EMERGENT PATTERNS IN SUSPENSIONS OF MOLECULAR MOTORS

I. Pagonabarraga
University of Barcelona

1. Introduction

2. Collective transport of molecular motors

3. Cooperative motion under confinmenet

4. Conclusions

1. Introduction

Understanding motion at mesoscale

How does collective transport emerges in systems which
move due to internal consumption of energy?

Generic features due to
dynamic coupling to the environment?

Basic physical mechanisms
(over)simplified geometries / models
neglect any specific coupling

1. Introduction

Molecular motors

Protein complexes: make use of ATP to change their conformation
Mechano/chemical coupling: generate net motion

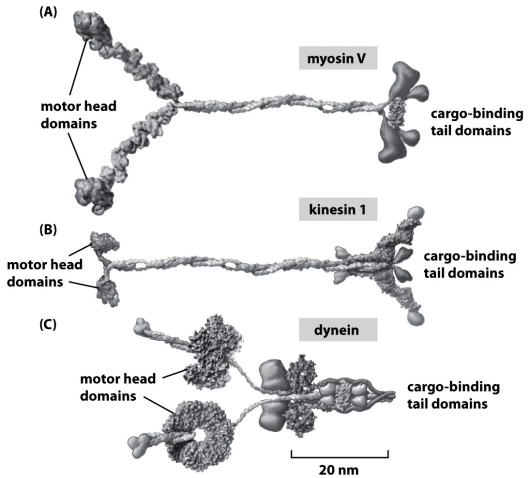


Figure 16.2 Physical Biology of the Cell (© Garland Science 2009)

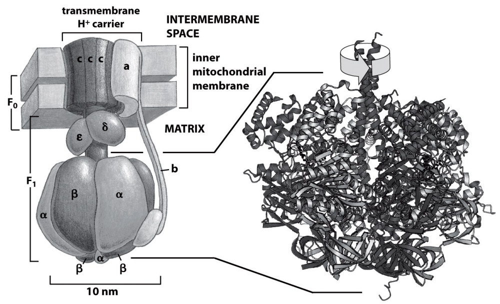


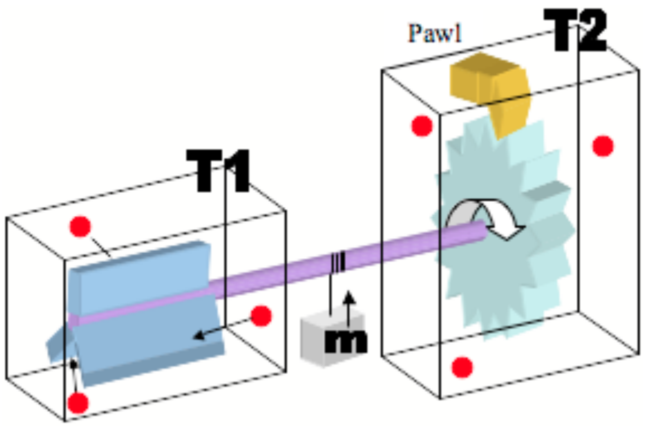
Figure 16.13b Physical Biology of the Cell (© Garland Science 2009)

Attract to biopolymers
polarity induces motion: net displacement

1. Introduction

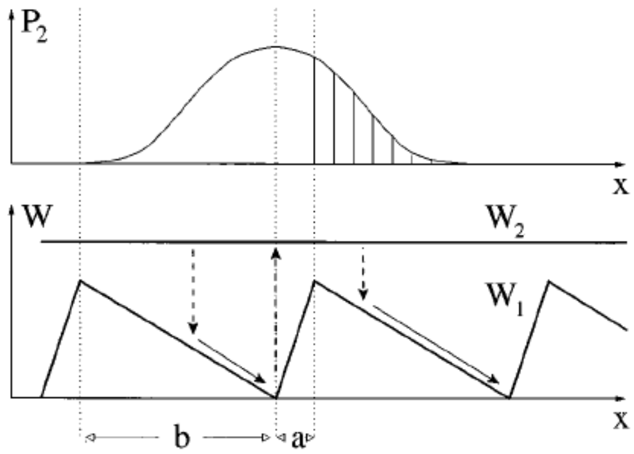
How do molecular motors move?
Brownian ratchet analogy

Asymmetry + no-detailed balance



Smoluchowsky

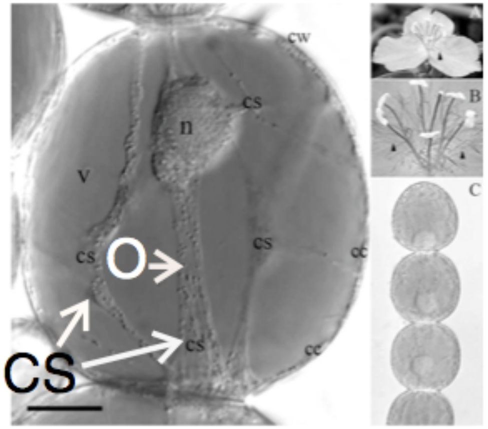
Feynman



Asymmetry in jump rates
biases diffusion in less bound state

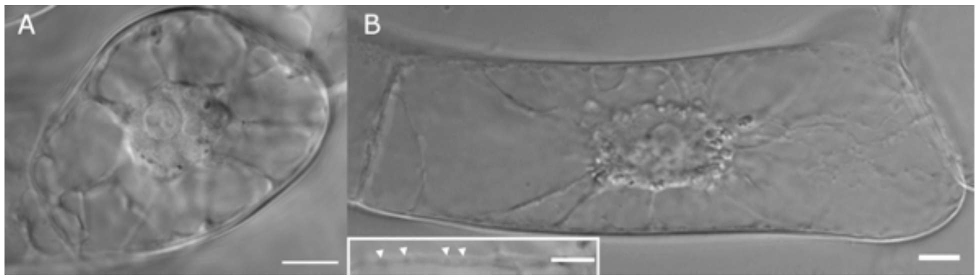
2. Collective transport

Tradescantia virginiana



Inside cells
molecular motors
  cooperative transport
    role of embedding solvent?

