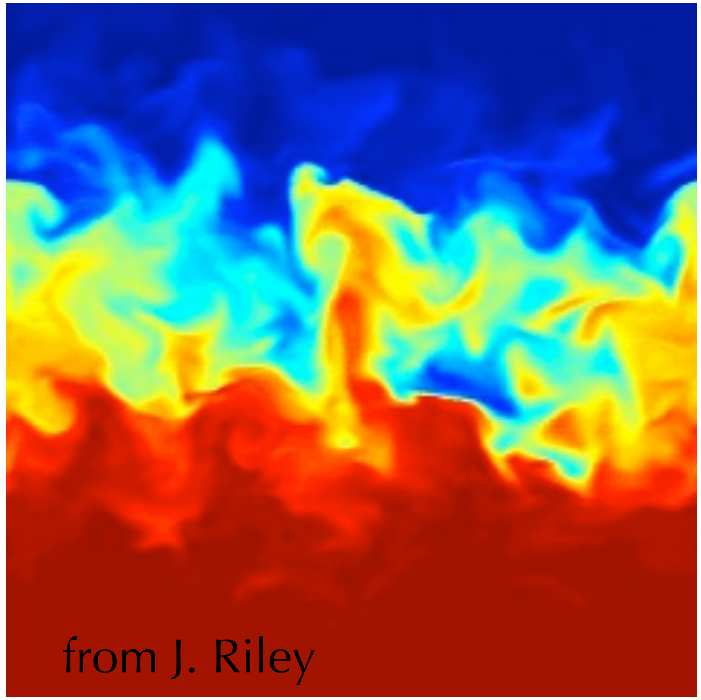
Anomalous Transport: from Billiards to Nanosystems, Sperlonga, September 2010

Turbulent Transport/Mixing

- **Industrial/Natural problems:** passive or active transport of species by a turbulent flow

- **Effect of turbulence:**
enhance mixing/dispersion
(w.r.t. molecular diffusion)

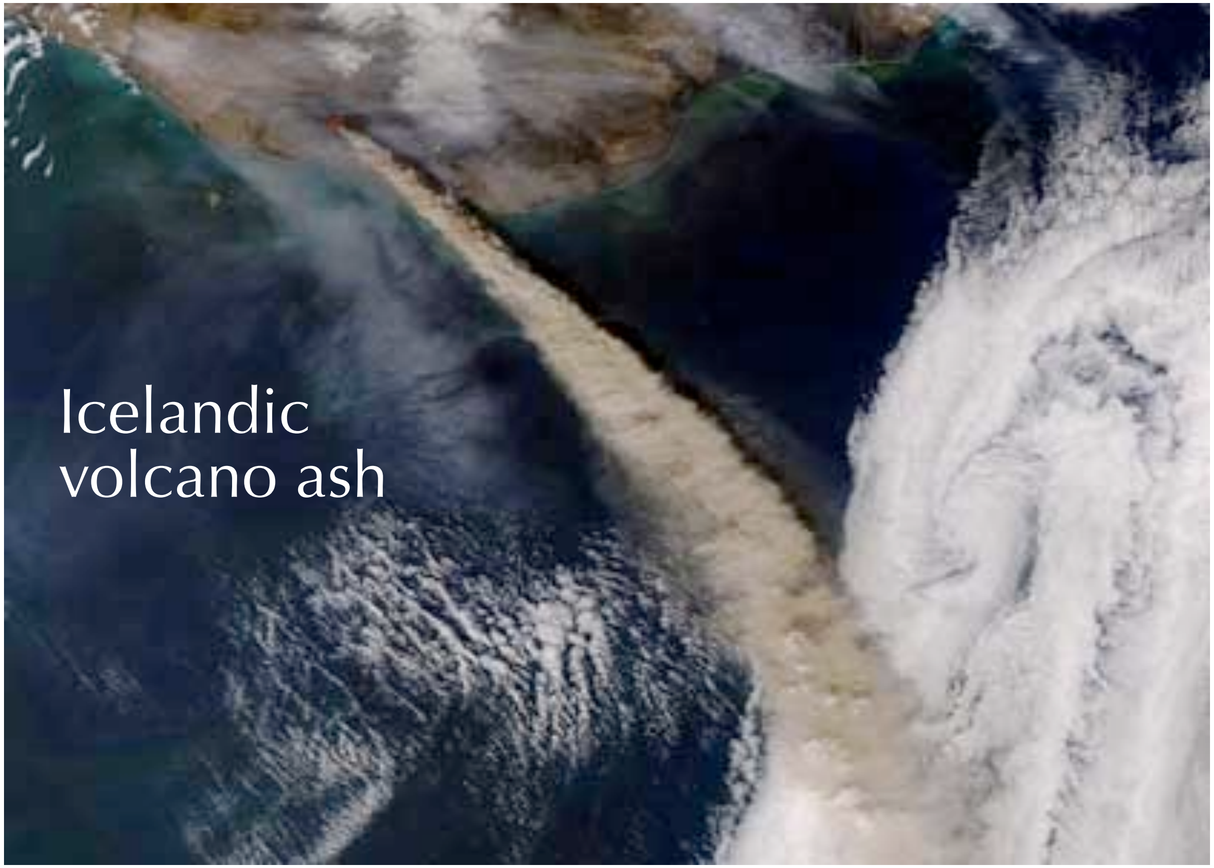
  Eddy diffusivity
  (~mean-field effect)



- **Quantifying fluctuations?** What are the mechanisms leading to the presence of very high concentrations?

Fluctuations in turbulent transport

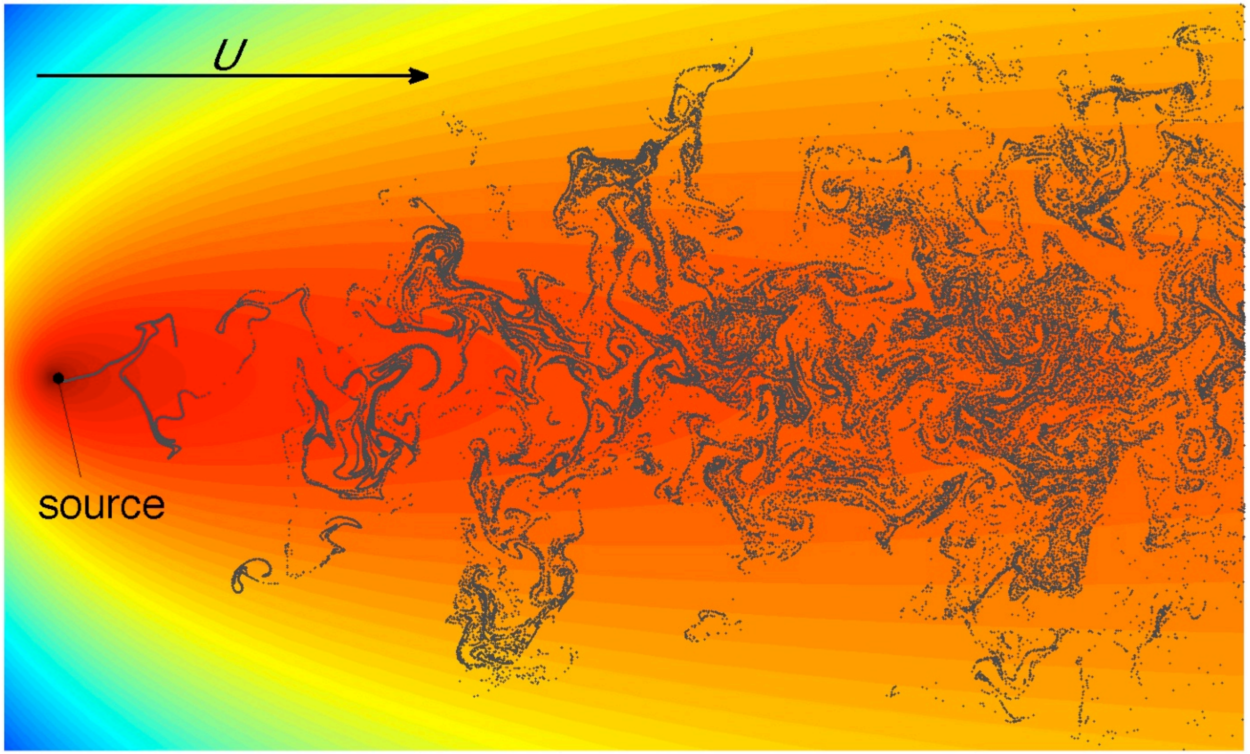
- Fluctuations are important for risk assessments



