EXTREME DEVIATIONS IN TURBULENT PAIR DISPERSION

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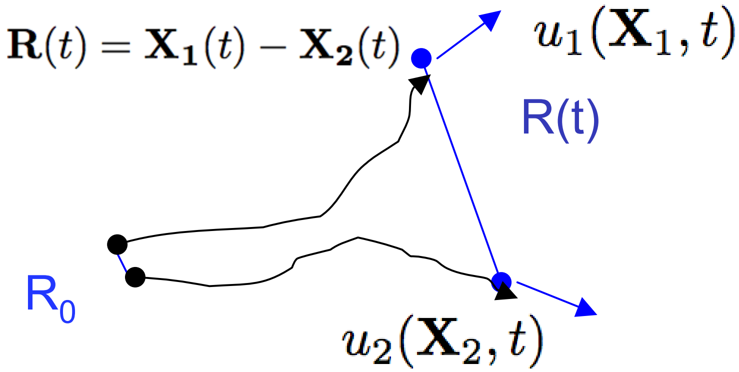
How pairs of tracers separate in turbulent flows ?



Richardson diffusive model requires:

- *fast decorrelation of the velocity*
  to adopt a diffusive-like eq.

- *a single critical exponent for velocity fluctuations*
  to have global time self-similarity

- *infinite scaling range*
  to have asymptotic solution valid

<u>3D turbulent flows have none of these requirements</u>

1. what's the applicability of Richardson model?
2. what's the origin of extreme events in pair dispersion?
3. what about the inertial particles separation distribution?

Applications

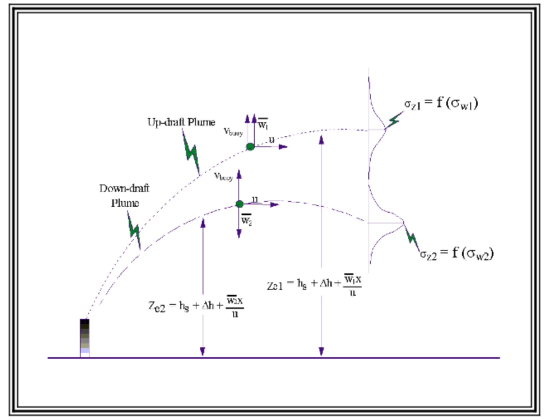
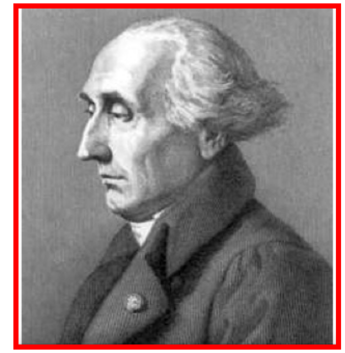


Figure 15: AERMOD's pdf approach for plume dispersion in the CBL. AERMOD approximates the skewed distribution by superimposing two Gaussian distributions, the updraft and downdraft distributions.



At small diffusivities κ

Eulerian approach

$$\partial_t c + \mathbf{u} \cdot \nabla c = \kappa \nabla^2 c + S$$

$$< c(\mathbf{x}, t) >=$$

$$< c(\mathbf{x_1}, t) c(\mathbf{x_2}, t) >=$$

$$\int_{s_1 \le t} \int_{s_2 \le t} \int_V \int_V p_2(\mathbf{X_1} = \mathbf{x_1}, \mathbf{X_2} = \mathbf{x_2}, t | \mathbf{Y_1}, \mathbf{Y_2}, s) S(\mathbf{Y_1}, s_1) S(\mathbf{Y_2}, s_2) d\mathbf{Y_1} ds_1 d\mathbf{Y_2} ds_2$$

Lagrangian approach

$$\frac{d\mathbf{X}}{dt} = \mathbf{u}(\mathbf{X}, t) + \sqrt{2\kappa}\, \eta(t)$$

$$\int_{s \le t} \int_V p(\mathbf{X} = \mathbf{x}, t | \mathbf{Y}, s) S(\mathbf{Y}, s) d\mathbf{Y} ds$$

Time scales in diffusion

Diffusion is a macroscopic behaviour emerging at long times t > t_{Mac} from many different microscopic ones. It requires:

$$t_{mic} \ll \tau_{cor} \ll t_{Mac}$$

time of microscopic forces

observation time

Standard diffusion $\boxed{\langle \Delta x^2 \rangle \equiv 2 D_0 \Delta t}$ *Diffusion equation, Gaussian PDF*

If correlation time is long (no time separation), deviations appear:

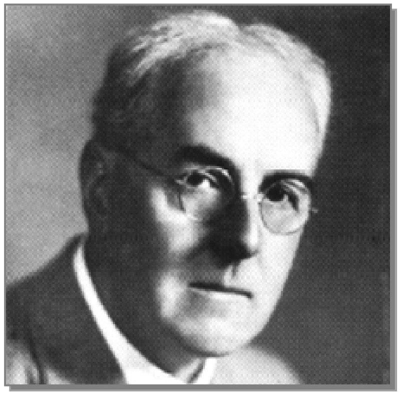
Anomalous diffusion $\boxed{\langle \Delta x^{2q} \rangle \simeq D_\alpha \Delta t^{q\alpha} \qquad \alpha \neq 1}$ *Fokker-Planck eq.*
Stretched exp PDF

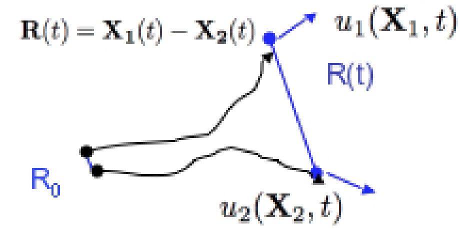
If self-similarity of the diffusion process is lost:

Strong Anomalous $\boxed{\langle \Delta x^{2q} \rangle \simeq D_\zeta \, \Delta t^{\zeta(q)} \qquad \zeta(2q) \neq 2\zeta(q)}$

No diffusive-like eq.

Richardson Law (1926) for isotropic & homogeneous flows



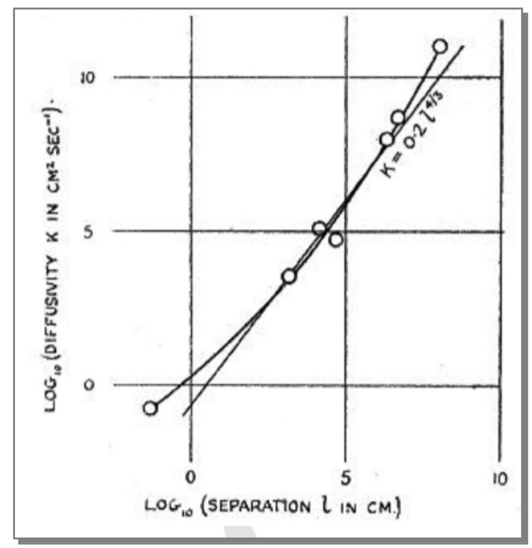


The PDF to observe a pair with distance R satisfies a *diffusion* eq.

$$\boxed{\frac{\partial P(R,t)}{\partial t} = \frac{1}{R^{d-1}} \left[\frac{\partial}{\partial R} R^{d-1} D_{Ric}(R) \frac{\partial}{\partial R} P(R,t) \right]}$$

$$\boxed{D_{Ric}(R,t) \simeq R^{4/3}}$$

scale-dependent eddy diffusivity



Proc. Roy. Soc. A 756, 1926

Richardson distribution is non-Gaussian but self-similar

$$P(R,t) \simeq \frac{R^2}{t^{9/2}} \exp(-C R^{2/3}/t)$$

- Second order moment: superdiffusive
  Constant “g” is assumed universal

$$\langle R^2(t) \rangle = g \, \epsilon \, t^3$$

- High order moments

$$\langle R^{2q}(t) \rangle = g_{2q} \, \epsilon^q \, t^{3q}$$

Richardson PDF vs. Kolmogorov-Obukhov theory

$$D_{Ric}(R,t) \equiv \frac{1}{2}\frac{d}{dt}\langle \mathbf{R}^2 \rangle \equiv \langle \delta_R \mathbf{u}(\mathbf{R},t)\cdot\mathbf{R}(t)\rangle \equiv \int_0^t \langle \delta_R \mathbf{u}(\mathbf{R}(t),t)\delta_R \mathbf{u}(\mathbf{R}(s),s)\rangle ds$$

If *local correlation time τ(R) is* ***so short*** that rel. separation have not much changed:

$$\boxed{D_{Ric}(R,t) \simeq \tau(R)\langle \delta_R \mathbf{u}(\mathbf{R}(t),t)^2\rangle = k_0 \epsilon^{1/3} R^{4/3}}$$

We used **self-similar** Kolmogorov scaling:

$$\langle \delta_r \mathbf{u}(\mathbf{r}_{||})^2\rangle \propto \epsilon^{2/3} r^{2/3}; \qquad \tau_r \propto (r^2/\epsilon)^{1/3}$$

ε = the mean kinetic energy dissipation

In the inertial range of scales **with IF cut-off --> 0, and UV cut-off to ∞**

$$P(R >> R_0, t >> t_0) = \frac{A}{\epsilon^{3/2} t^{9/2}}\, exp\left(-\frac{9R^{2/3}}{4k_0\epsilon^{1/3}t}\right)$$