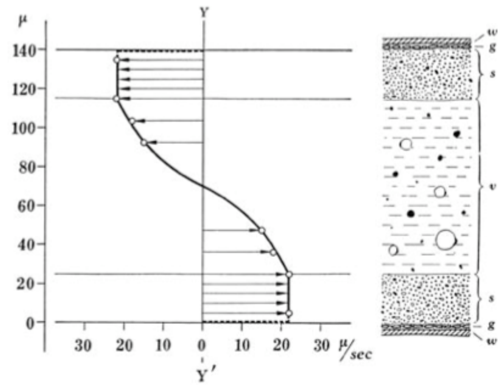
Tobacco BY-2



Esseling et al. 2007

Cytoplasmatic streaming

Role actin/myosin



Shimmen 2007

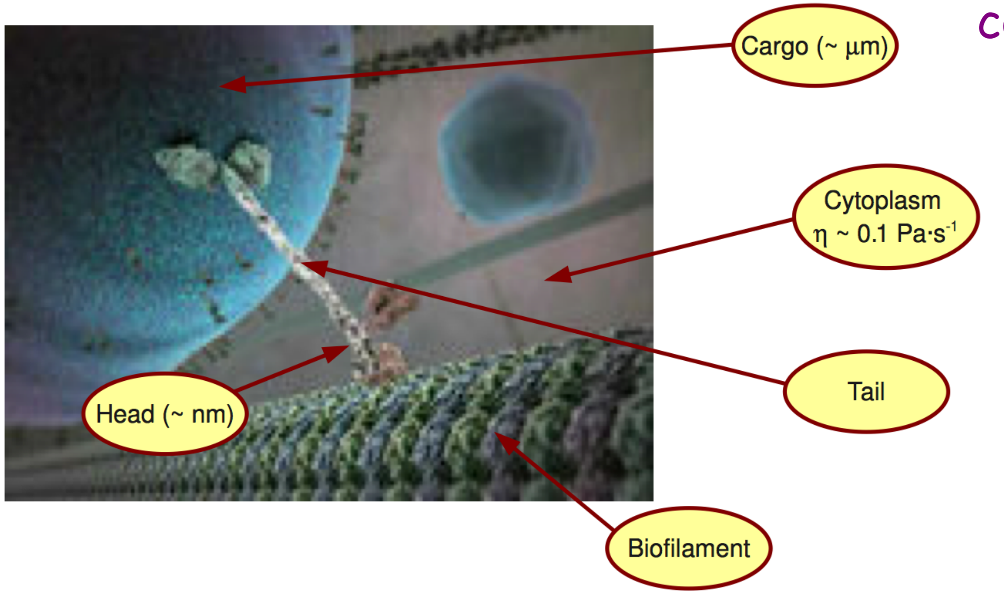
Relevance of hydrodynamic coupling?
  passive transport?

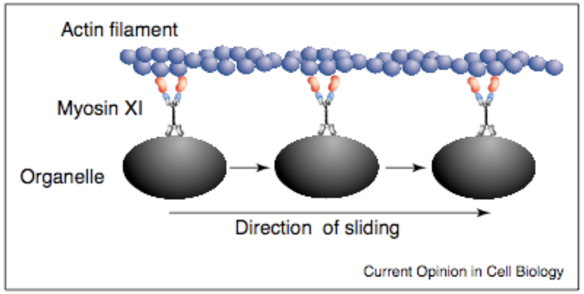
Forces small
  collective flow (restricted geometries)

2. Collective transport

Relevance of hydrodynamic coupling?
passive transport?

Forces small
collective flow (restricted geometries)



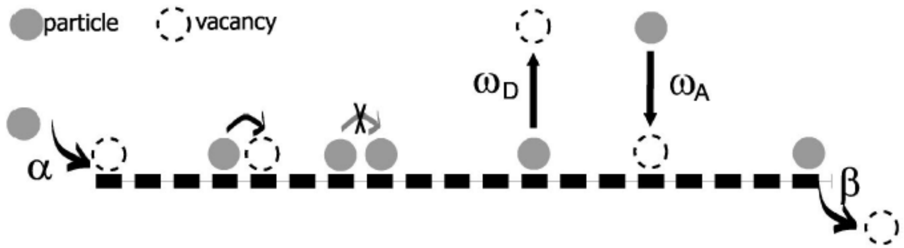


Cytoplasmatic streaming
induced by molecular motors?

Up to 100 μm/s
Single motor 7 μm/s

Collective motion of molecular motors

Higher coarse graining
TASEP model + Langmuir kinetics

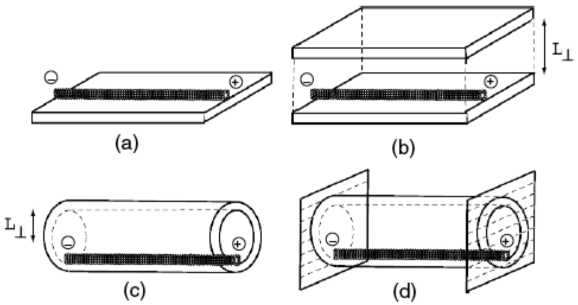


Kinetic equation
occupation variables

$$\hat{j}_i(t) = \hat{n}_i(t)[1 - \hat{n}_{i+1}(t)]$$

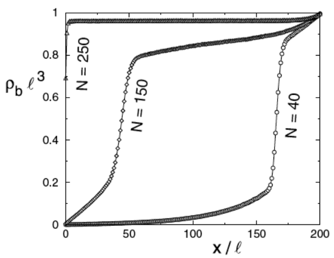
$$\frac{d}{dt}\hat{n}_i(t) = \hat{n}_{i-1}(t)[1 - \hat{n}_i(t)] - \hat{n}_i(t)[1 - \hat{n}_{i+1}(t)] \quad + \omega_A[1 - \hat{n}_i(t)] - \omega_D\hat{n}_i(t)$$

Under confinement

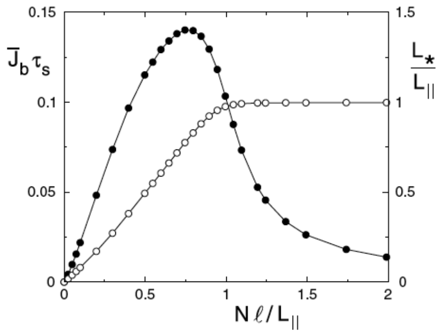


Presence of confining ends
accumulates motors
generates shock waves
inhomogeneous motor flux

