Roberto Artuso

Stickiness, correlations and large deviations

Sperlonga, September 22, 2010

Collaborators

Cesar Manchein - Curitiba

Lucia Cavallasca - Como

Giampaolo Cristadoro - Bologna

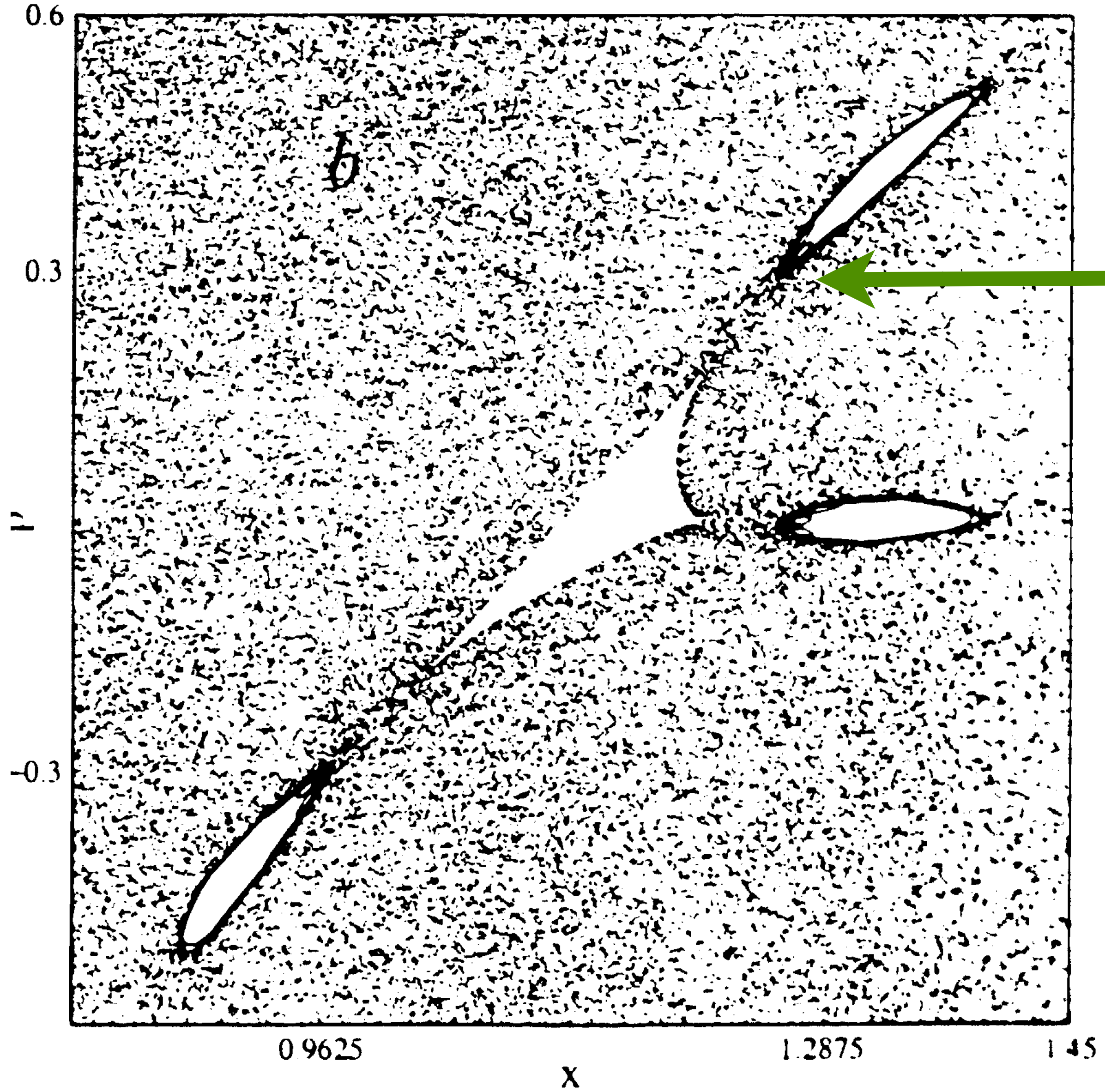
Acknowledgements

Xavier Leoncini - Marseille

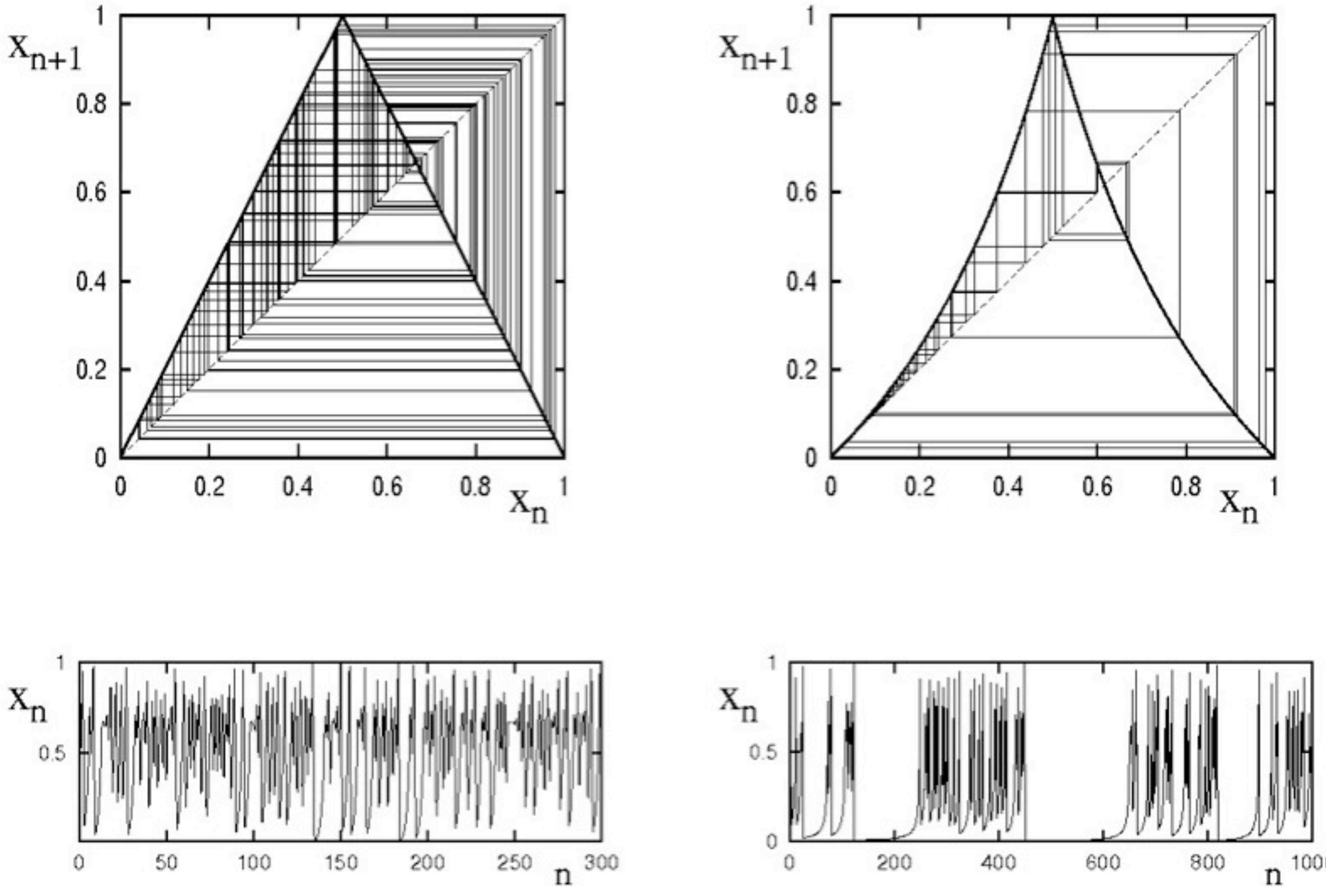
Sandro Vaienti - Marseille

What's this?

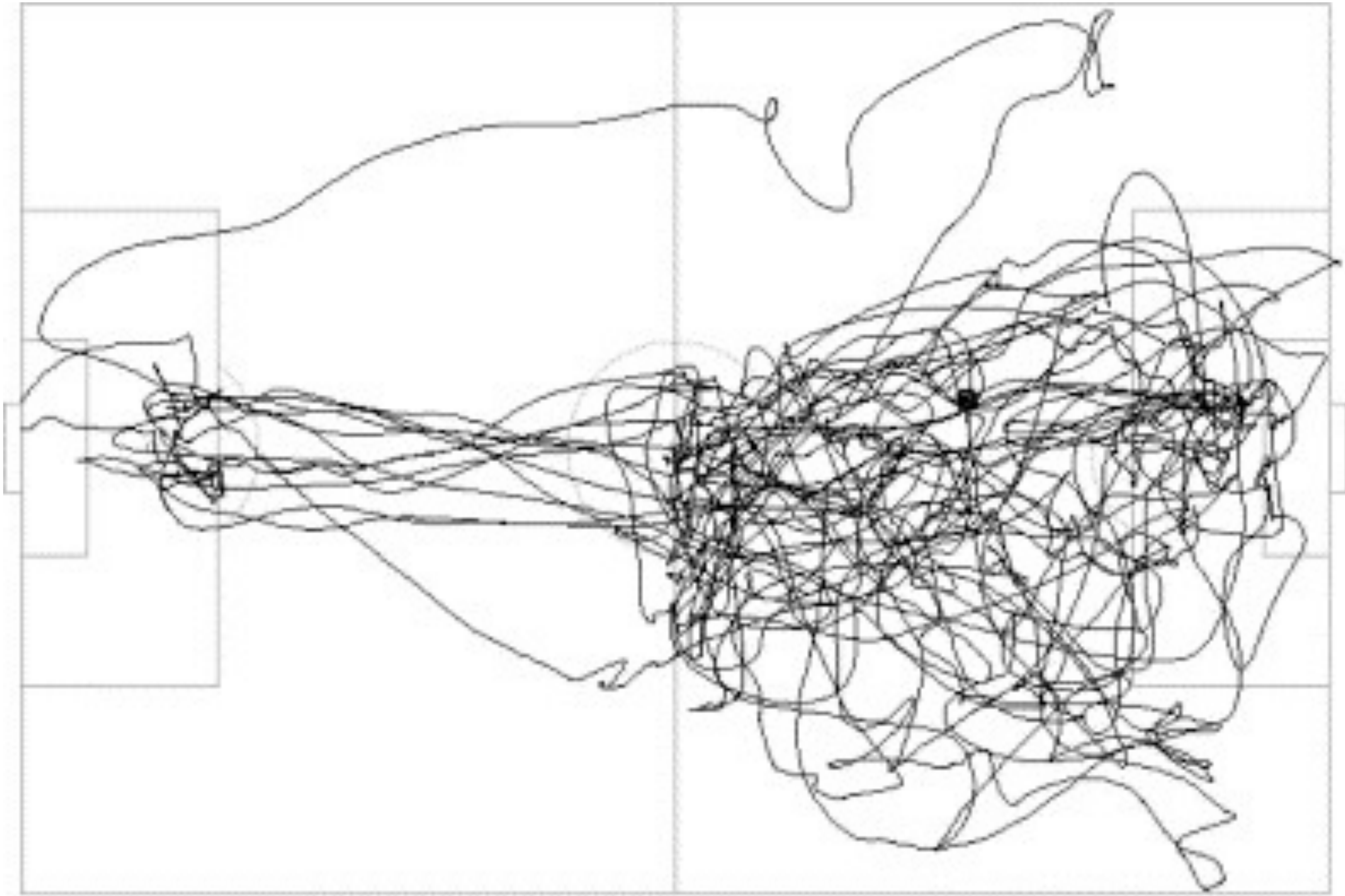
Correlation decay for weakly chaotic dynamical systems