More generally

Falkovich, Gawedzki, Vergassola RMP (2001)

Richardson's approach is exact for the evolution of a particle pair in a stochastic self-similar and delta-correlated in time, incompressible velocity field, ex. Kraichnan model:

$$D_{||}(R) \propto R^{\xi} \qquad 0 \leq \xi \leq 2$$

- Asymptotic solution for $\xi \neq 2$: *stretched exponential*

$$P(R,t) \approx \frac{C\,R^2}{\langle R^2 \rangle^{3/2}} \exp\left[-b\left(\frac{R}{\langle R^2(t)\rangle^{1/2}}\right)^{2-\xi}\right] \qquad \langle R^2(t)\rangle \simeq t^{2/(2-\xi)}$$

- Asymptotic solution for $\xi = 2$: *log-normal*

$$P(R,t|R_0,0) \propto \exp\left[-\frac{[\ln(R/R_0) - \lambda(t-t_0)]^2}{4\Gamma(t-t_0)}\right] \qquad < R^2(t) > = R_0^2\, e^{(2\lambda+4\Gamma)t}$$

HINTS

Sokolov, PRE (1999)

Turbulent dispersion is **non stationary** & with **variable increments,** correlating on all scales…very far from standard diffusion

Can we quantify how wrong is to assume a diffusion approach?

- Velocity fluctuations at scale R:

$$\delta_R u(R) \approx U_0 \left(\frac{R}{L_0}\right)^{1/3}$$

- Typical time scale associated:

$$\tau(R) \approx T_0 \left(\frac{R}{L_0}\right)^{2/3}$$

- Lagrangian time along traject. :

$$\tau_{Lag}(R) \approx \frac{R}{\delta_R u(R)} \approx \frac{R_0^{1/3}}{U_0} R^{2/3}$$

For diffusive approach to be valid

$$\boxed{Ps = \frac{\tau(R)}{\tau_{Lag}(R)} \approx \frac{T_0 U_0}{R_0} << 1}$$

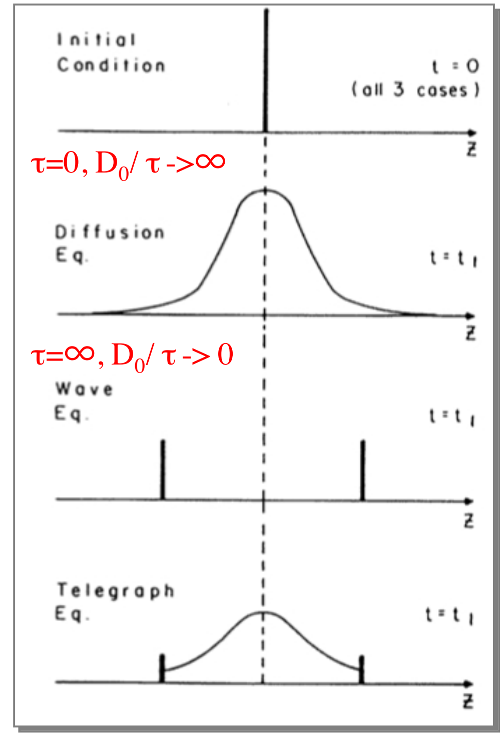
*In d=3 turbulence **Ps≈ 1**, and Kolmogorov type of scaling is the borderline case where diffusion approach breaks down…*

Probls: infinite speed propagation & time locality

Masoliver, Weiss Eur. J. Phys. (1996)

- Richardson diffusive approach P(R,t) > 0 at any R at time t….
Telegrapher eq. is the simplest generalization with constant <u>*bounded speed*</u> *(persistent random walk)*

$$\boxed{\tau \frac{\partial^2 P(R,t)}{\partial t^2} + \frac{\partial P(R,t)}{\partial t} = D_0 \frac{\partial^2 P(R,t)}{\partial R^2}}$$



- Altern., Fokker-Planck eq. can be modified to include time non-locality of diffusive contribution & cure infinite speed

$$\boxed{\frac{\partial P(R,t)}{\partial t} = \int_0^t K(t-s)\left[\frac{1}{R^{d-1}}\frac{\partial}{\partial R}R^{d-1}\frac{\partial}{\partial R}P(R,s)\right]ds}$$

Ilyin, Procaccia, Zagorodny CMP (2013)