Diffusive backflow balances active flow

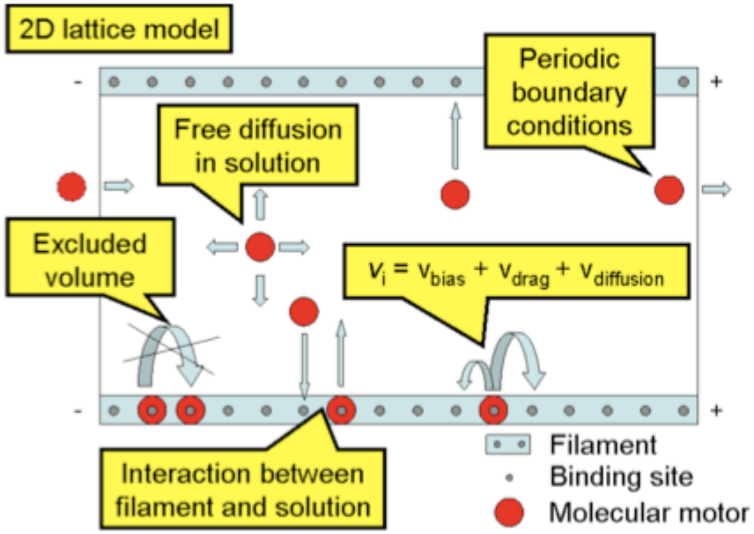


Traffic jams



2. Collective transport

Simple
lattice model



Introduce HI
simplest approach
disregard effects near field
substrate

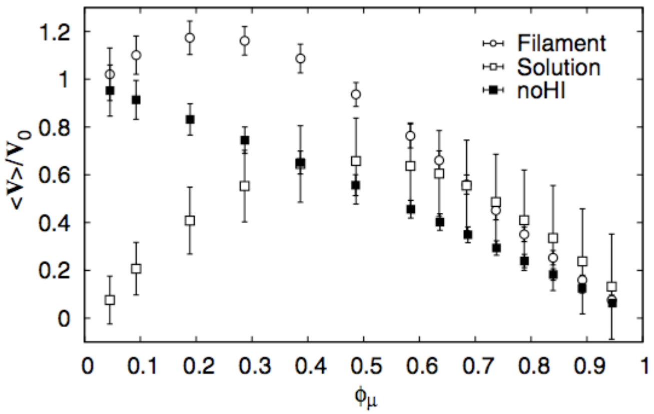
Particles follow ASEP
no molecular detail

1D geometry + PBC

Instantaneous coupling

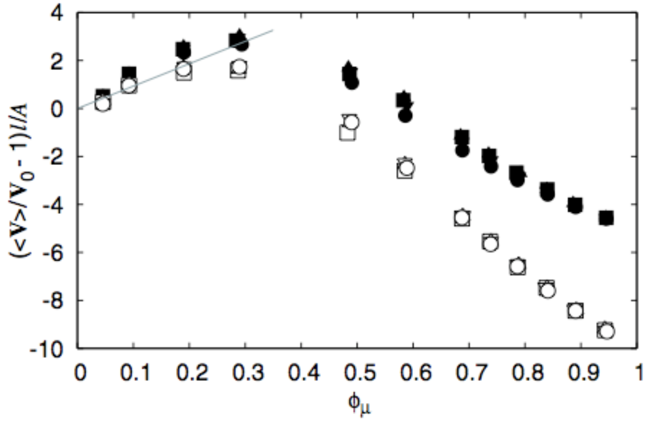
Coupling active motors/suspended organelles
convection-diffusion

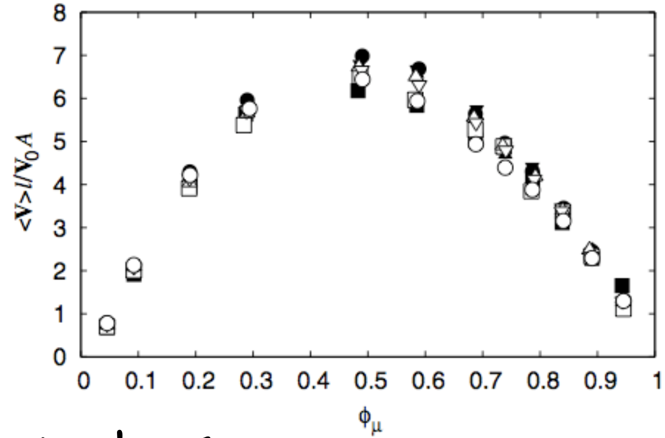
2. Collective transport



Motor/organelle velocities
induced suspension flow

Competition Hydrodynamic/excluded volume





Simple scaling with hydrodynamci parameters

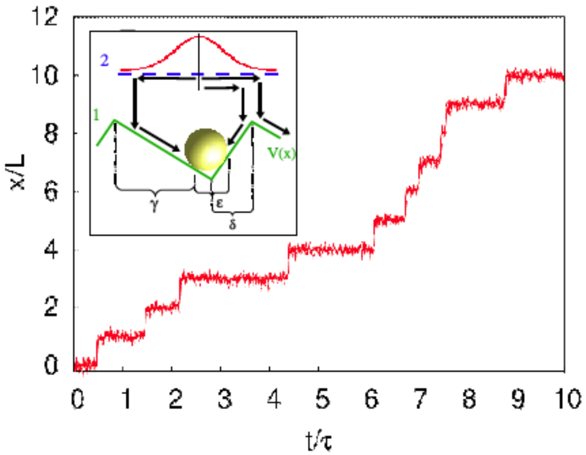
Hydrodynamics enhances transport

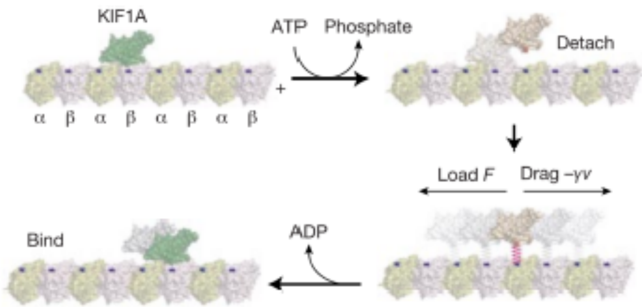
Houtman et al, EPL 2007

2. Collective transport

Motors step rather than slide

Need for a detailed description of motor displacement along a biofilaments?

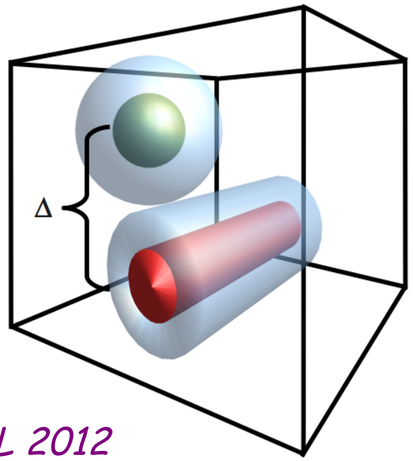




Ratchet model vs TASEP

Combined motion +
coupling to solvent

Simplified geometry



Malgaretti et al. PRL 2012

2. Collective transport

Lowe-Andersen thermostat (LAT):
C.P.Lowe, Europhys. Lett. 47, 145, (1999).