sticking enhances correlations



x_{n+1}
x_n
0
0.2
0.4
0.6
0.8
1
x_n
0.5
n
0
50
100
150
200
250
300
400
600
800
1000



Available online at www.sciencedirect.com

SCIENCE DIRECT®

Computer Vision and Image Understanding 101 (2006) 122–135

Computer Vision and Image Understanding

www.elsevier.com/locate/cviu

Tracking soccer players aiming their kinematical motion analysis

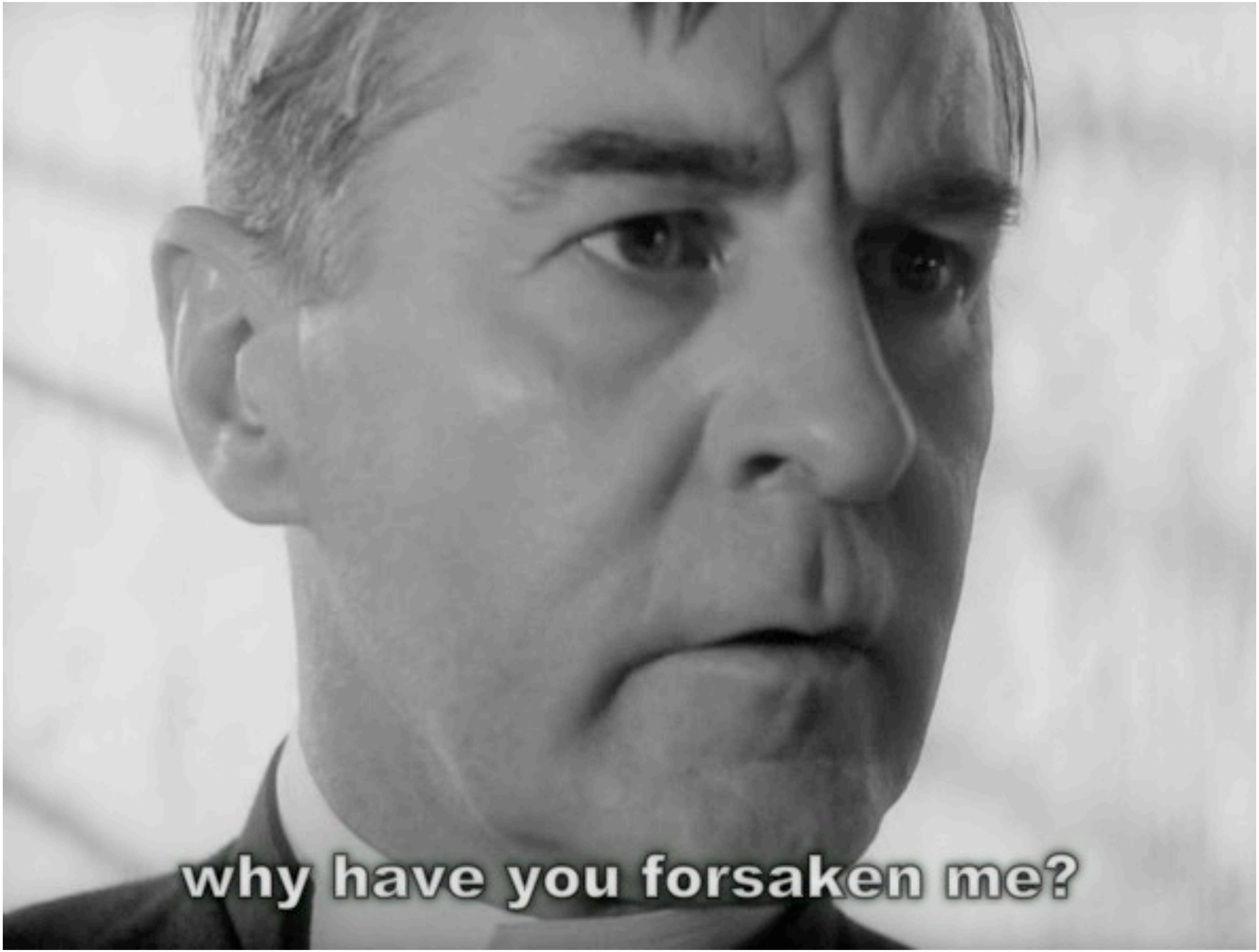
Pascual J. Figueroa [a], Neucimar J. Leite [a,*], Ricardo M.L. Barros [b]

Estimates and computation of correlations

Spectral methods: Perron-Frobenius operator, zeta functions techniques

Inducing: Poincaré recurrences, waiting time distributions, Markov towers

Adding noise: reading asymptotic noiseless limit by transients


why have you forsaken me?

"Generic" correlation decay?

Crawford, Cary, Collet, Isola... modes and rates of decay depend crucially on smoothness properties of observables