- Note also that any diffusivity kernel

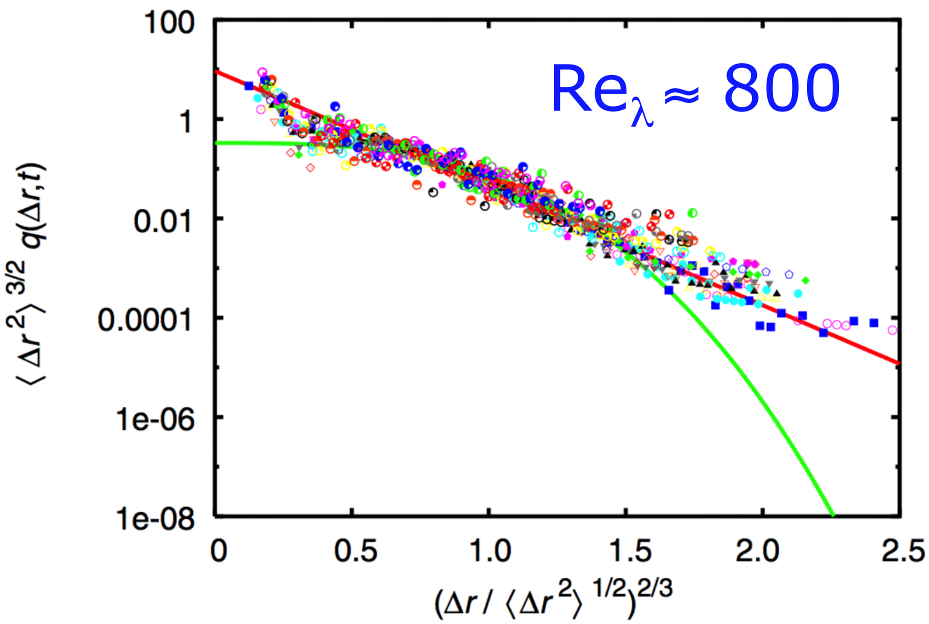
$$D_{||}(R,t) \propto R^{\alpha}t^{\beta}, \quad 3\alpha + 2\beta = 4 \quad \Longrightarrow \quad \begin{cases} \langle R^2 \rangle \propto t^3 \\ P(R,t) \propto \dfrac{AR^2}{(\epsilon^{1/3}t)^{3(\beta+1)/(2-\alpha)}} \exp\left(-C\,\dfrac{R^{2-\alpha}}{t^{1+\beta}}\right) \end{cases}$$

Laboratory experiments of pair dispersion : 3d *isotropic and homogeneous* turbulence

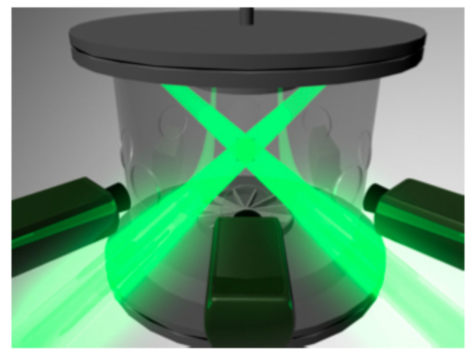
Richardson diffusive approach is *wrong*, but a non-markovian model is not yet there.

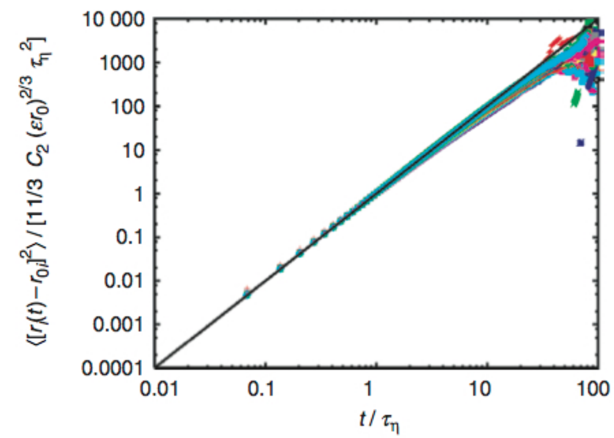
Let's look at data…

$f(Hz)$	R_λ	u'_r (ms^{-1})	u'_z (ms^{-1})	ϵ (m^2s^{-3})	$\eta(\mu m)$	L/η	τ_η (ms)	T_L/τ_η	FPS	δt (ms)	$\tau_\eta/\delta t$
0.30	200	0.039	0.026	7.09×10^{-4}	192	365	36.8	51	1000	1.00	37
0.43	240	0.056	0.038	2.03×10^{-3}	146	479	21.3	61	1600	0.625	34
0.62	290	0.083	0.054	6.26×10^{-3}	111	630	12.3	74	3000	0.333	37
0.90	350	0.121	0.080	2.01×10^{-2}	84	830	7.11	88	5000	0.200	36
1.29	415	0.181	0.116	6.17×10^{-2}	64	1090	4.12	106	9000	0.111	37
1.86	500	0.262	0.169	0.196	49	1433	2.39	127	27000	0.037	65
3.50	690	0.487	0.315	1.24	30	2337	0.897	176	27000	0.037	24
5.00	815	0.669	0.440	3.39	23	3087	0.544	208	27000	0.037	15