$$\vec{r}_i(t+\Delta t) = \vec{r}_i(t) + \Delta t \vec{v}_i(t) + \frac{1}{2}\Delta t^2 f_i^C(t)$$

$$\vec{v}_i(t+\Delta t) = \vec{v}_i \qquad \Gamma\Delta t < \xi$$

$$\vec{v}_j(t+\Delta t) = \vec{v}_j \qquad \Gamma\Delta t < \xi$$

$$\text{"Bath" collision}\left\{\begin{aligned} \vec{v}_i(t+\Delta t) &= \vec{v}_i + \frac{\mu_{\mathrm{ij}}}{m_i}\left(\theta_{ij}\sqrt{\frac{kT}{\mu_{ij}}} - (\vec{v}_i - \vec{v}_j)\cdot\hat{r}_{ij}\right)\hat{r}_{ij} \qquad \Gamma\Delta t \geq \xi \\ \vec{v}_j(t+\Delta t) &= \vec{v}_j - \frac{\mu_{\mathrm{ij}}}{m_j}\left(\theta_{ij}\sqrt{\frac{kT}{\mu_{ij}}} - (\vec{v}_i - \vec{v}_j)\cdot\hat{r}_{ij}\right)\hat{r}_{ij} \qquad \Gamma\Delta t \geq \xi \end{aligned}\right.$$

Here Γ is a bath collision frequency (plays a similar role to γ/m in DPD)

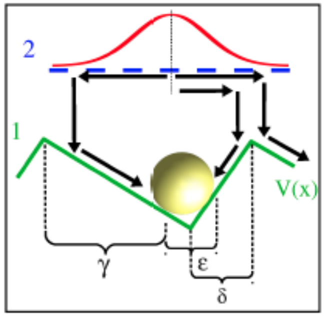
- Bath collisions are processed for all pairs with $r_{ij}<r_c$
- The current value of the velocity is always used in the bath collision (hence the lack of an explicit time on the R.H.S.)
- The quantity ξ is a random number uniformly distributed in the range 0-1
- Reduced mass for particles *i* and *j*, $\mu_{ij}=m_i\, m_j/(m_i+m_j)$

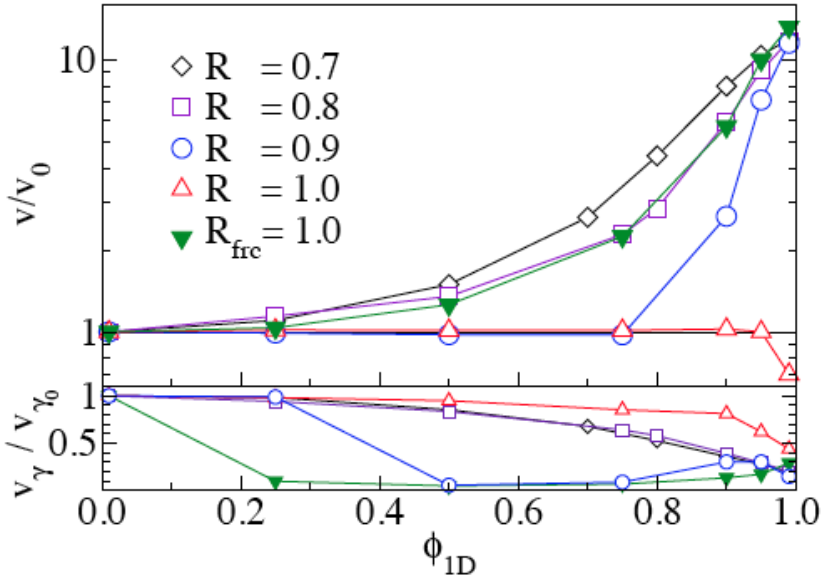
2. Collective transport

Correlated trajectories due to pull/push

Excluded volume

substrate periodicity

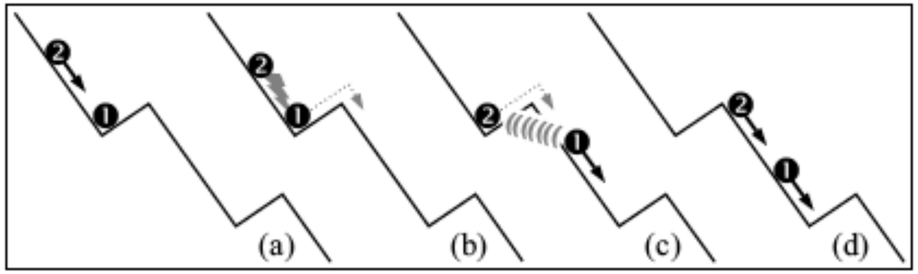




Relevance of forcing during free diffusion

Determine velocity enhacement

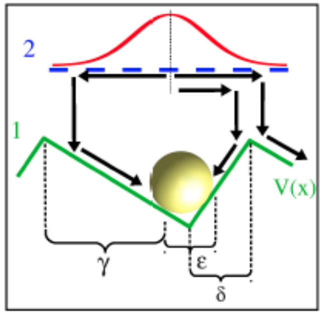
Commensurate motors do not see each other