Physica 6D (1983) 223–232
North-Holland Publishing Company

DECAY OF CORRELATIONS IN A CHAOTIC MEASURE-PRESERVING TRANSFORMATION*

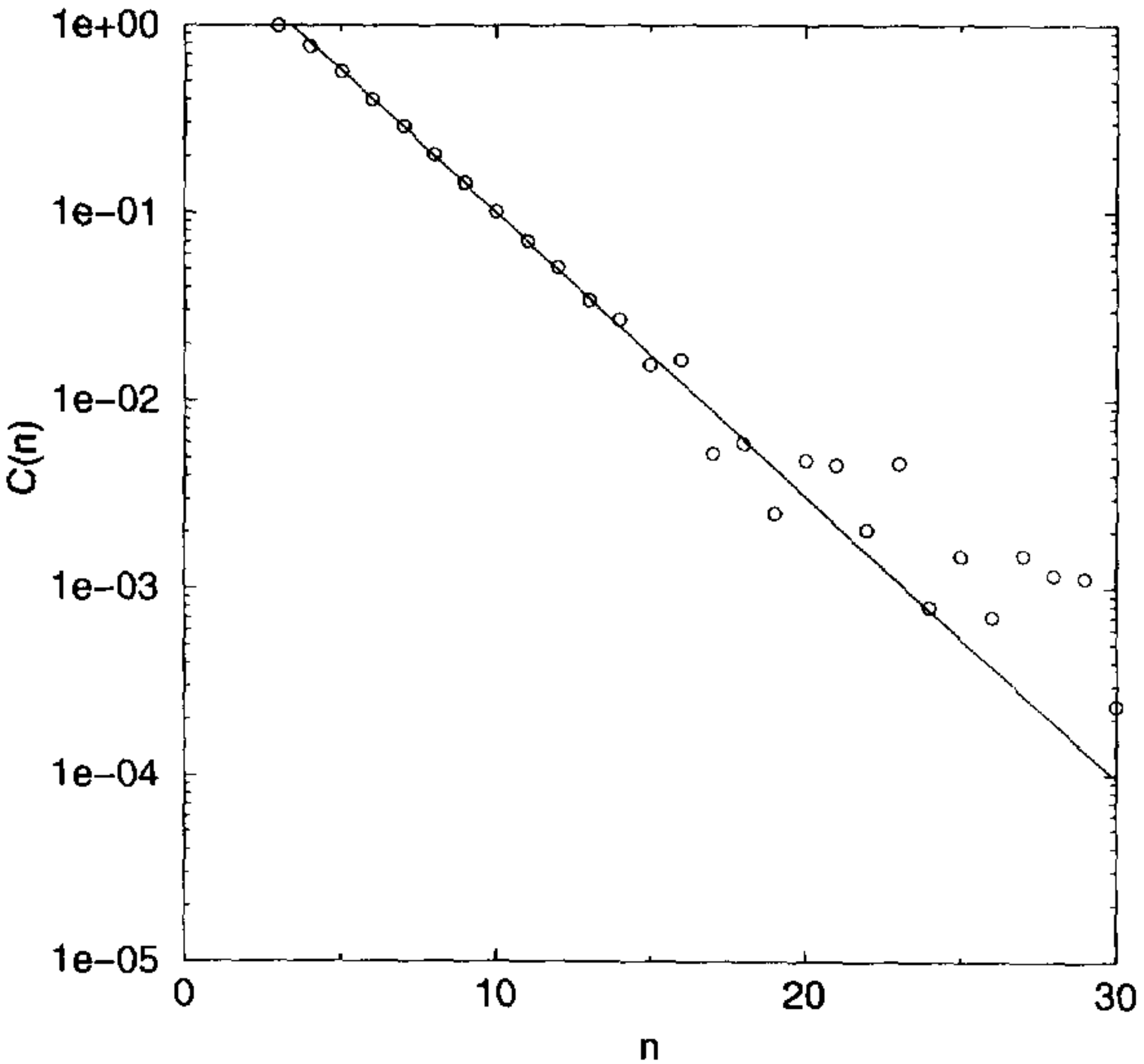
John David CRAWFORD and John R. CARY†

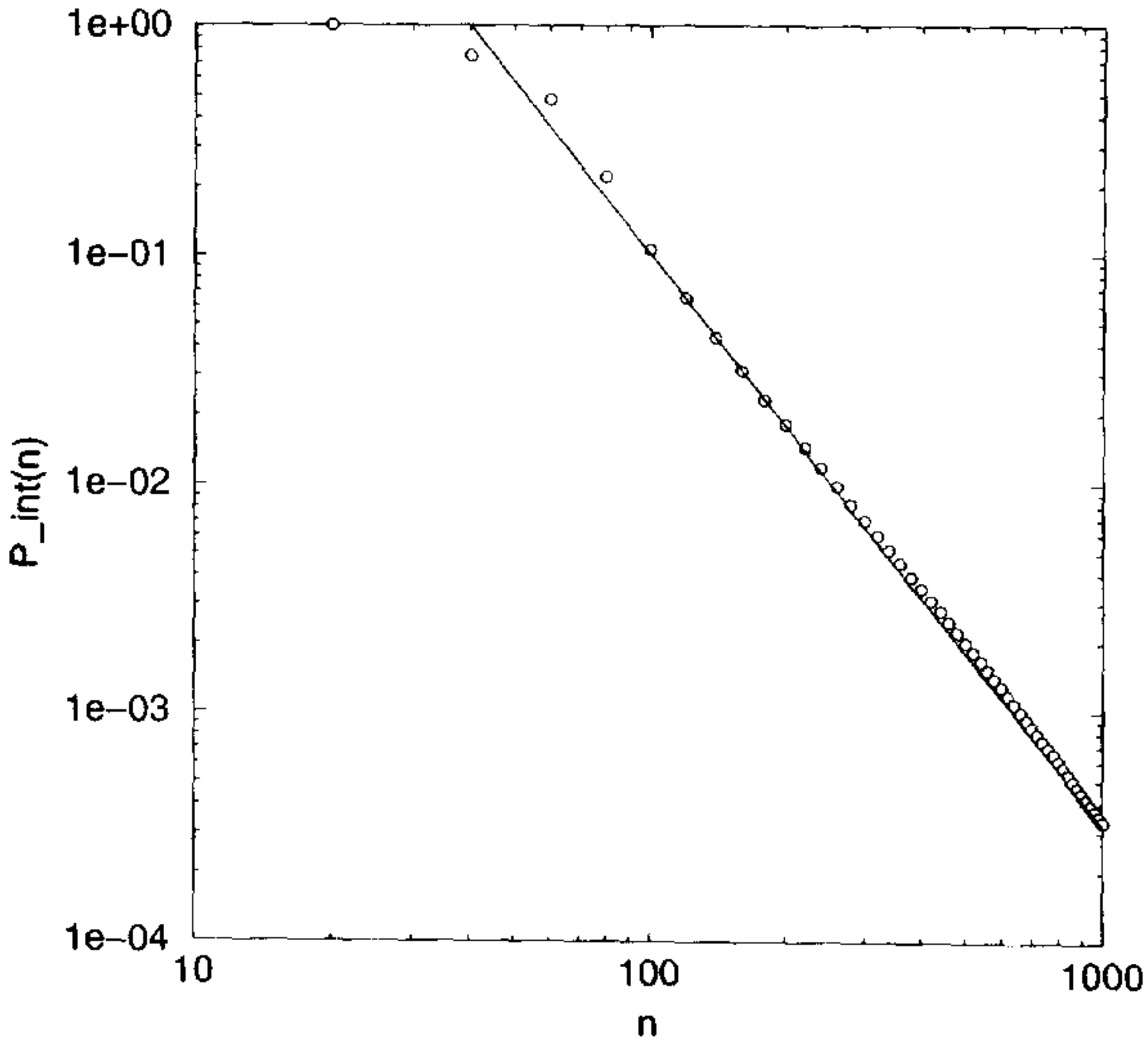
Lawrence Berkeley Laboratory University of California Berkeley, CA 94720, USA

Received 8 March 1982

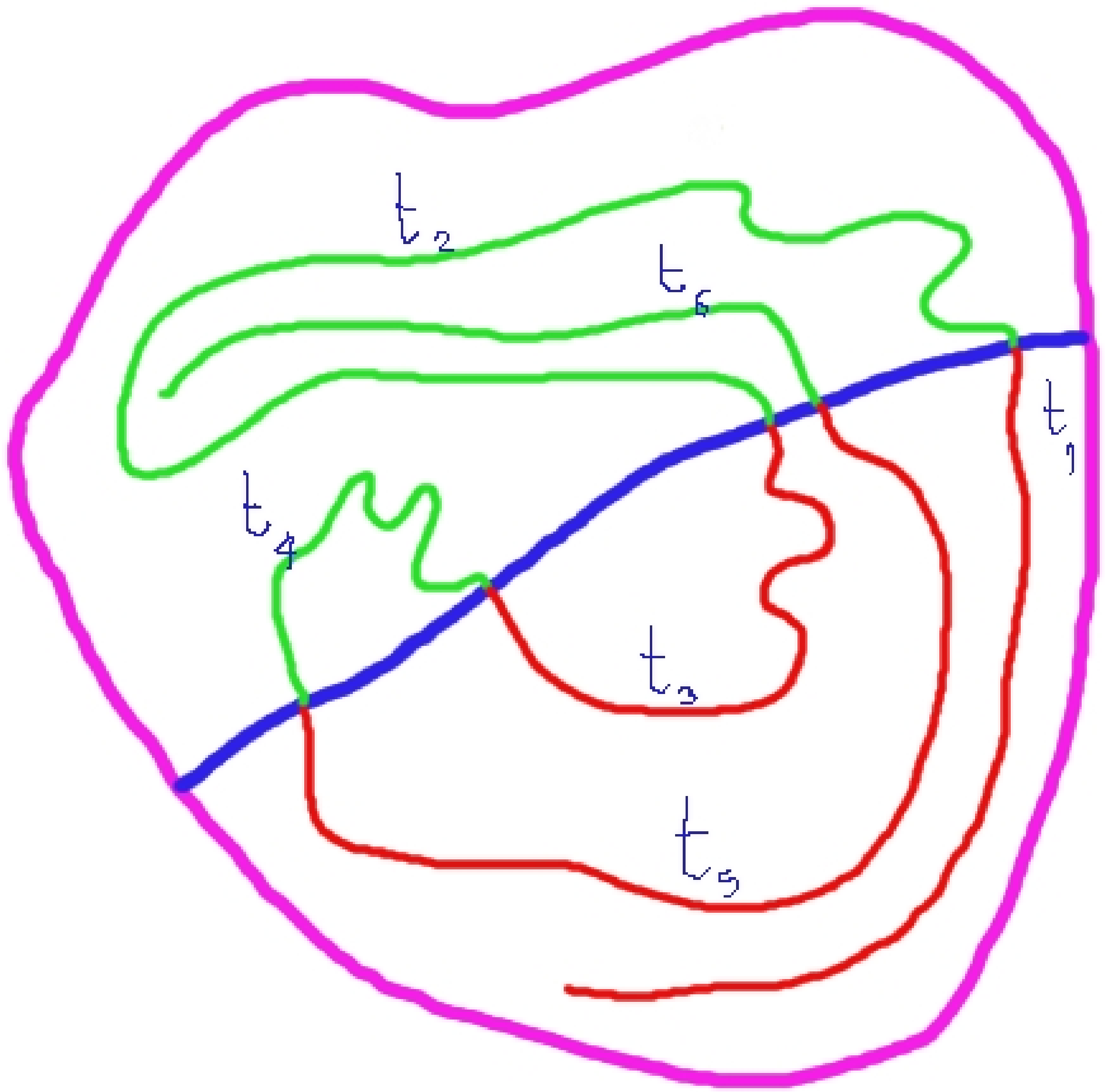
For a chaotic, area-preserving map on the torus, we study the decay of correlations in detail. Taking as observables the square-integrable functions, we find examples of decay rates which are algebraic, exponential, and faster than exponential. For correlations that decay exponentially the rate is sensitive to the choice of function. The implications for numerical experiments of this nonuniformity in the decay are discussed.

noisiness of numerical data





Residence times distribution



$\psi(t)$

Waiting times and correlations

Correlation(n) ~ Probability that two points, chosen at random n-steps apart belong to the same residence sequence

$$C(n) \sim \frac{1}{\langle n \rangle} \left(\psi(n) + 2\psi(n+1) + 3\psi(n+2) \cdots \right)$$

$$C(n) \sim \frac{1}{\langle n \rangle} \int_n^\infty dt \int_t^\infty d\tau \, \psi(\tau)$$