Ouellette et al., NJP 2006

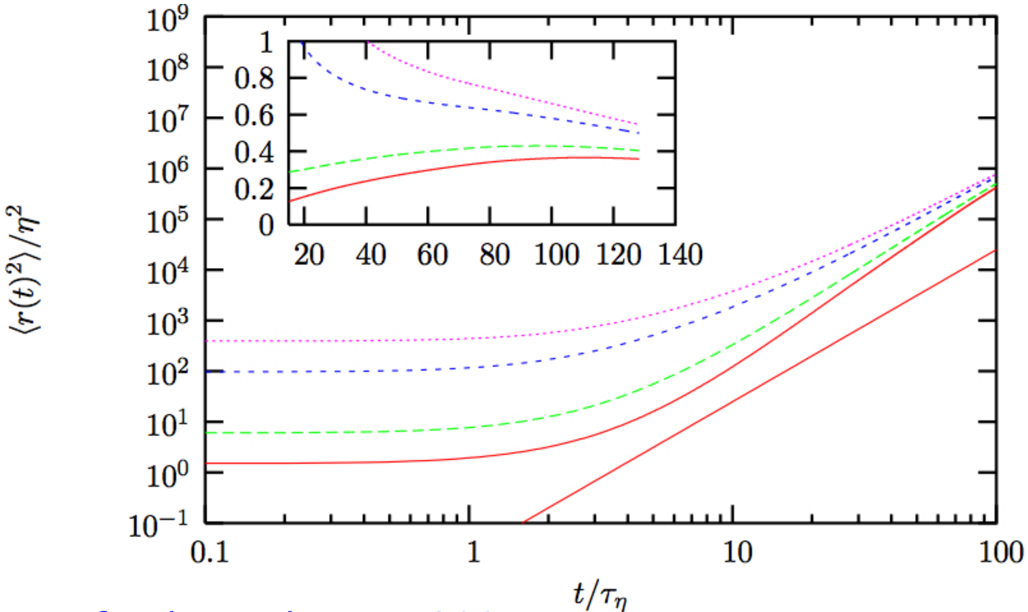




The $<R^2> = g\ \varepsilon\ t^3$ is hardly observed,

Richardson PDF is observed in a narrow range

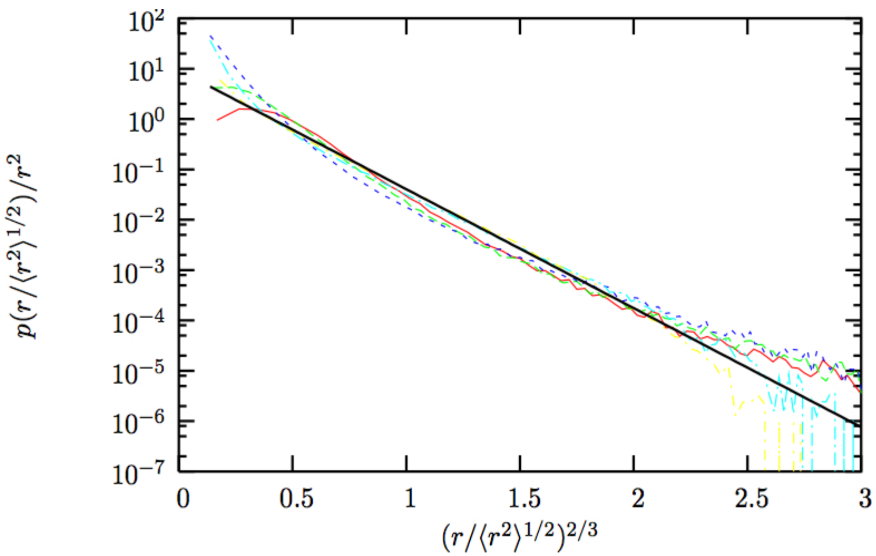
Numerical simulations vs experiments



Biferale et al., PoF 2005

$Re_\lambda \approx 300$

Model	Richardson universal *g*
Numerics 3D (Boffetta Sokolov,…)	0.5
Experiment 3D (Ott Mann)	0.5
Theory LDHI (Kraichnan)	2.42
Stochastinc models (Borgas & Sawford)	$0.8 \leq g \leq 1.8$



Numerical simulations are competitive for Lagrangian Turb.