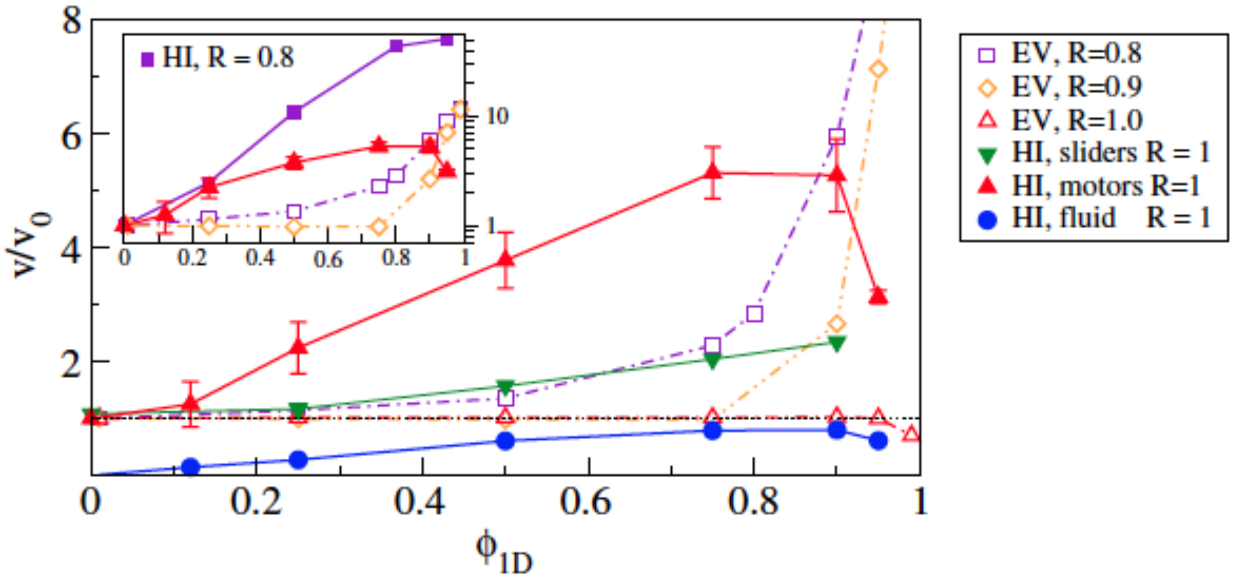
Forced colloids

Lutz et al 06

2. Collective transport



Minimize excluded volume interactions



Correlated trajectories due to
hydrodynamic pull/push

Bias due to induced flow during diffusion

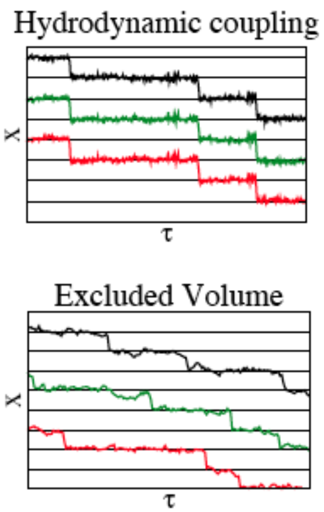
Long-range hydrodynamic coupling subdominant

Forces small
motion due to collective flow

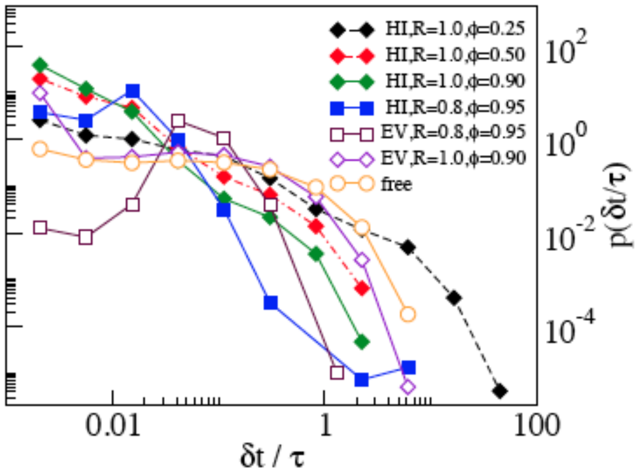
Hydrodynamics enhances transport
suspended particles

2. Collective transport

Correlated trajectories due to
hydrodynamic pull/push

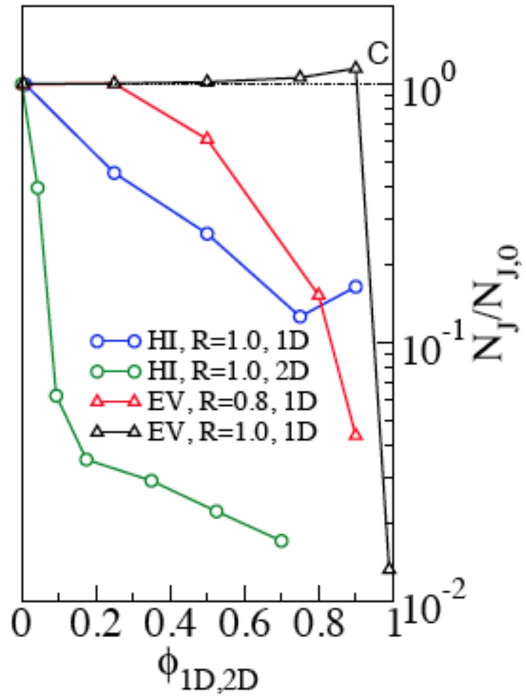


Dwelling times consecutive motors



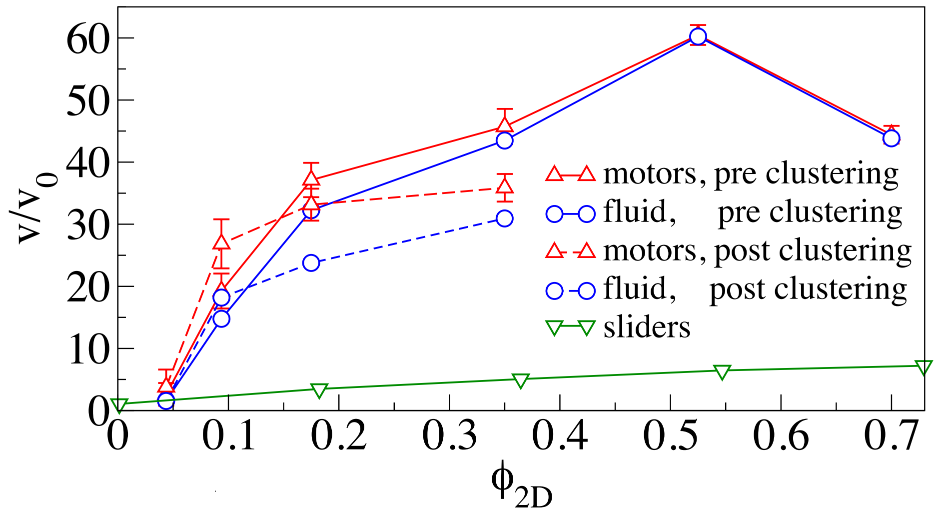
Correlated jumps

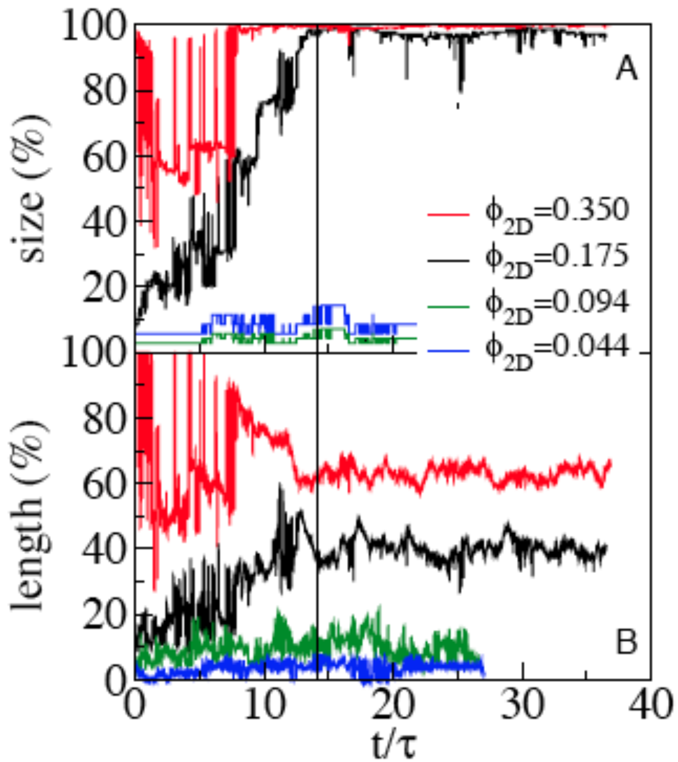
Decrease longer dwelling times



Hydrodynamic coupling
decrease in energy cost
energy used to move isolated motor
exploited by neighbours