Channon, Lebowitz, Chirikov, Shepelyansky, Karney

P. Dahlqvist, R. Artuso / Physics l

Baladi. Eckmann, Ruelle, Dahlqvist, RA

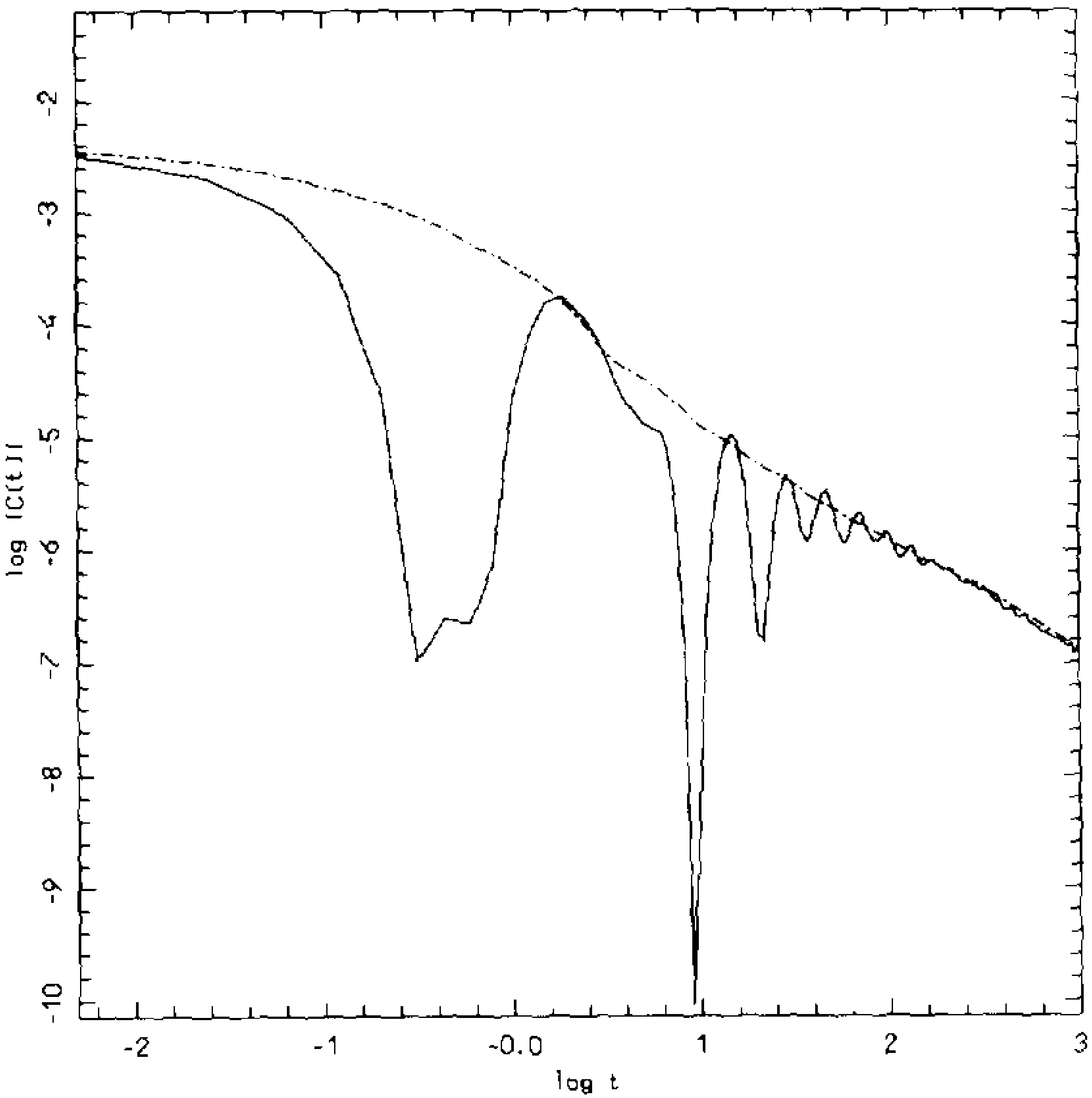


Fig. 2. Experimental correlation function (full line), for $R = 0.318$. The dash-dotted line represents the BER approximation (15), using a numerical $p(\Delta)$.

Short time dynamics is far more complex but asymptotic behavior is correctly reproduced

$$C_{AA}(t) = P_0(t)\,\langle A^2\rangle + [1 - P_0(t)]\,\langle A\rangle^2 - \langle A\rangle^2$$
$$= P_0(t)\,V(A), \qquad (11)$$

Area-preserving map

Prototype example: parabolic fixed point

$$T(x,y) = \begin{cases} 2x - \sin(x) + y \\ y + x - \sin(x) \end{cases}$$

(0,0) is a parabolic fixed point

$\sim x^3$ z=2 intermittency?

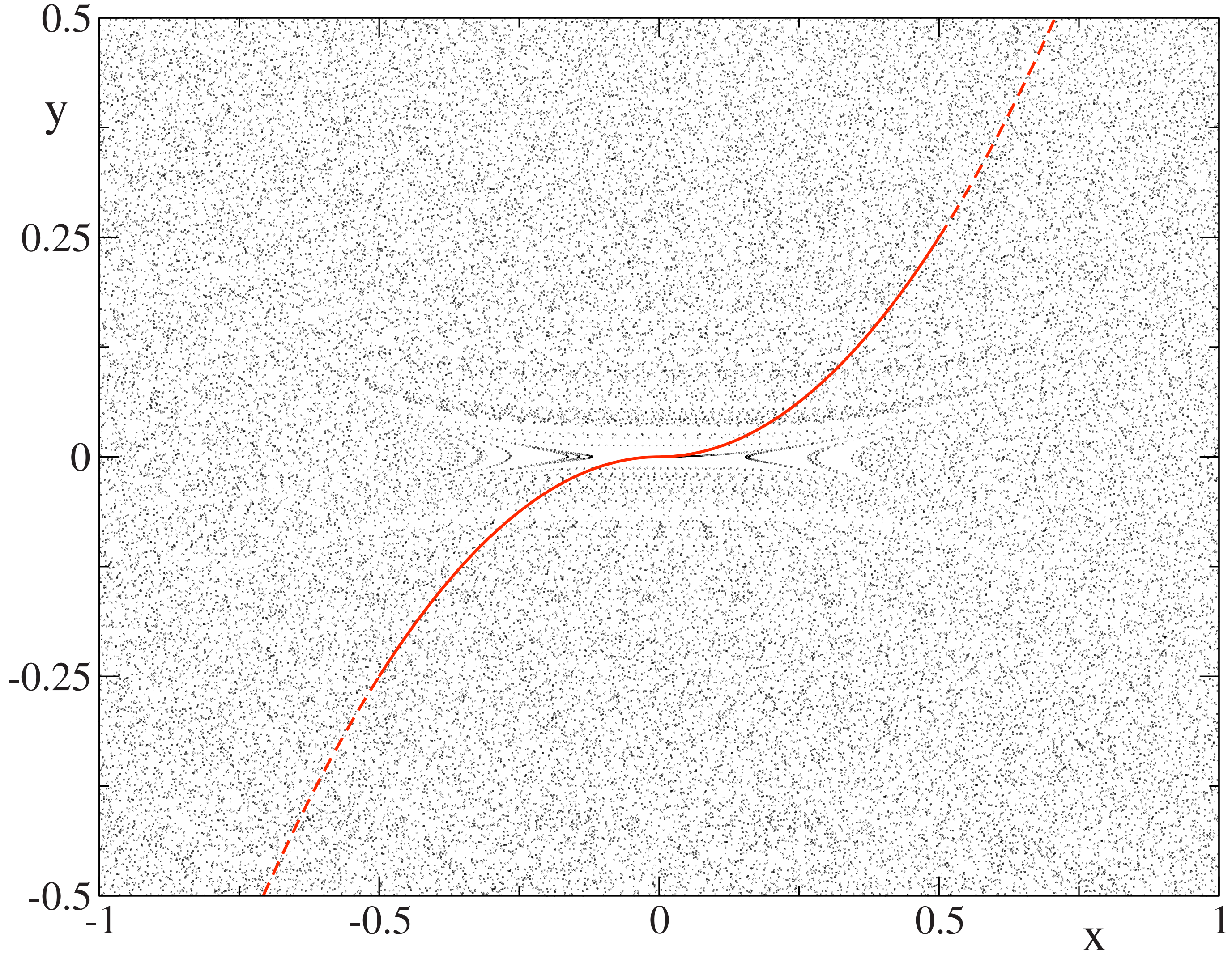
Lewowicz, MacKay, RA, Prampolini, Liverani

1-parameter family

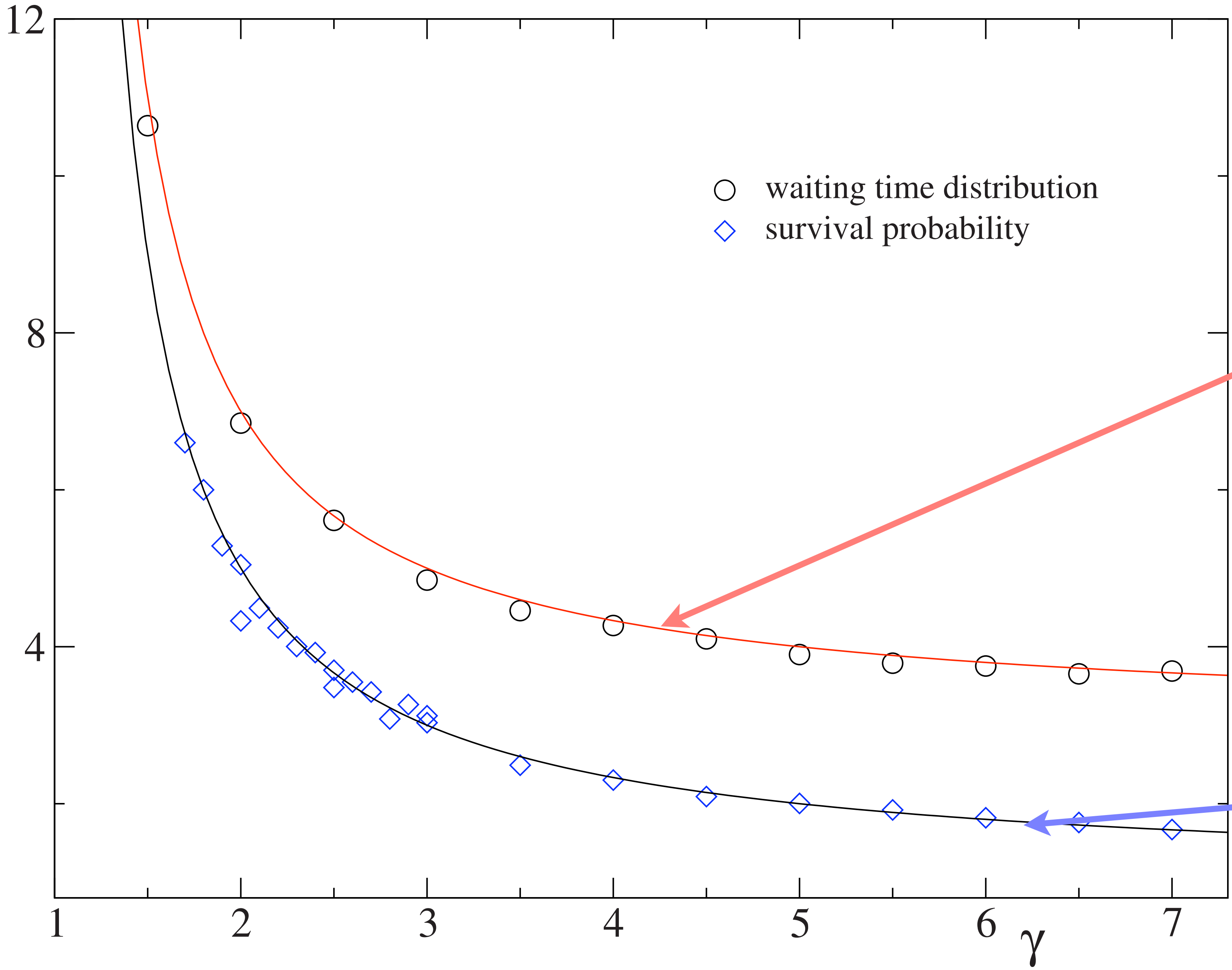
(Frigerio, Guarneri; RA, Cavallasca, Cristadoro)