They show a discrete agreement with Richardson model.

POINT SOURCE EMISSION

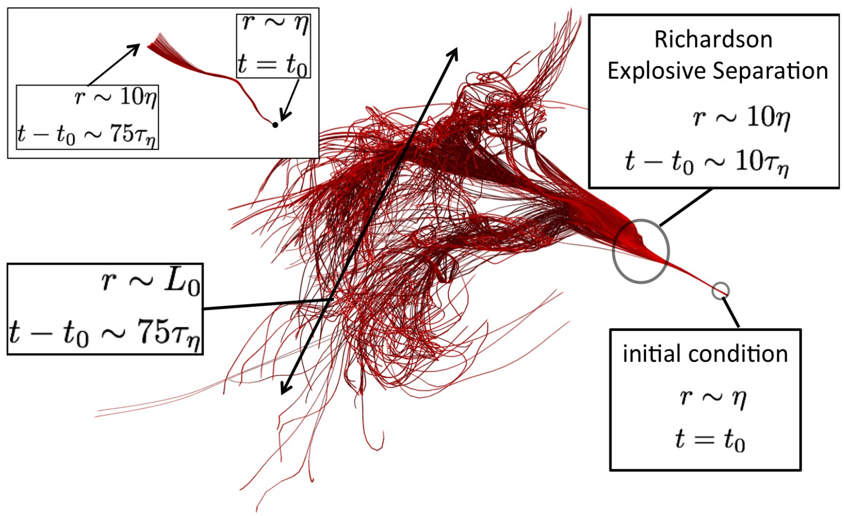
Navier-Stokes eqs. in a cubic domain 1024^3 resolution, periodic BC.
Self-similarity of the flow is broken: Multifractal Statistics

$$\langle [\delta_r u(r)]^q \rangle \approx c_q r^{\zeta_q}, \qquad \zeta_{2q} < 2\zeta_q$$

$$\langle [\delta_r u(r)]^3 \rangle = -\frac{4}{5}\epsilon r$$

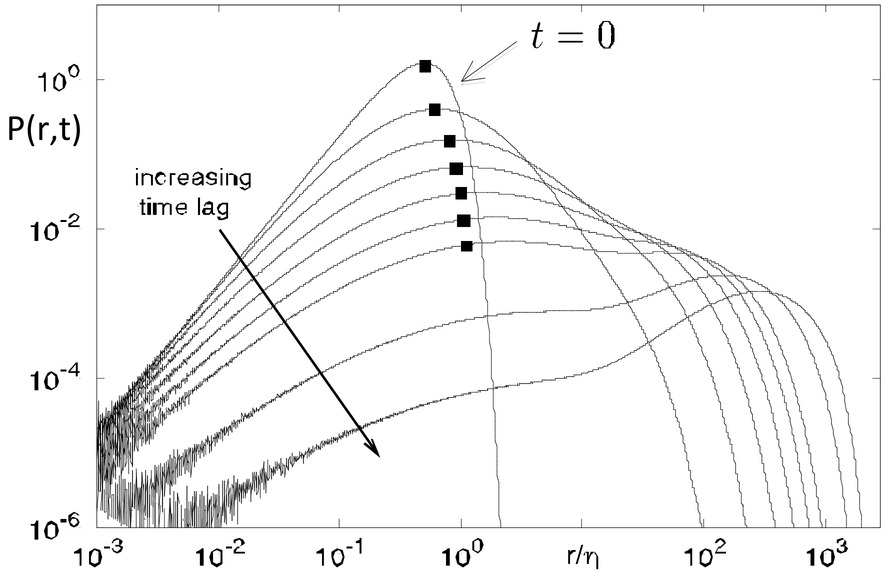
Flow is seeded with 10^{11} pairs emitted from many point sources.