2. Collective transport





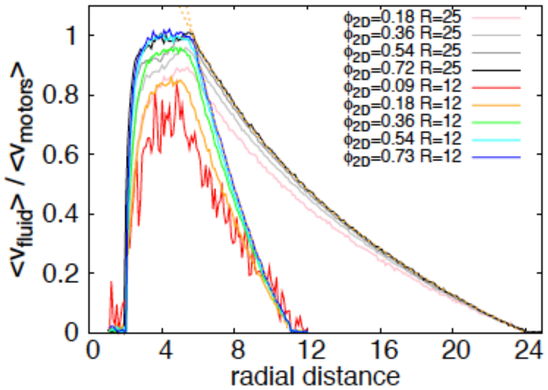
Cluster formation: ring stability?

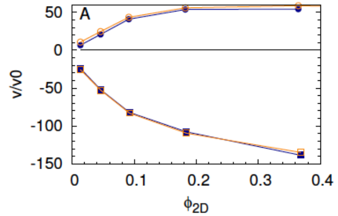
Shock wave?
Connection with traffic jamms?

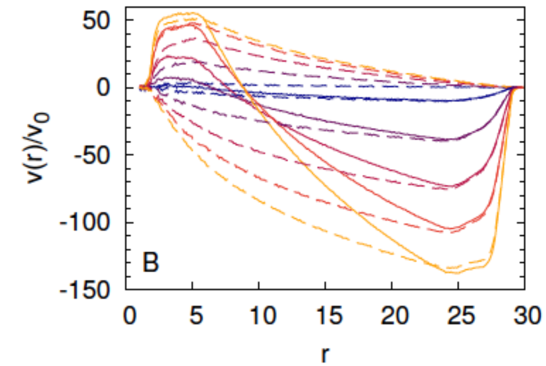
Fraction of motors in largest cluster

2. Collective transport

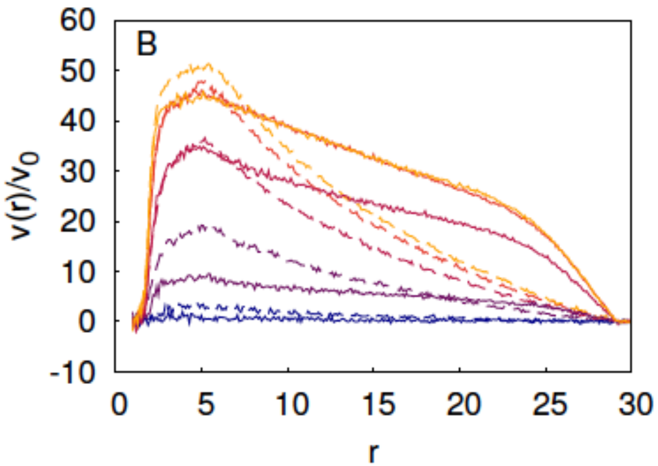
Molecular motors generate cytoplasmatic flow

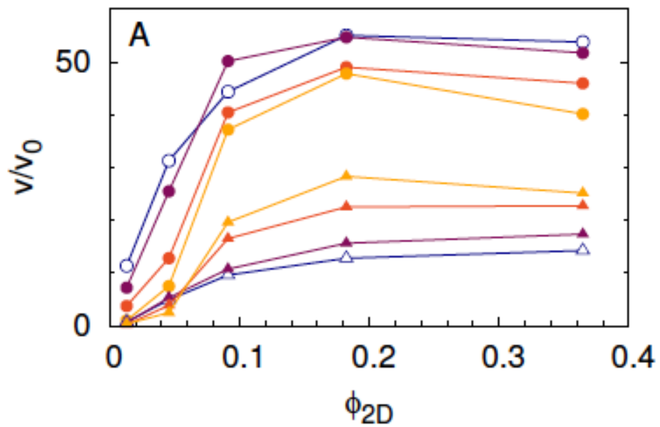






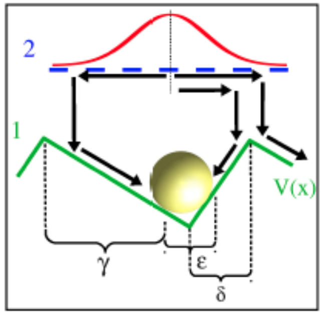
Drives suspended organelles for free





Interference / Coupling with motors

3. Confined Brownian ratchets



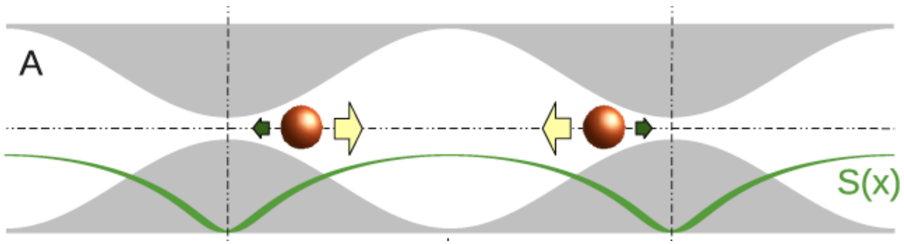
Brownian ratchet
  Moving in a confined, variable, environment

Asymmetric, variable geometric constraints
  Induce ratchet motion of colloids

Relevance of entropic ratchet transport for active motors?

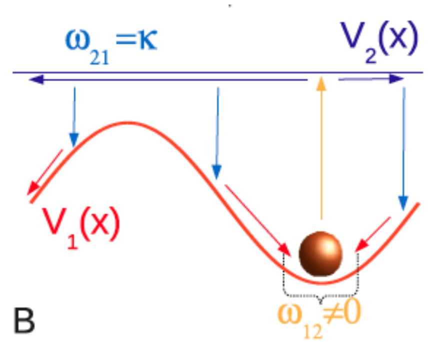
3. ConfinedBrownian ratchets

In elongated, variable channels

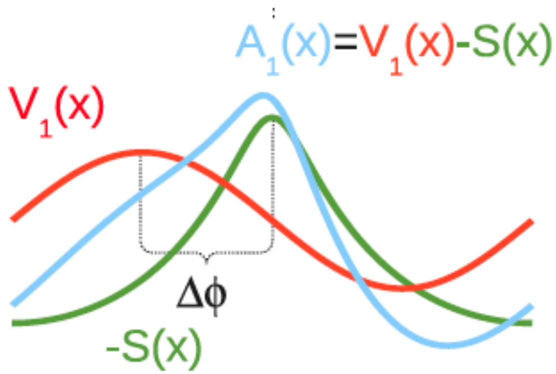


Equilibrate along cross section

Identify local entropy:
available space



Motor activity
less bound state explores
cross section



Effective longitudinal potential

Asymmetric A(x)
out of symmetric ones?