- **Models/Observations:** space and/or time averages

Mean vs. meandering plumes

- One source of fluctuations = the turbulent transport itself



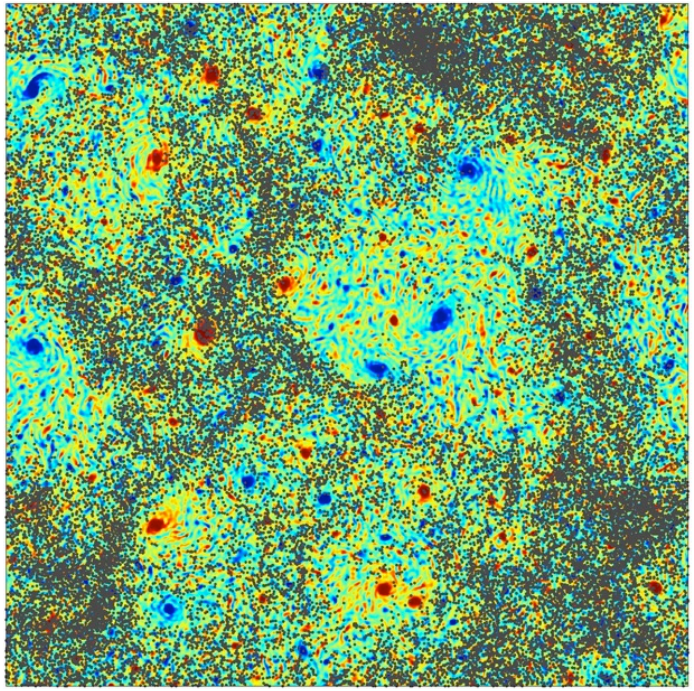
- Concentration PDFs have tails rather far from Gaussian (trapping events for the random walk of particles in the random environment of subjacent turbulent fluctuations?)

Universality??

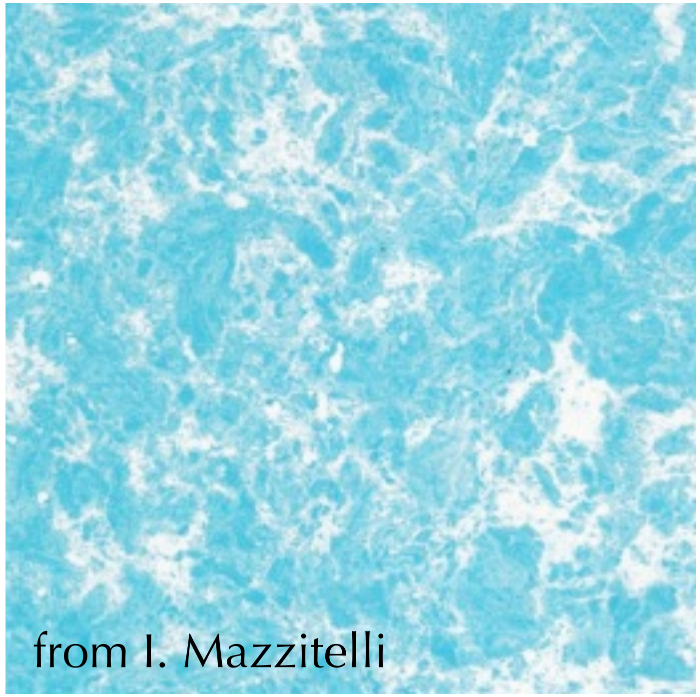
Particles finite mass

- **Most particles are not tracers but have inertia**

Heavy particles are ejected from eddies



Light particles cluster in their cores



Preferential concentration

Very heavy particles

- Spherical particles much much heavier than the fluid, feeling no gravity, evolving with moderate velocities: **one of the simplest model**

$$\ddot{\boldsymbol{X}} = -\frac{1}{\tau}\left(\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X}, t)\right)$$

↑ Prescribed velocity field (random or solution to NS)

Stokes time:

$$\tau = \frac{2\,\rho_p\,a^2}{9\,\rho_f\,\nu}$$

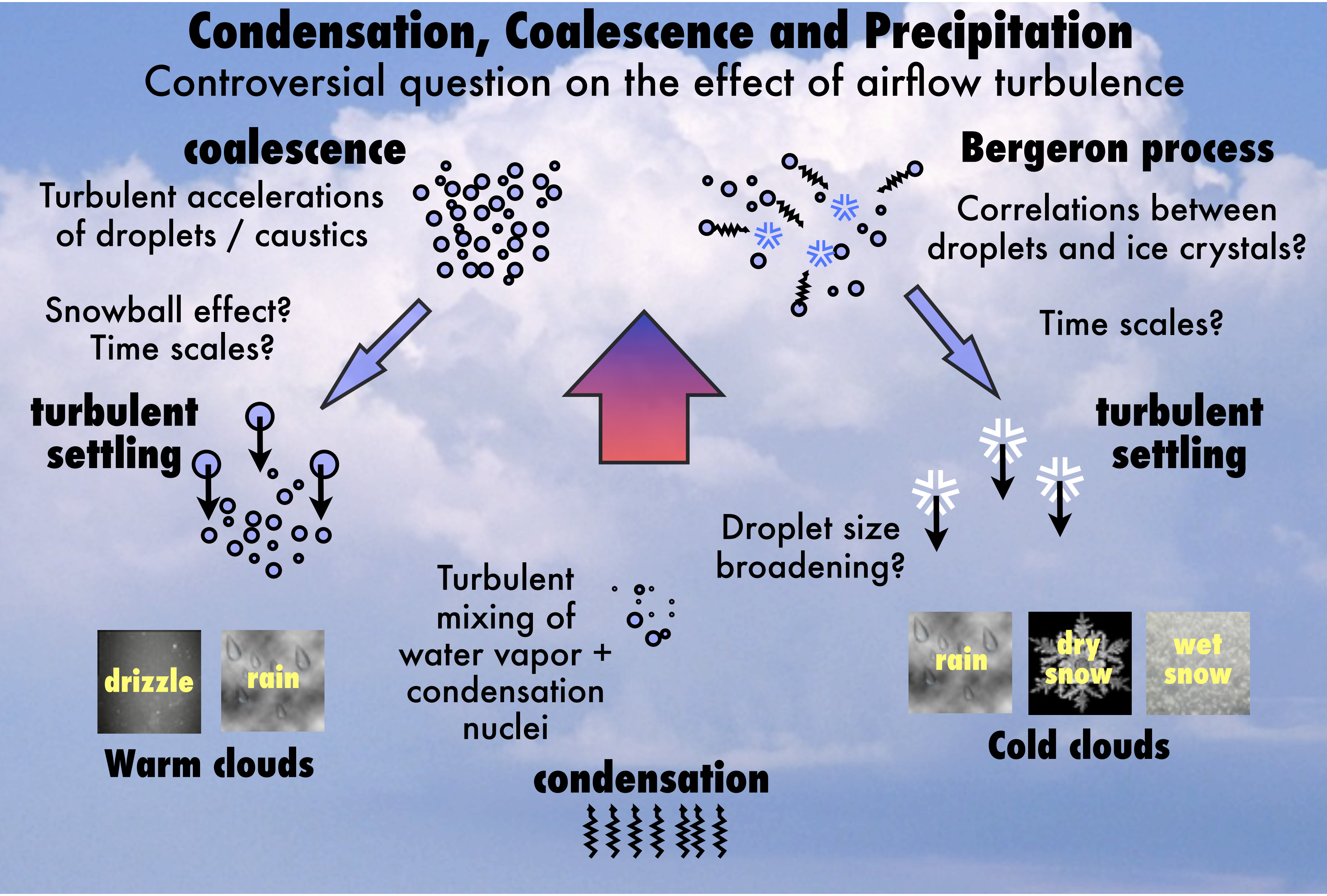
Inertia measured by the Stokes number

$$\mathrm{St} = \tau/\tau_{\mathrm{f}}$$

- **Dissipative dynamics** (even if $\boldsymbol{u}(\boldsymbol{x}, t)$ is incompressible)

  Lagrangian averages correspond to an SRB measure (with support on the attractor) that depends on time and on the realization of the fluid velocity field.

Warm clouds


Condensation, Coalescence and Precipitation
Controversial question on the effect of airflow turbulence
coalescence
Turbulent accelerations of droplets / caustics
Bergeron process
Correlations between droplets and ice crystals?
Snowball effect?
Time scales?
Time scales?
turbulent settling
turbulent settling
Droplet size broadening?
Turbulent mixing of water vapor + condensation nuclei
drizzle
rain
Warm clouds
rain
dry snow
wet snow
Cold clouds
condensation

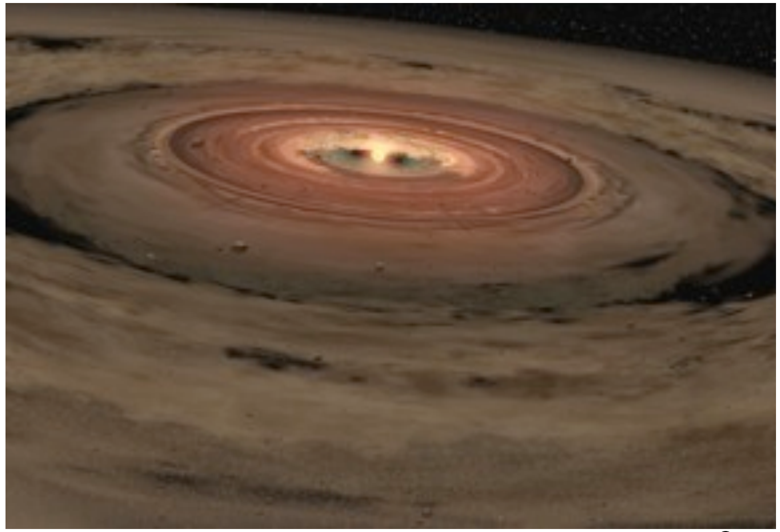
Planet formation

protostar nebula

gravitational collapse

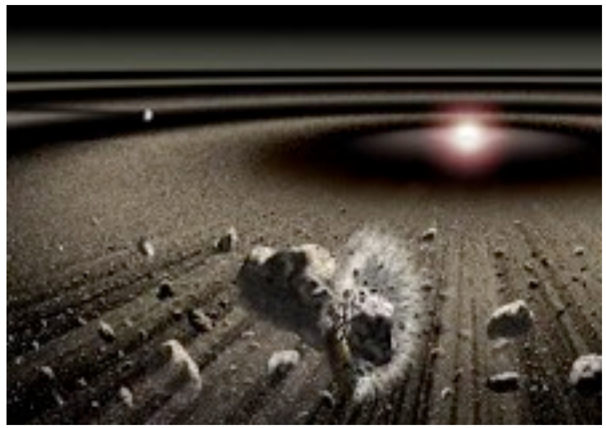
differential rotation + momentum dissipation ⇒ migration toward the equatorial plane

circumstellar disk



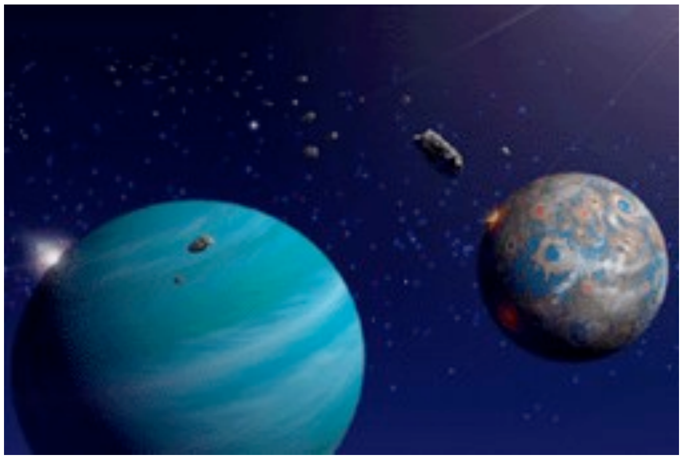
Development of **turbulence** in the gas motion + **accretion** of dust particles

creation of medium-size bodies (mm to m)
Time scales?



gravitational interactions + collisions between large bodies (1m to moons)