$$\frac{d\mathbf{X}}{dt} = \mathbf{u}(\mathbf{X}, t)$$



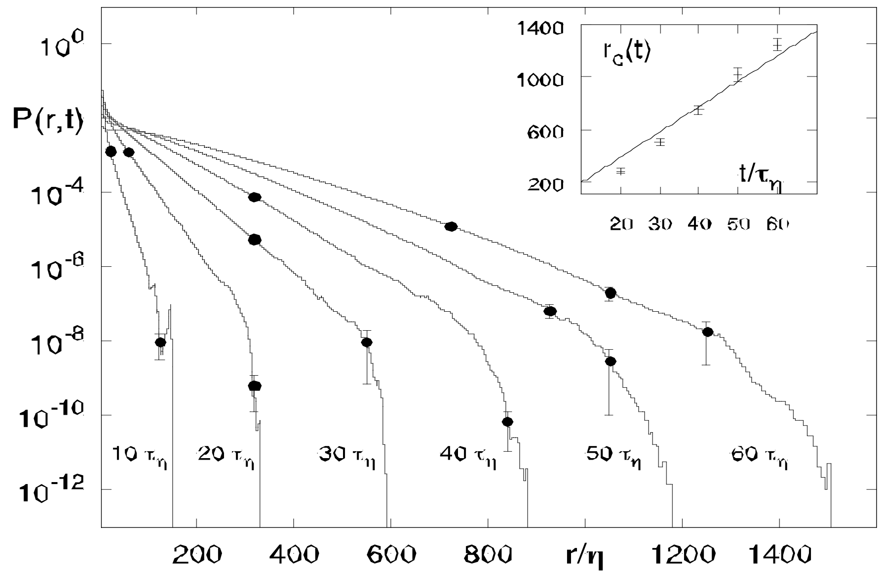
Slowest and fastest separation events

Slowly separating pairs



- Presence of a peak at sub-diffusive separations at every time.
- This behavior is due to emissions that do not separate efficiently in the flow (Lyapunov exponent fluctuates and can become very small).