3. Active Brownian ratchets

Diffusion along a channel

$$\frac{\partial}{\partial t} C(x,y,t) = D \frac{\partial}{\partial x} e^{-\beta U(x,y)} \frac{\partial}{\partial x} e^{\beta U(x,y)} C(x,y,t) + D \frac{\partial}{\partial y} e^{-\beta U(x,y)} \frac{\partial}{\partial y} e^{\beta U(x,y)} C(x,y,t)$$

Smoluchowsky equation

Local equilibrium density

$$\rho(y;x) = e^{-\beta U(x,y)} / e^{-\beta A(x)} \qquad e^{-\beta A(x)} = \int \mathrm{d}y\, e^{-\beta U(x,y)}$$

Effective longitudinal diffusion $G(x,t) = \int \mathrm{d}y\, C(x,y,t)$

Fick-Jacobs equation

$$C(x,y,t) \simeq G(x,t)\, \rho(y;x) \qquad \partial_x h \ll 1$$

3. Confined Brownian ratchets

Diffusion in an effective free energy

$$\frac{\partial}{\partial t} G(x,t) \cong D \frac{\partial}{\partial x} e^{-\beta A(x)} \frac{\partial}{\partial x} e^{\beta A(x)} G(x,t)$$

Hard-core confining potential
Channel section

$$e^{-\beta A(x)} = \mathcal{A}(x)$$

Free energy is entropic

Fick-Jacobs equation

Zwanzig 92

3. Confined Brownian ratchets

Simplified model

Effective entropy

$$S(x) = k_B \ln 2h(x)$$

Asymmetrically controlled cross section

$$h(x) = \gamma + \beta \left[\sin(2\pi x + \phi) + \Lambda \sin(4\pi x + \phi) \right]$$

$$\Delta S_m = \ln \frac{\gamma + \beta(1+\Lambda)}{\gamma - \beta(1+\Lambda)}$$

Identify minimum, maximum aperture + phase

$$h_{min} = \gamma - \beta(1 + \Lambda).$$

3. Confined Brownian ratchets

Two state molecular motor

Asymmetric potential through λ

$$V_1(x) = V_0\left[\sin(2\pi x) + \lambda\sin(4\pi x)\right], \partial_x V_2 = 0$$

Two population diffusion + jumping rates

$$\begin{aligned} \partial_t p_1(x) + \partial_x J_1 &= -\omega_{12}(x)p_1(x) + \omega_{21}(x)p_2(x) \\ \partial_t p_2(x) + \partial_x J_2 &= \omega_{12}(x)p_1(x) - \omega_{21}(x)p_2(x) \end{aligned}$$

Diffusion fluxes in effective free energy

$$J_{1,2}(x) = -D(x)\big(\partial_x p_{1,2}(x) + p_{1,2}(x)\partial_x A_{1,2}(x)\big)$$