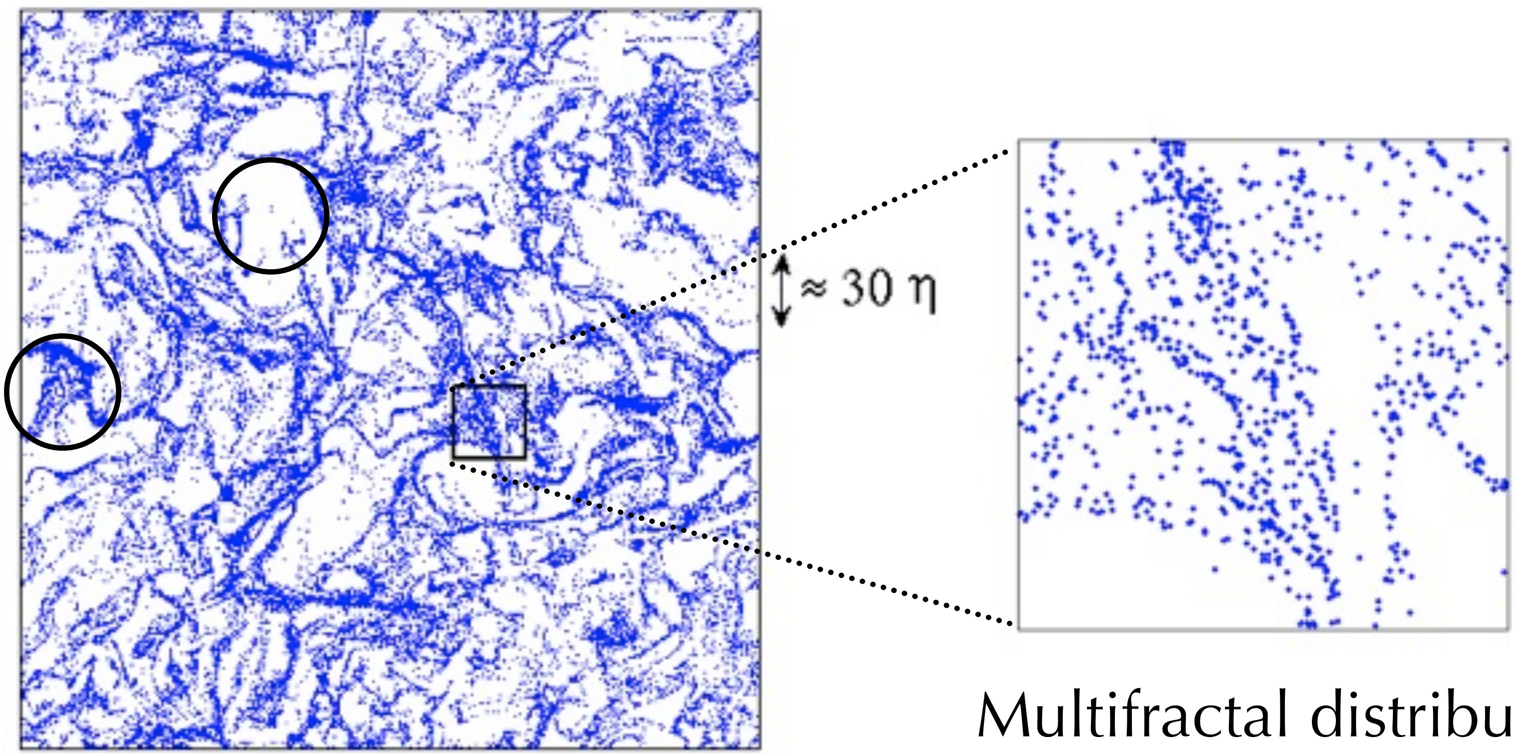
planetary system



Preferential concentration

- Observed for a long time in experiments
  Eaton & Fessler (1994); Douady, Couder, & Brachet (1991)

- **Quantifying them is important for**
  - ∗ the rates at which particles interact
  - ∗ the fluctuations in the concentration of a pollutant
  - ∗ the possible feedback of the particles on the fluid



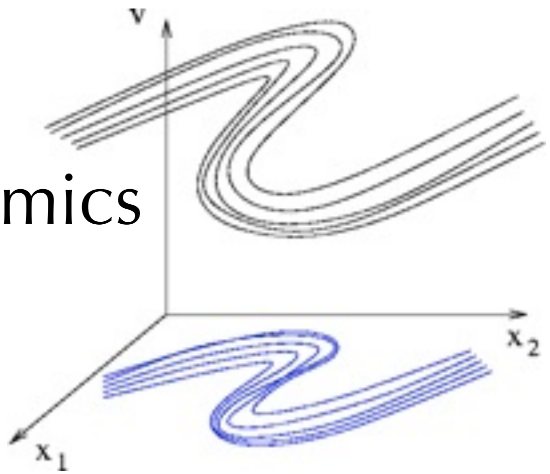
Inertial-range clusters and voids

Multifractal distribution at dissipative scales

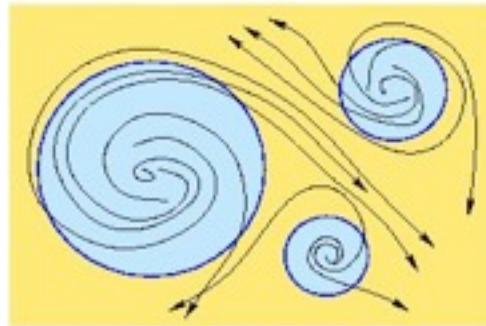
Phenomenology

- **Different mechanisms:**

Dissipative dynamics
$\Rightarrow$ **attractor**



Ejection from **eddies**
by centrifugal forces



- **Theory:** requires elaborating models to disentangle these two effects. For instance:
  - flows with no structures (uncorrelated in time) to isolate the effects of a dissipative dynamics
  - coarse-grained closures to understand ejection from eddies

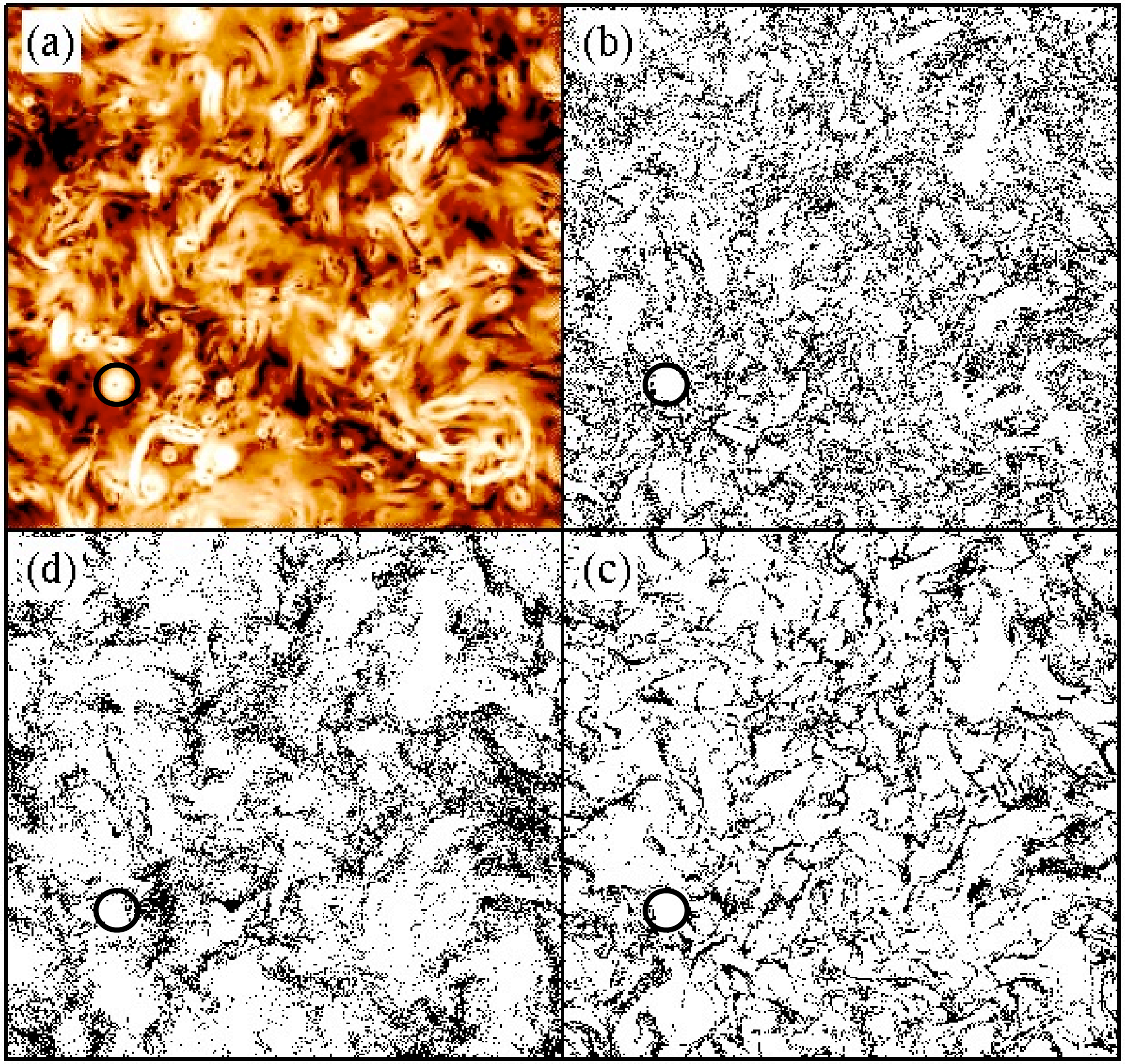
- Numerics show that these effects act at different scales