$$T_z(x,y) = \begin{cases} x + f_z(x) + y & \text{on } \mathbb{T} \\ y + f_z(x) & \text{on } \mathbb{T} \end{cases}$$

$$f_z(x) \sim x^z$$



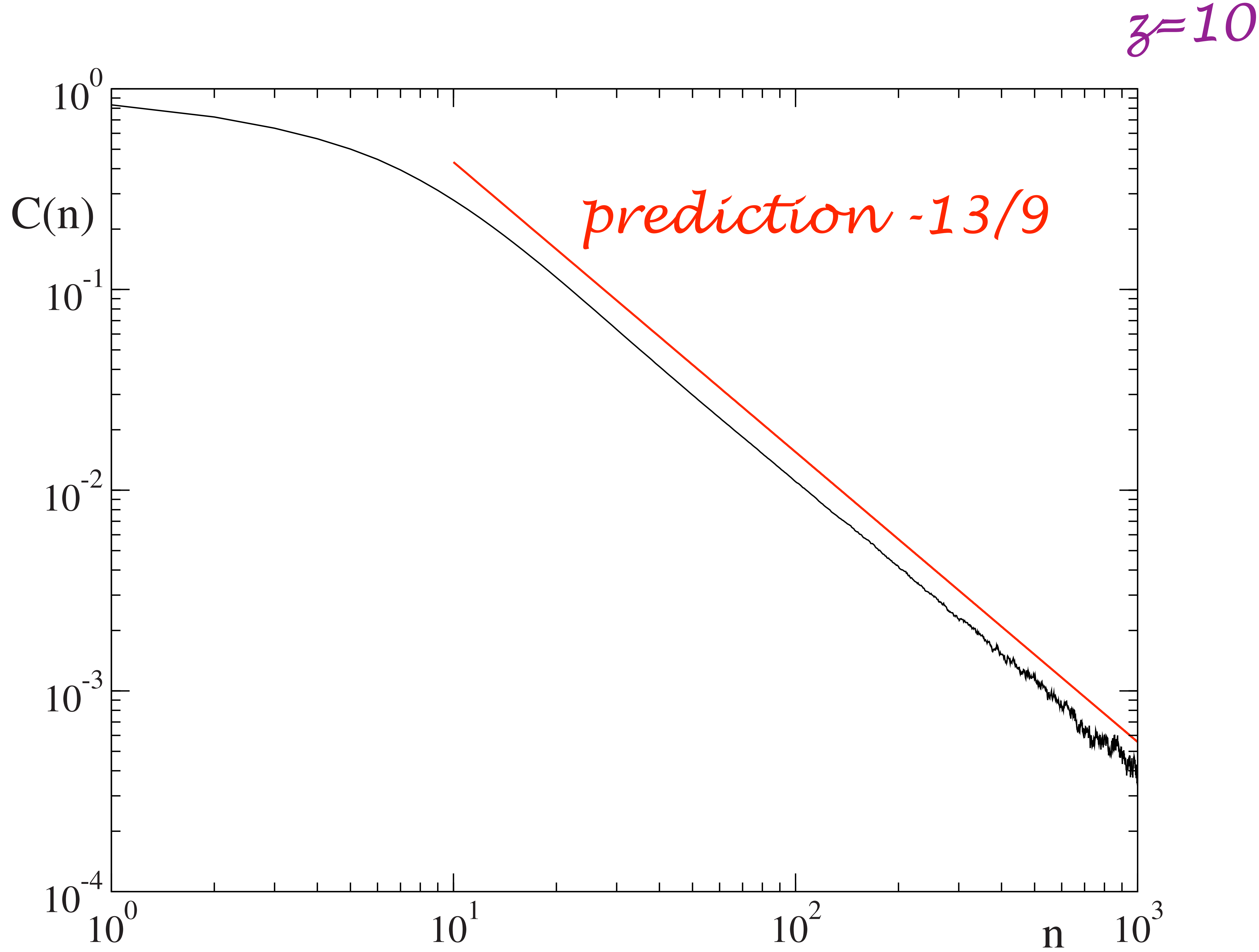
RA, Cavallasca,Cristadoro



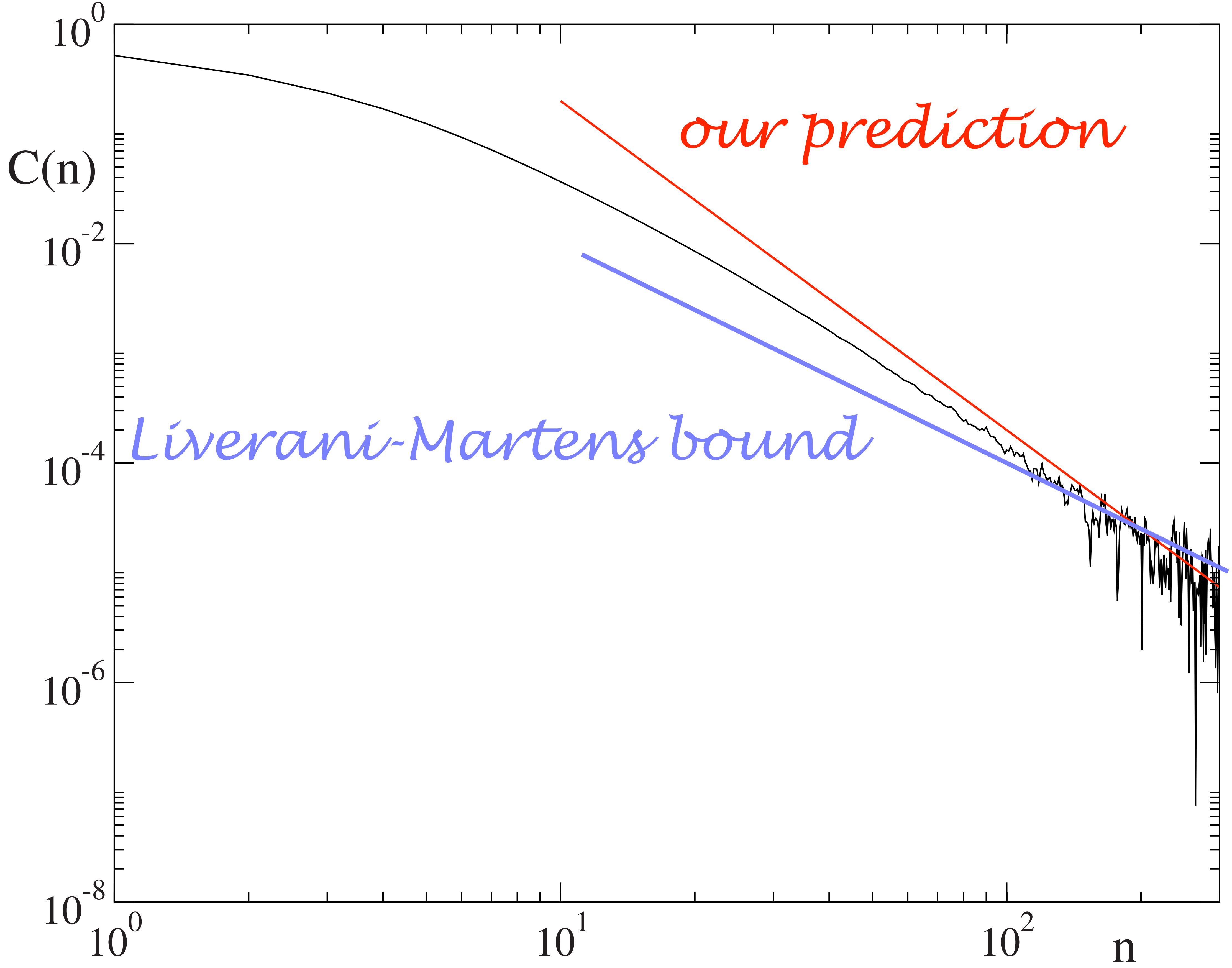
$$\psi_\Omega(n) \sim n^{-\frac{3z+1}{z-1}}.$$