Jump rates: break detailed balance

$$\omega_{12} = k_{12}$$

$$\omega_{21,p}(x) = k_{21}$$

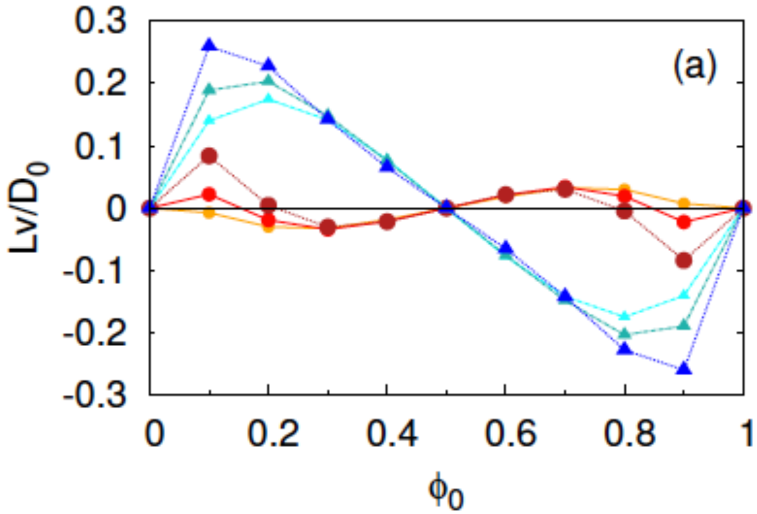
$$\omega_{21,np}(x) = k_{21}/h(x)$$

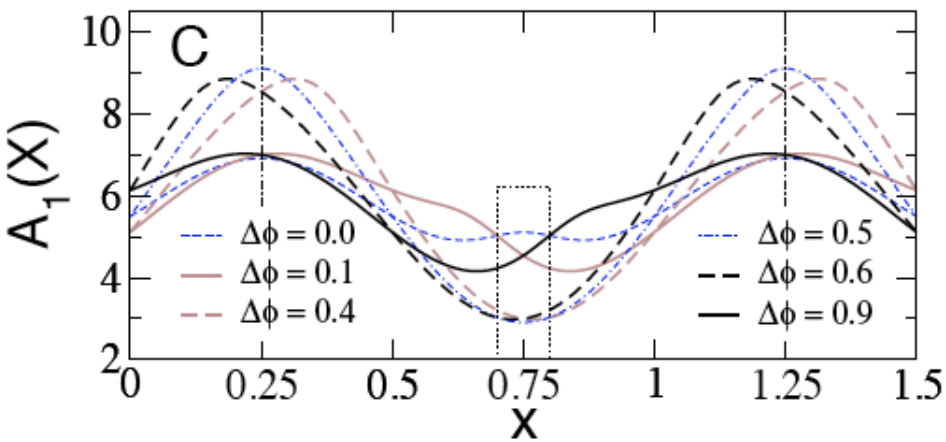
3. Confined molecular motors

Symmetric confinement + symmetric ratchet $\lambda = \Lambda = 0$

Effective free energy?
sensitive to dephasing

Cooperative ratcheting

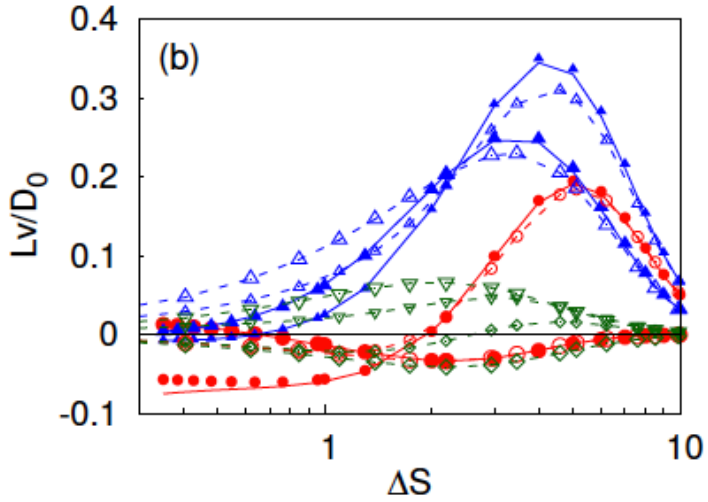




Asymmetric free energy
closest path from jumping position to energy maximum

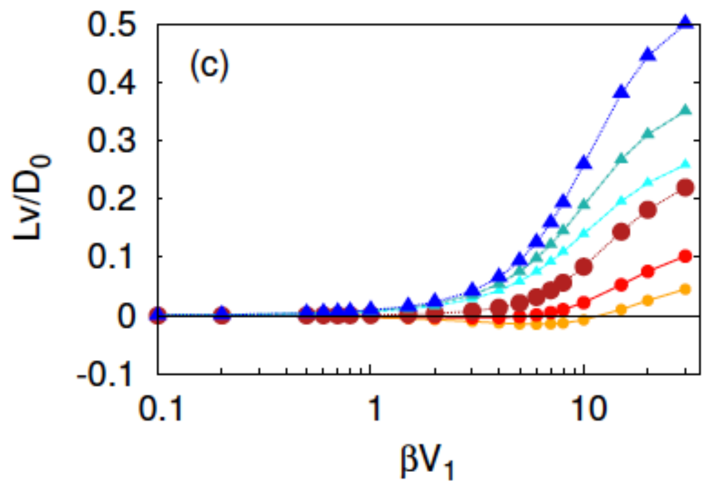
3. Confined molecular motors

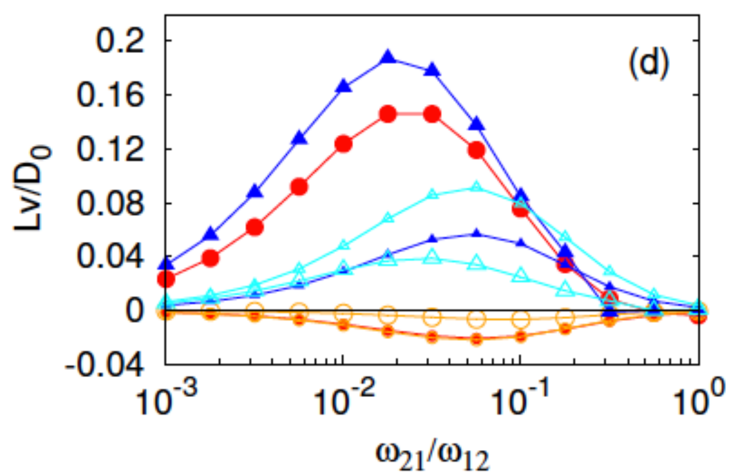
Sensitivity to entropic confinement



Enhanced transport
Optimal parameters

Velocity inversion

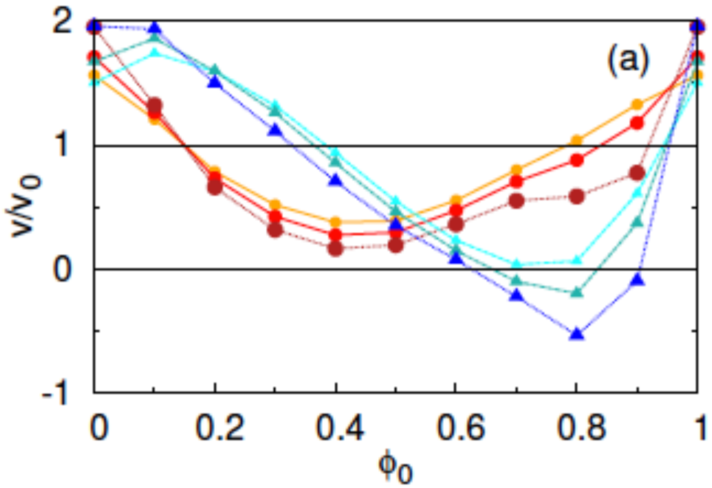




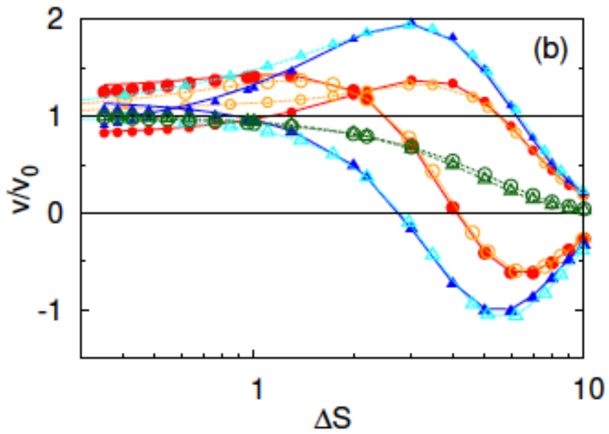
Malgaretti et al. PRE 2012

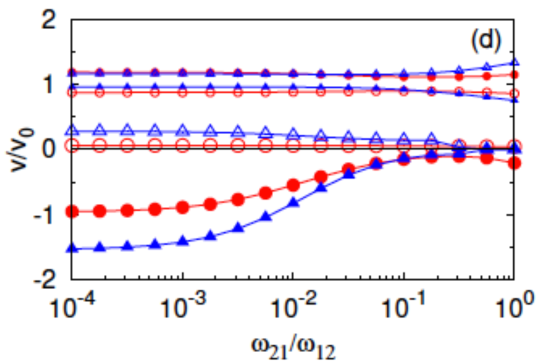
3. Confined Brownian ratchets

Symmetric confinement
Asymmetric Motor
Enhanced transport
Optimal parameters



Velocity inversion





Particle segregation
low concentrations

Malgaretti et al. PRE 2012, JCP 2013

4. Conclusions

Hydrodynamics as a mechanism for cooperation

release internal energy / metabolism

Relevance of hydrodynamics in molecular motors

simple couplings set up aggregation schemes

collective transport and patterns at different scales

Increase in collective motion molecular motors

correlated jumps

more efficient that continuous shift

Relevance of activty and entropic constraints

collective rectification

active segregation

Acknowledgements

Molecular motors

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Cooperative ratchets

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University of Barcelona

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