Particles in turbulent flow

Real flow contain structures and particle distribution correlates with the vortices

Modulus of acceleration

$R_\lambda = 185$



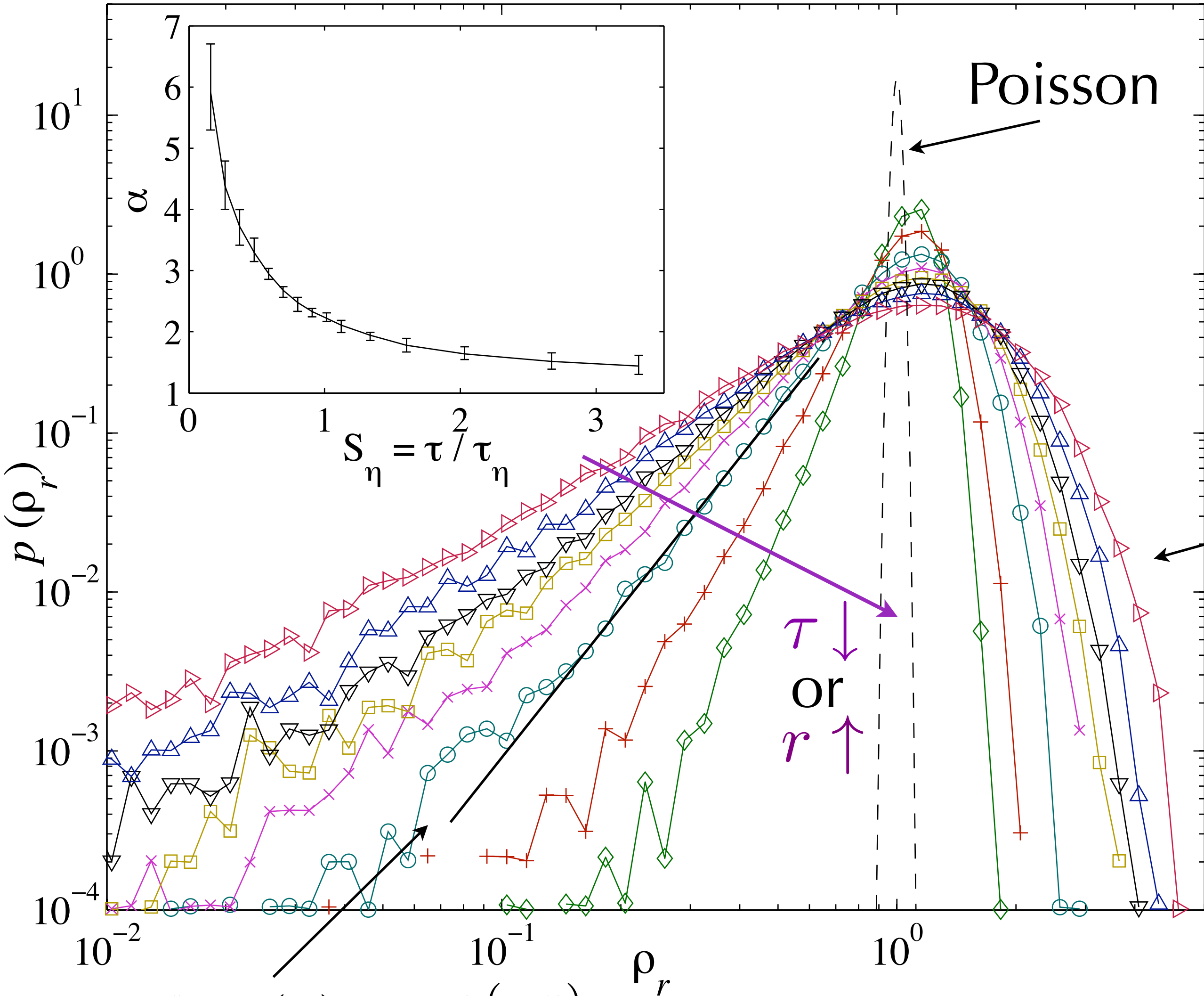
$\mathrm{St} = 0.16$

JB, Biferale *et al.* (2007)

$\mathrm{St} = 3.3$

$\mathrm{St} = 0.8$

Coarse-grained density



Tails faster than exponential

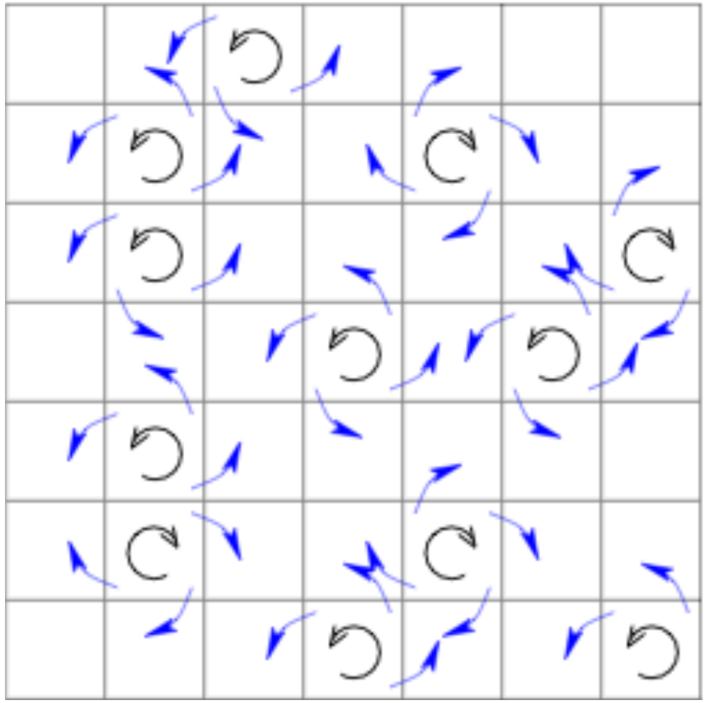
Algebraic tails $p(\rho) \propto \rho^{\alpha(\tau,r)}$ (signature of voids)