and

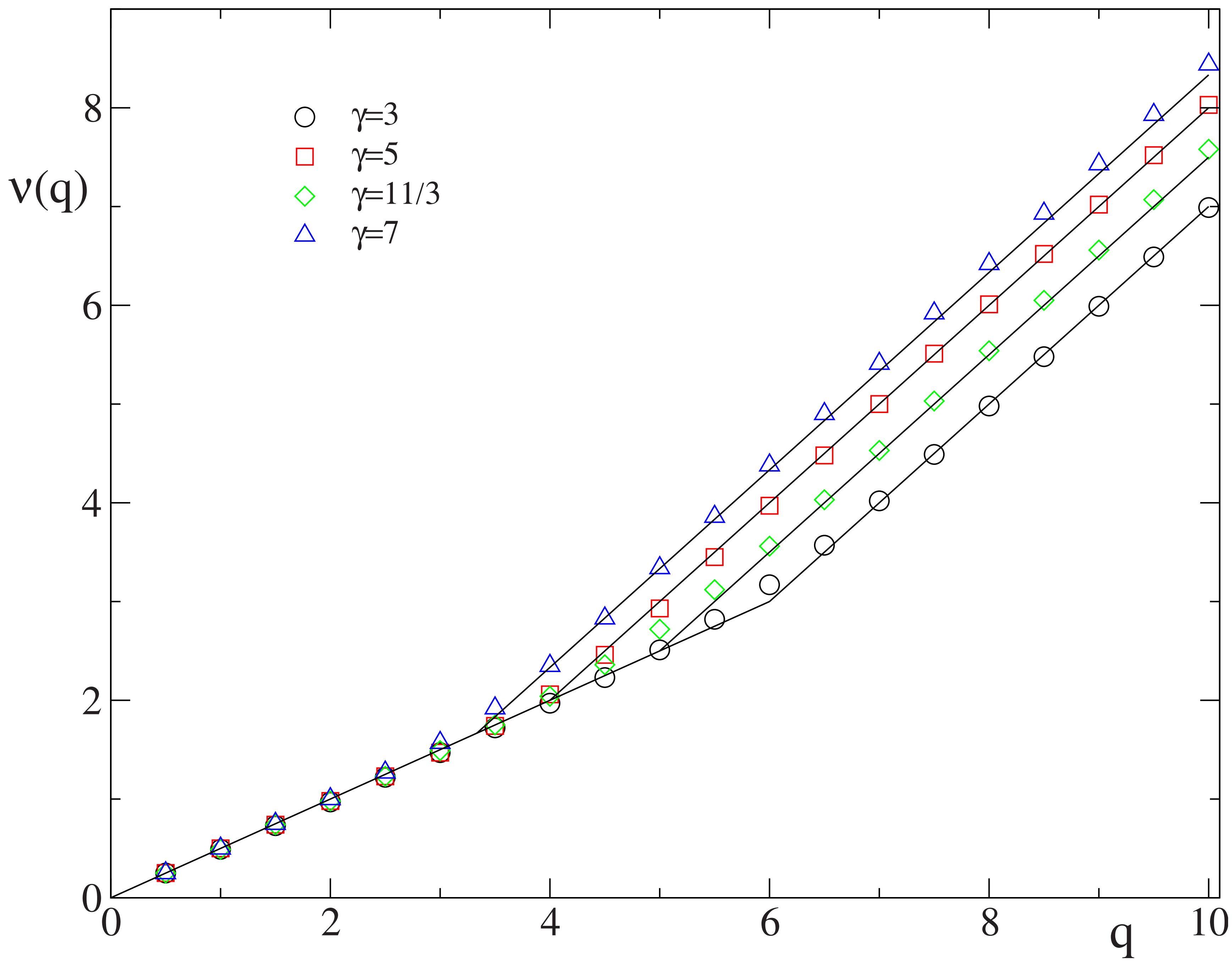
$$C_g(n) \sim n^{-\frac{z+3}{z-1}}$$


z=10
C(n)
prediction -13/9
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^0
10^1
10^2
10^3
n

z=3


C(n)
n
our prediction
Liverani-Martens bound
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^0
10^1
10^2

Associated non trivial transport properties



Statistics of Finite-time Lyapunov

-Detection of small islands

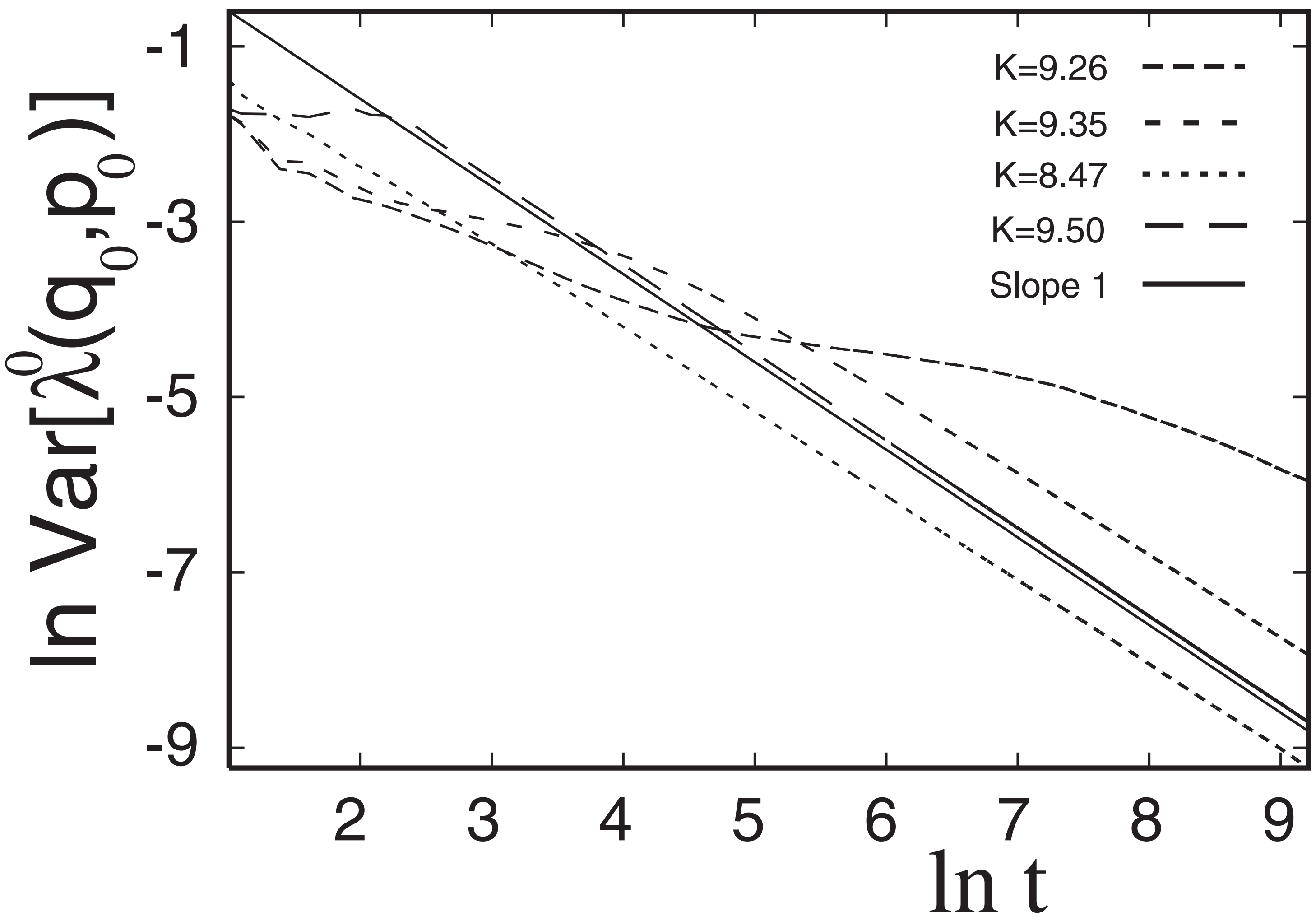


FIG. 6. The scaling of the variance for an ensemble of 40000 initial conditions uniformly distributed in the square $\mathcal{A}$ $=\{(0.10,0.10),(0.15,0.15)\}$.

Tomsovic, Lakshminarayan

Large deviations and correlations

Originally proposed for 1d intermittent maps

$$\lambda_n(x_0) = \frac{1}{n} \ln \left| \left. \frac{df^{(n)}(x)}{dx} \right|_{x_0} \right|$$

with corresponding probability distribution P_n

Fix a threshold $\tilde{\lambda} < \lambda$

$$\mathcal{M}_{\tilde{\lambda}}(n) = \int_{-\infty}^{\tilde{\lambda}} d\lambda_n \, P_n(\lambda_n).$$

The idea is to connect the shrinking of such a tail to correlation decay

$$\mathcal{M}_{\tilde{\lambda}}(n) \sim \frac{1}{n^{\xi}}$$

$$\Downarrow$$

$$\mathcal{C}(n) \leq \frac{1}{n^{\xi-1}}$$

Alves, Luzzatto, Pinheiro

Birkhoff averages and correlations

$$\mu\left(x \,\middle|\, |n^{-1}\sum_{k=0}^{n-1}\phi(f^{(k)}(x)) - \overline{\phi}| > \epsilon\right) \leq C_{\phi,\epsilon}\frac{1}{n^{\xi}}$$