- Find models belonging to the same universality class

Mass transport model

JB, R. Chétrite 2007

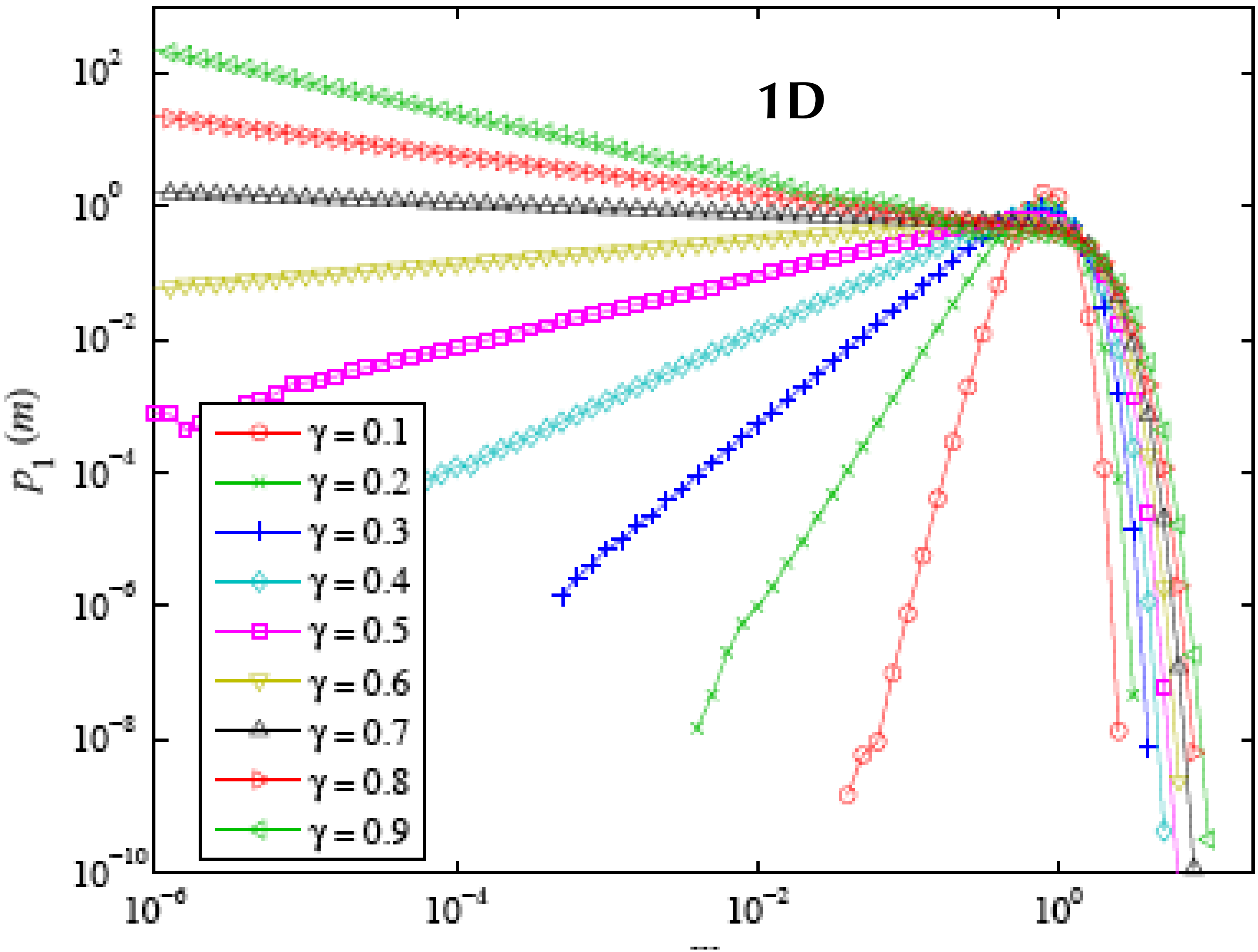
- Discreteness in time and space
- At each time step some cells are randomly chosen (with probability p) to be rotating cells. They eject a fraction γ of their mass to their neighbors



- Parameters = p (probability to rotate) and γ (ejection rate)

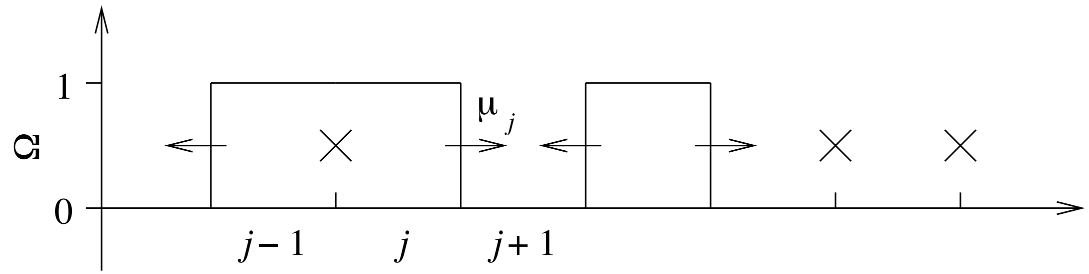
One-cell mass distribution

- PDF of mass in a given cell is very similar to that of DNS



Algebraic left tail

- One-dimension



- Left tail: $p(m) \propto m^{\alpha(\gamma)}$

  Events ejecting a lot of mass: when a cell remains ejecting for a long time.

$$N \text{ times} \begin{cases} m_0 \approx 1 \\ \vdots \\ m_N \approx (1-\gamma)^N \end{cases} \qquad \text{Prob} = p^N (1-p)^{2N}$$

$$\text{Prob}\,\{\text{mass} > m\} \geq C\, m^{\beta} \qquad \beta = \frac{\log[p(1-p)^2]}{\log(1-\gamma)}.$$

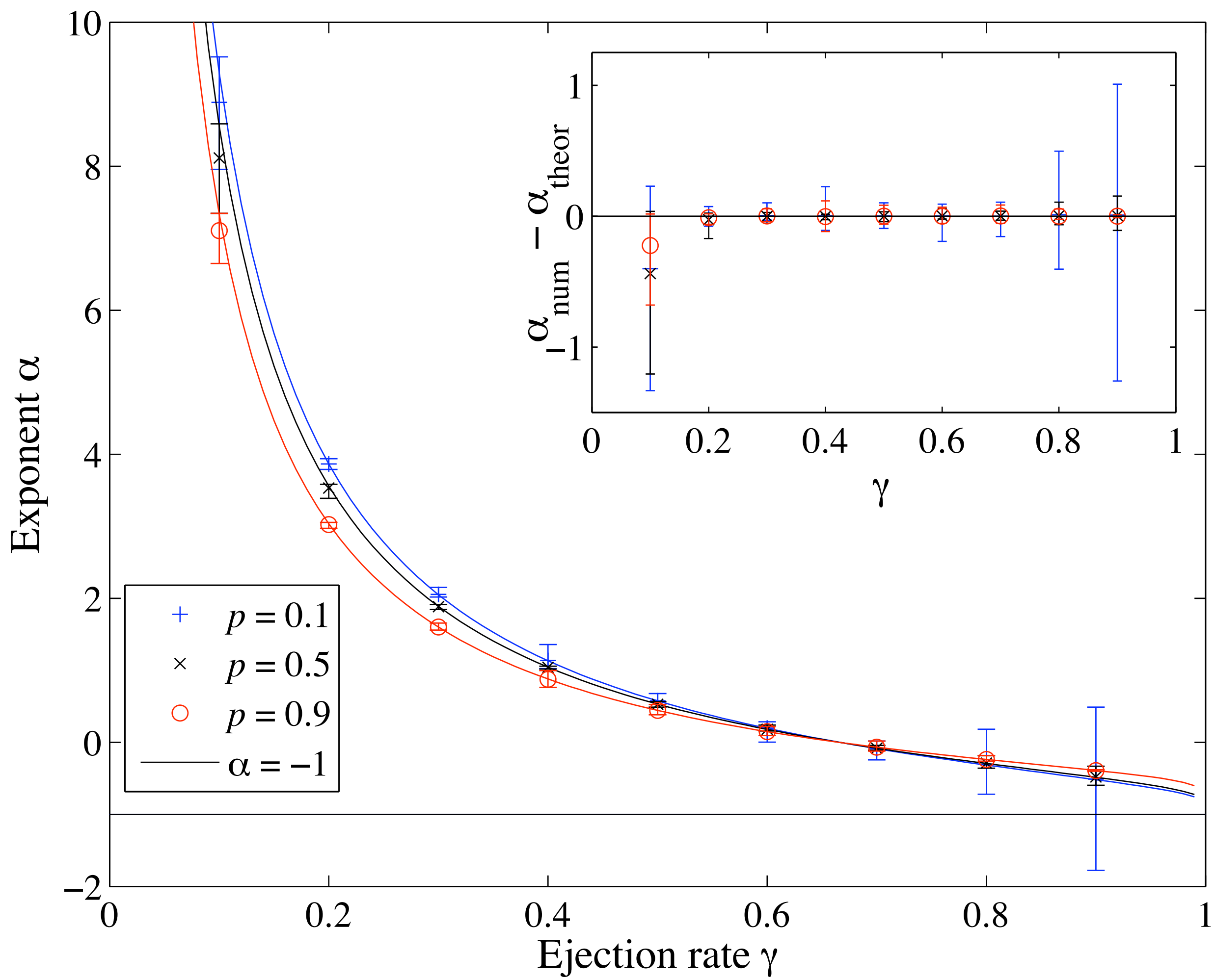
lower bound

- Dominant balance in the Markov equation

$$\frac{2p}{(1-\gamma/2)^{\alpha+1}} + \frac{(1-p)}{(1-\gamma)^{\alpha+1}} = 3\,.$$

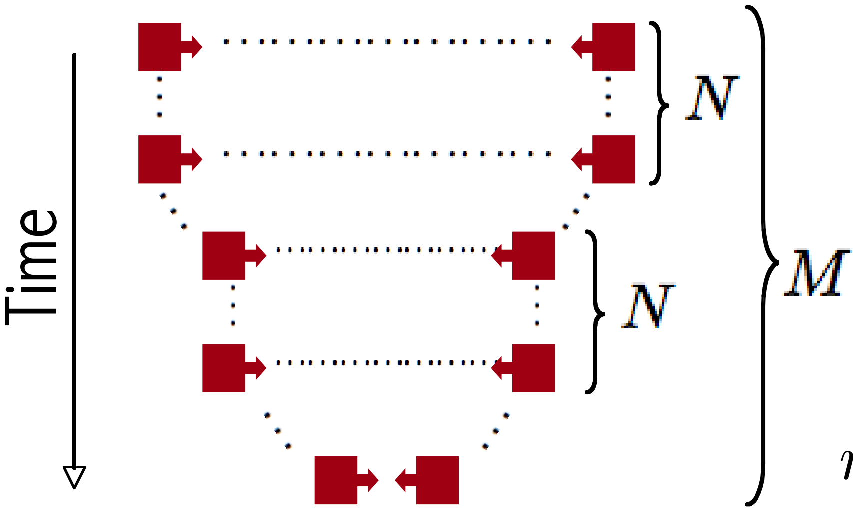
Algebraic left tail

$$\frac{2p}{(1-\gamma/2)^{\alpha+1}} + \frac{(1-p)}{(1-\gamma)^{\alpha+1}} = 3\,.$$



Super-exponential right tail

- Again 1D: realization leading to large masses



$$\text{Prob} = [p^2(1-p)]^{N\,M}$$

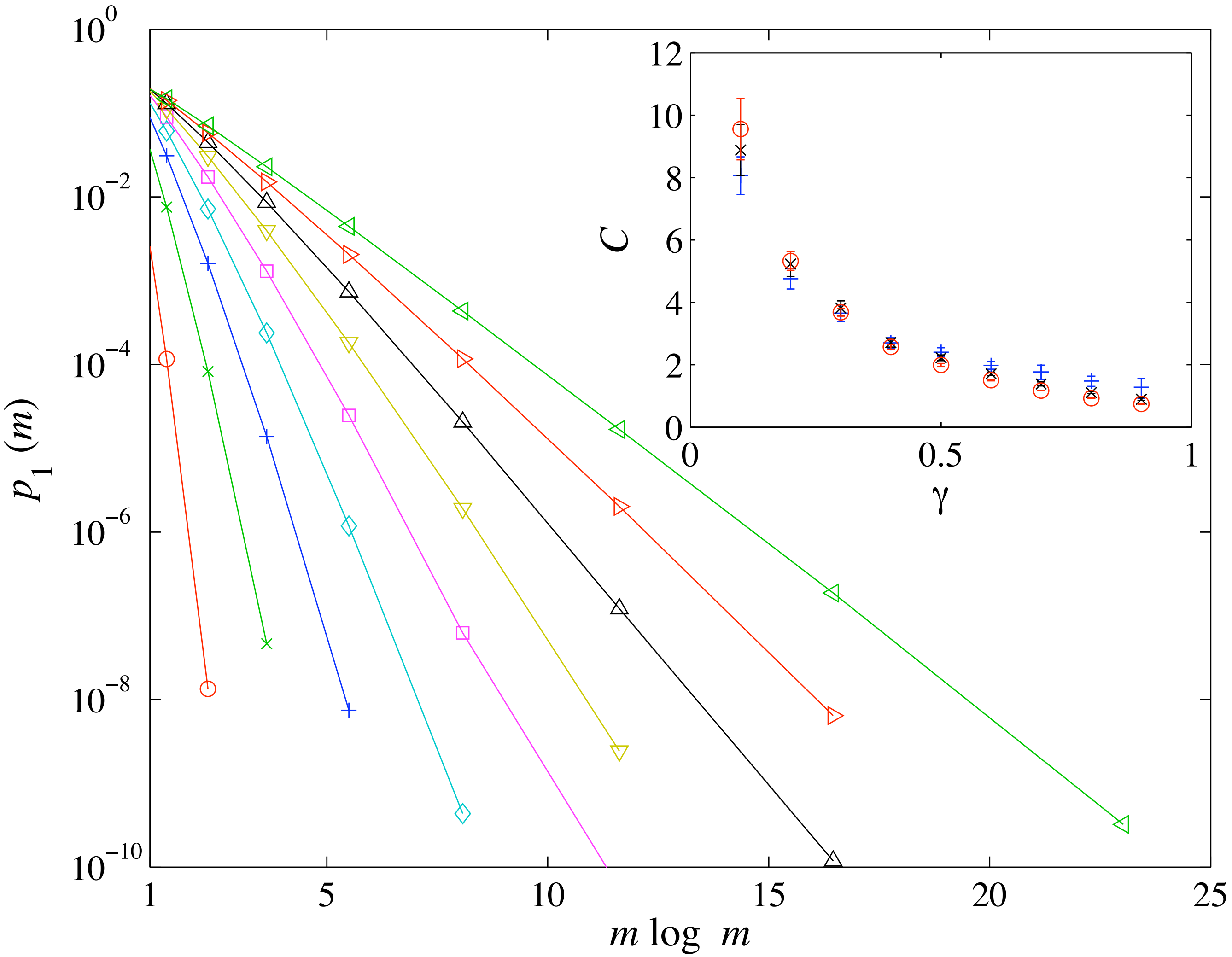
$$m_{N\,M} = \frac{1-[1-(1-\gamma)]^N]^M}{(1-\gamma)^N}$$

$$M = M(m,N) = \frac{\log\left[1-m(1-\gamma/2)^N\right]}{\log\left[1-(1-\gamma/2)^N\right]}$$