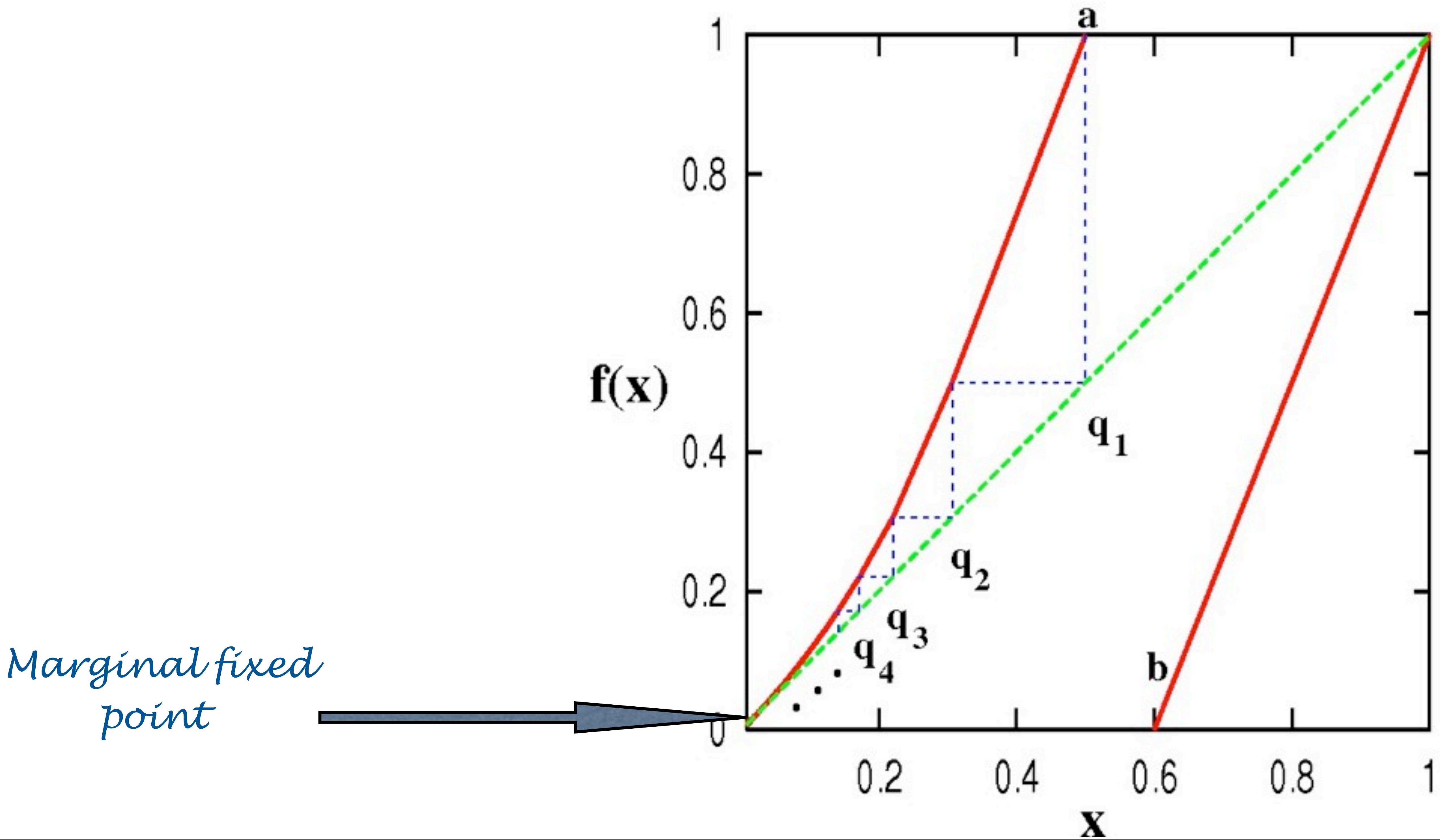
Polynomial large deviations for bounded functions

Melbourne

Pollicott, Sharp

An intermittent paradigm: Pomeau-Manneville

$$x_{n+1} = x_n + x_n^z$$



Periodic orbits instability growth

Hyperbolic orbits $\Lambda_n \sim \sigma^n$

Sticking orbits $\Lambda_{0^n 1} \sim n^{z/(z-1)}$

→ broaden the distribution of finite-time averages

→ change the spectral properties of PF

PHYSICAL REVIEW E **80**, 036210 (2009)

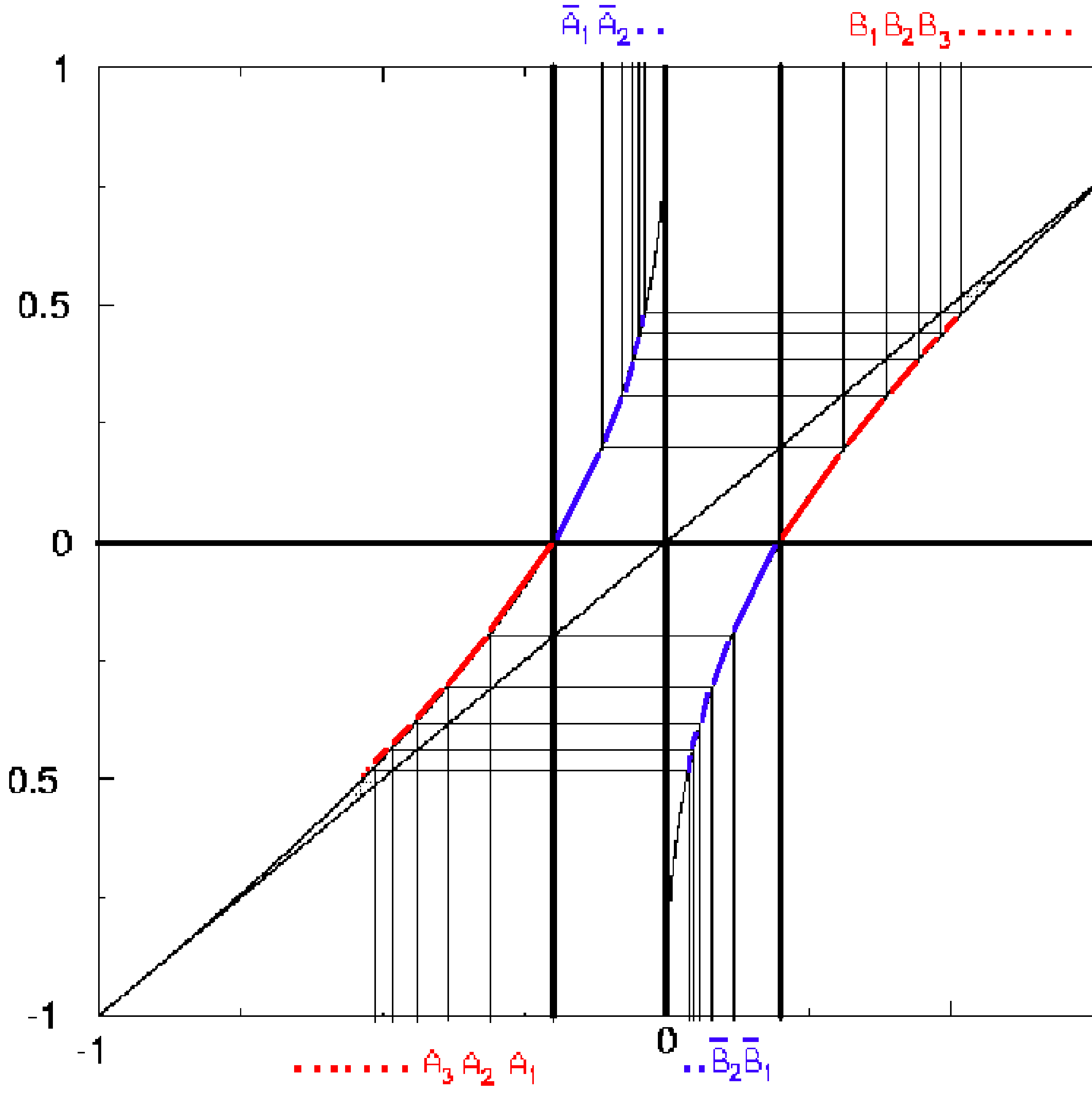
Instability statistics and mixing rates

Roberto Artuso[1,2,*] and Cesar Manchein[1,3,†]
[1] *Center for Nonlinear and Complex Systems and Dipartimento di Fisica e Matematica, Università degli Studi dell'Insubria, Via Valleggio 11, 22100 Como, Italy*
[2] *I.N.F.N. Sezione di Milano, Via Celoria 16, 20133 Milano, Italy*
[3] *Departamento de Física, Universidade Federal do Paraná, 81531-980 Curitiba, Paraná, Brazil*
(Received 3 June 2009; revised manuscript received 7 August 2009; published 21 September 2009)

We claim that looking at probability distributions of *finite time* largest Lyapunov exponents, and more precisely studying their large deviation properties, yields an extremely powerful technique to get quantitative estimates of polynomial decay rates of time correlations and Poincaré recurrences in the-quite-delicate case of dynamical systems with weak chaotic properties.

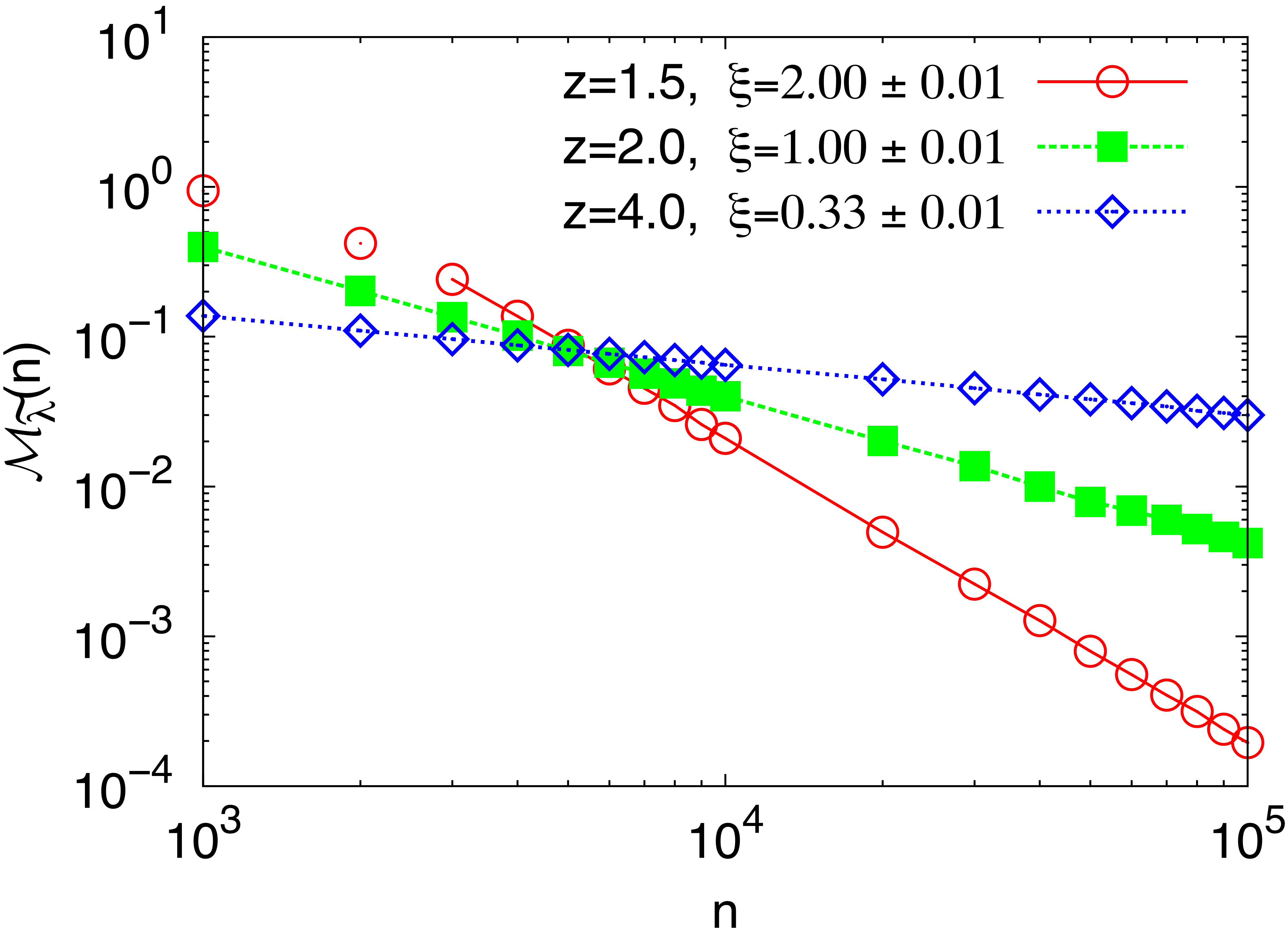
check of the performance of the method

Pikovsky map

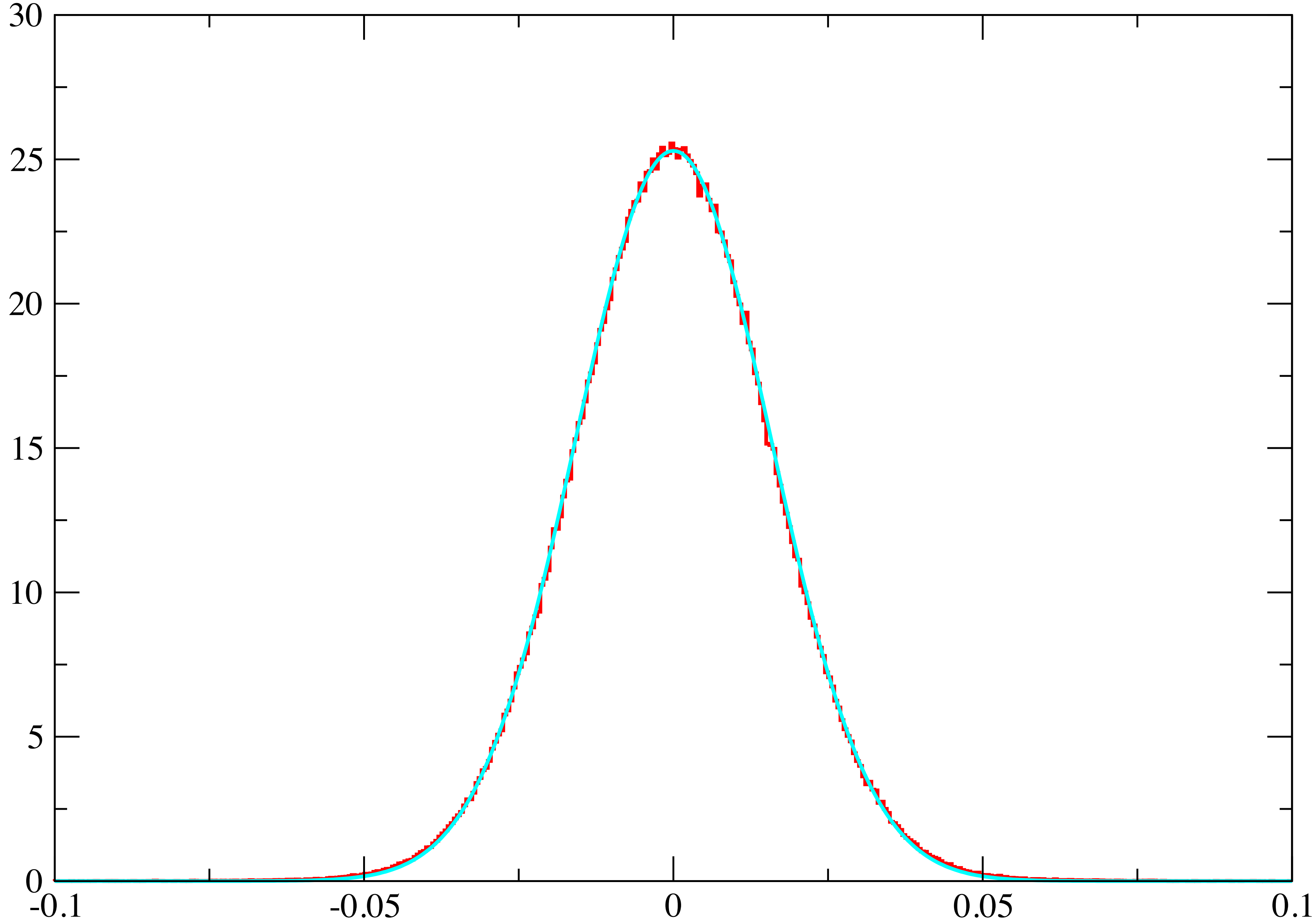


$$\mathcal{C}(n) \sim n^{-1/(z-1)}$$

Large deviations estimates



Pikovsky map in the regime of integrable correlations



Convergence to a normal law z<2

2d parabolic map

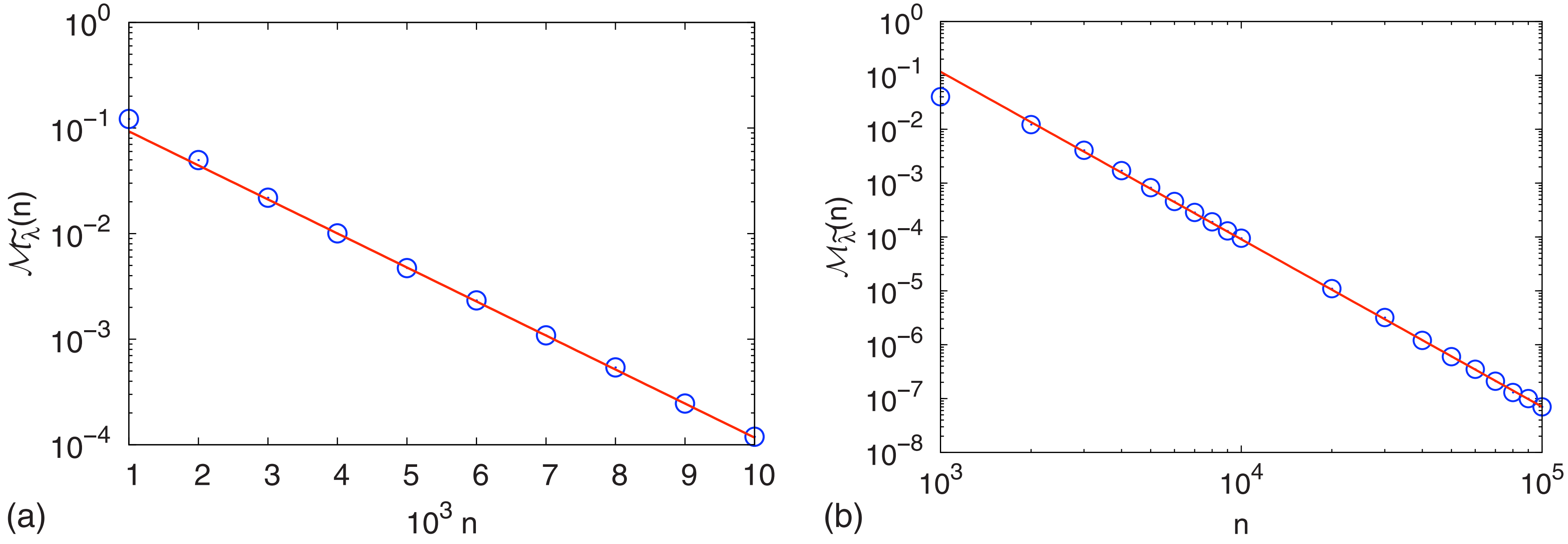


FIG. 4. (Color online) Decay $\mathcal{M}_{\tilde{\lambda}}(n)$ (symbols) together with a regression fit (full lines) for map (8) with (a) $\varepsilon=0.5$ and $\gamma=1$ (exponential rate decay 0.87 ± 0.01) and (b) $\varepsilon=0$ and $\gamma=1$ (polynomial rate decay 3.05 ± 0.05). Both fits were done by starting at $n=3000$.

$$y_{n+1} = y_n + f(x_n) \qquad \mod 2\pi,$$
$$x_{n+1} = x_n + y_{n+1} \qquad \mod 2\pi,$$

$$f(x_n) = [x_n - (1-\varepsilon)\sin(x_n)]^\gamma$$

Conclusions (if any)

The study of correlations for weakly chaotic systems is still a hard problem

Large deviations represent a mathematically sound approach to such a problem, and a numerically robust procedure