Optimization problem: find N maximizing the probability to have a mass m

$$\Rightarrow p(m) \propto \exp(-C\,m\ln m)$$

Super-exponential right tail



More general model

- **Continuous limit** $\mathrm{d}m_n = \gamma_{n+1} m_{n+1} + \gamma_{n-1} m_{n-1} - 2\gamma_n m_n$

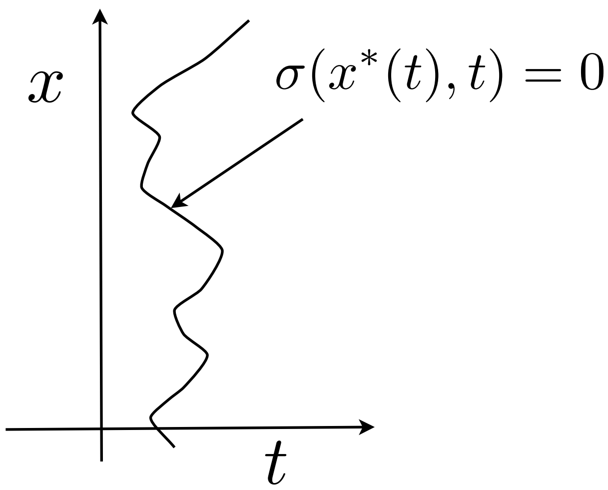
$$\left.\begin{aligned} (\mathrm{d}x^2/\mathrm{d}t)\,\gamma_n &\to \sigma^2(x,t)/2 \\ m_n/\mathrm{d}x &\to \rho(x,t) \end{aligned}\right\} \Rightarrow \boxed{\partial_t \rho = \frac{1}{2}\partial_x^2\left(\sigma^2\,\rho\right)}$$

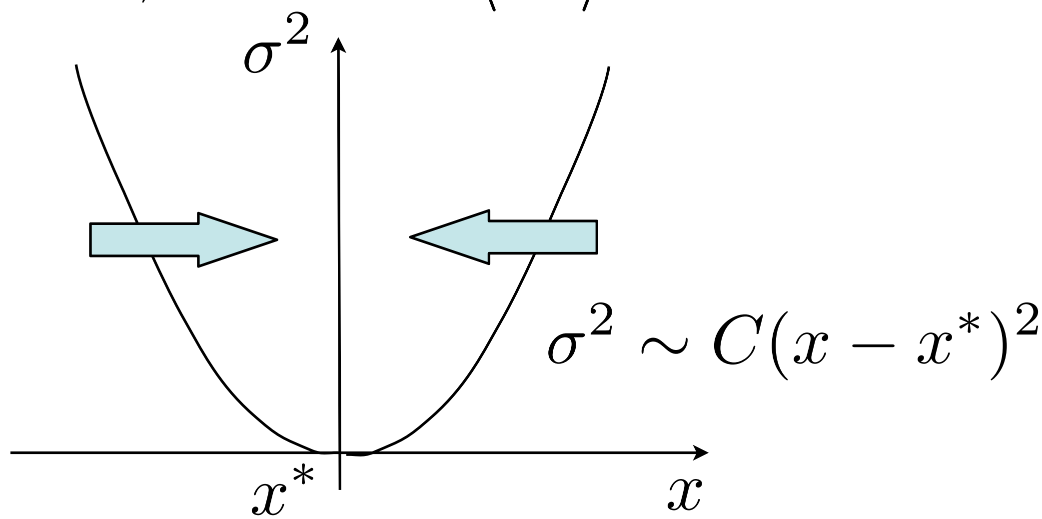
σ random ⇒ diffusion in a random environment

Green function ↔ transition probability for $\mathrm{d}X = \sigma(X,t)\,\mathrm{d}W_t$

- **High densities:** near the zeros of the diffusion coefficient σ

Example: $\sigma(x,t)$ smooth, generic, Gaussian $\langle \sigma^2 \rangle \gg 1$

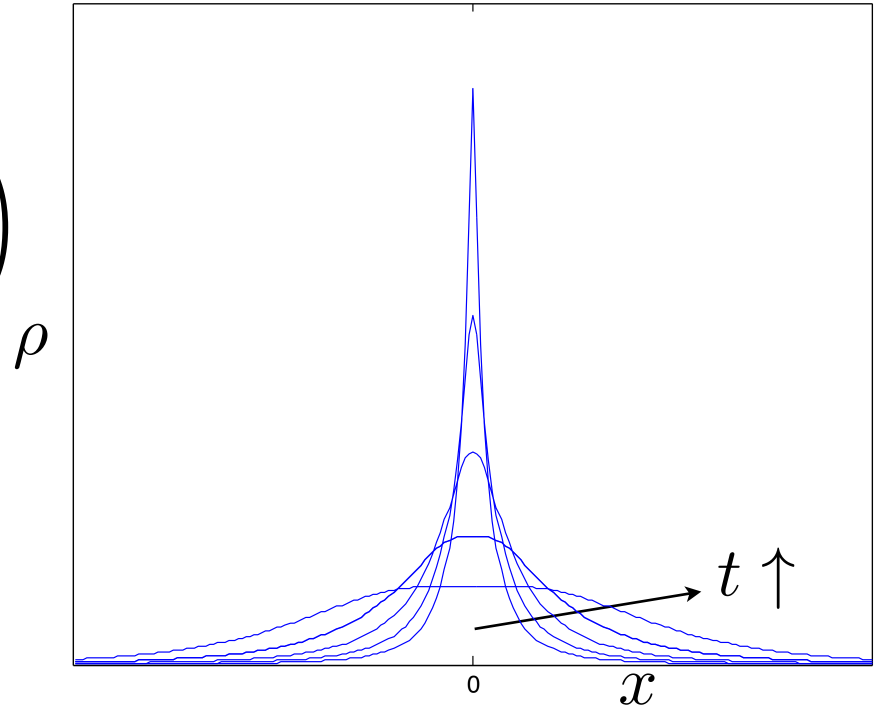




Large density tail

$$\partial_t \rho = \partial_x^2 \left(x^2 \rho\right)$$

$$\rho(x,t) = \frac{1}{2}\mathrm{e}^{2t}\mathrm{erfc}\left(\frac{\ln|x| + 3t}{2\sqrt{t}}\right)$$



Cumulative probability $P^{>}(\rho)$
= fraction of space-time where
the density is larger than ρ

$$\lambda = 2t/\ln\rho$$

$$P^{>}(\rho) \simeq \ln\rho \int_1^{\infty} \mathrm{e}^{-\mu(\lambda)\ln\rho}\mathrm{d}\lambda \qquad \mu(\lambda) = \frac{3}{2}\lambda - \sqrt{2\lambda(\lambda-1)}$$

Saddle-point: $P^{>}(\rho) \propto \rho^{-1}$

Universal intermediate asymptotics where $p(\rho) = -\frac{\mathrm{d}P^{>}}{\mathrm{d}x} \propto \rho^{-2}$

+ non-universal cut-off (distribution of zeros lifetime)

Conclusions

Summary

Clustering of heavy particles:
of two kinds, depending on the observation scale:

* multifractal in the dissipative range
* dependent on a rescaled contraction rate in the inertial range

Connection to random walks in random environments:
ejection models reproduce most features (in particular tails)

Open questions

* **Universality** of mass probability distribution?
* **Spatial/temporal correlations** (e.g. scale invariance) of the ejection/diffusion rate?