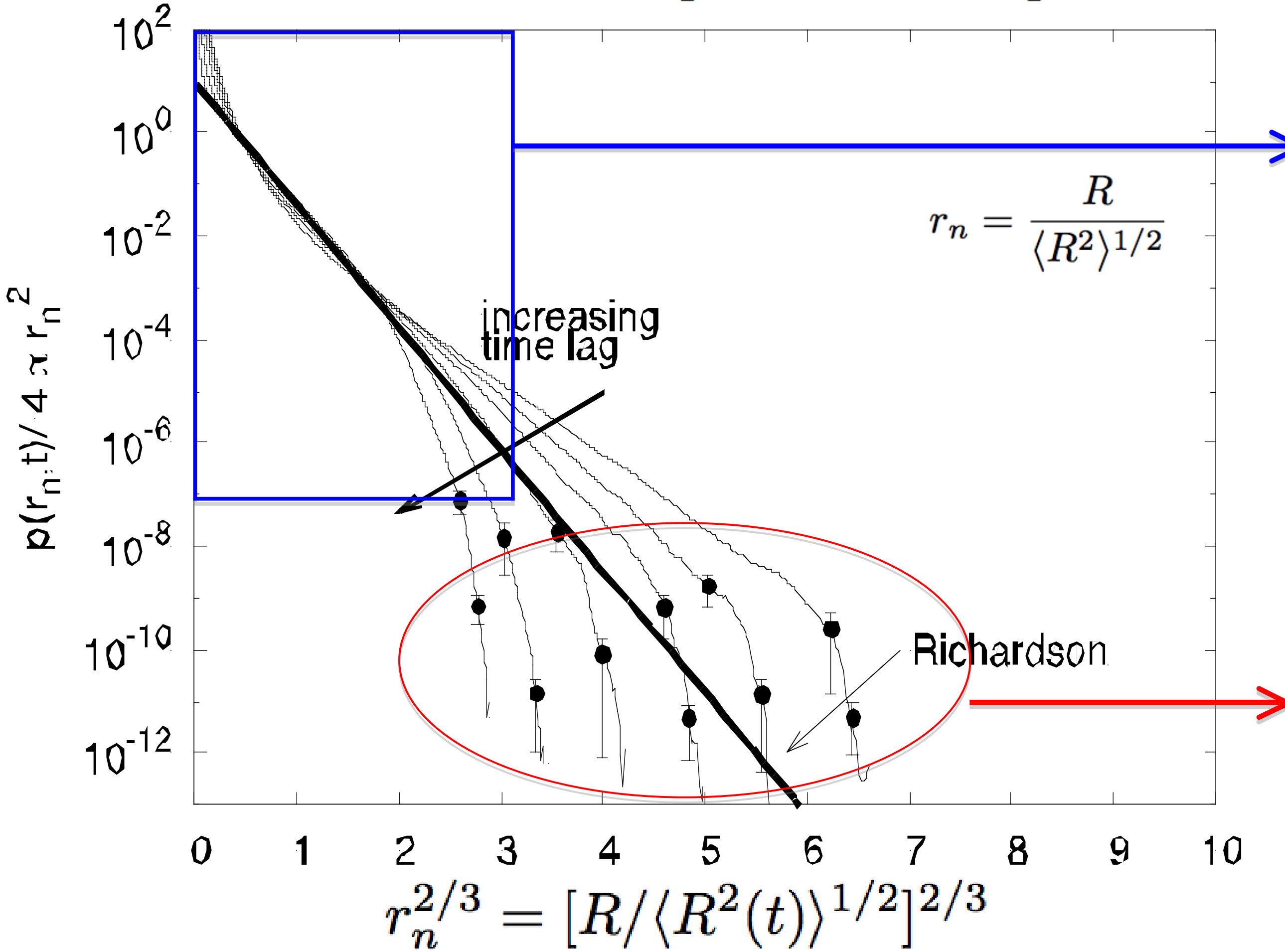
Fast separating pairs

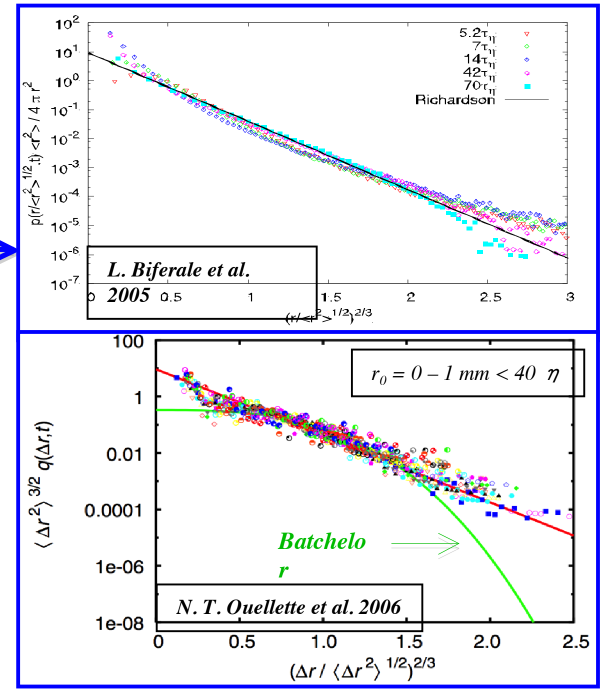


- Exponential-like tails with a sharp drop at a cut-off separation $r_c(t)$.
- This cut-off scale is the signature of tracers pairs experiencing a persistent high relative velocity, which is limited by U_{rms}

Comparison with Richardson's PDF

$$\mathcal{P}\left(\frac{R}{\langle R^2\rangle^{1/2}}\right) \simeq \frac{R^2}{\langle R^2\rangle} \exp\left[-A\left(\frac{R}{\langle R^2\rangle^{1/2}}\right)^{2/3}\right]$$





Strong departures from the ideal self-similar Richardson distribution

Deviations due to finite Reynolds effects, multifractality, temporal correlations

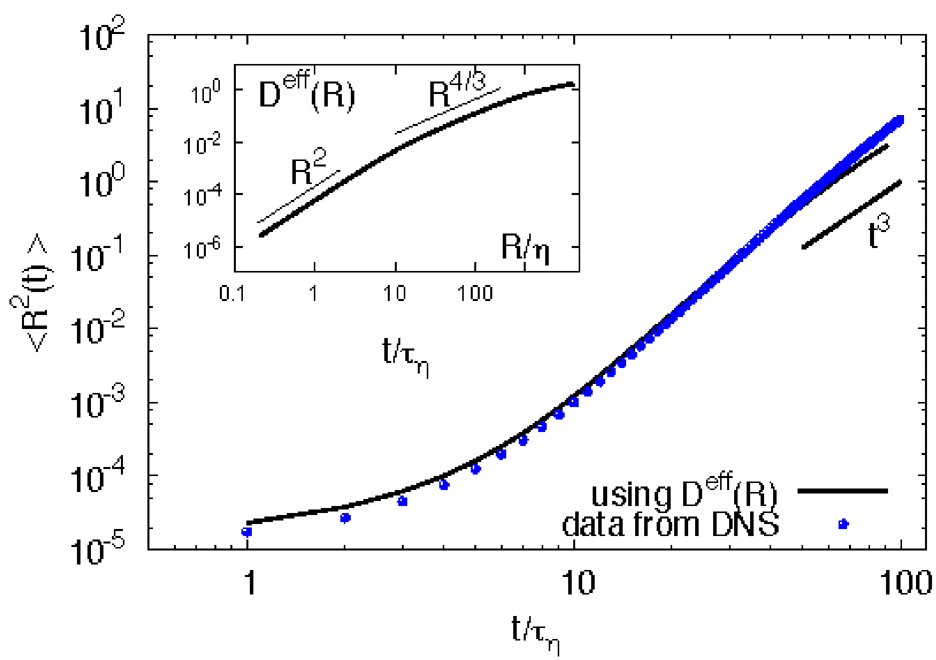
Eddy-Diffusivity Model with IF & UV cut-offs

Integrate Richardson equation using an Effective Turbulent Eddy-Diffusivity that takes into account the small and large scale behaviours

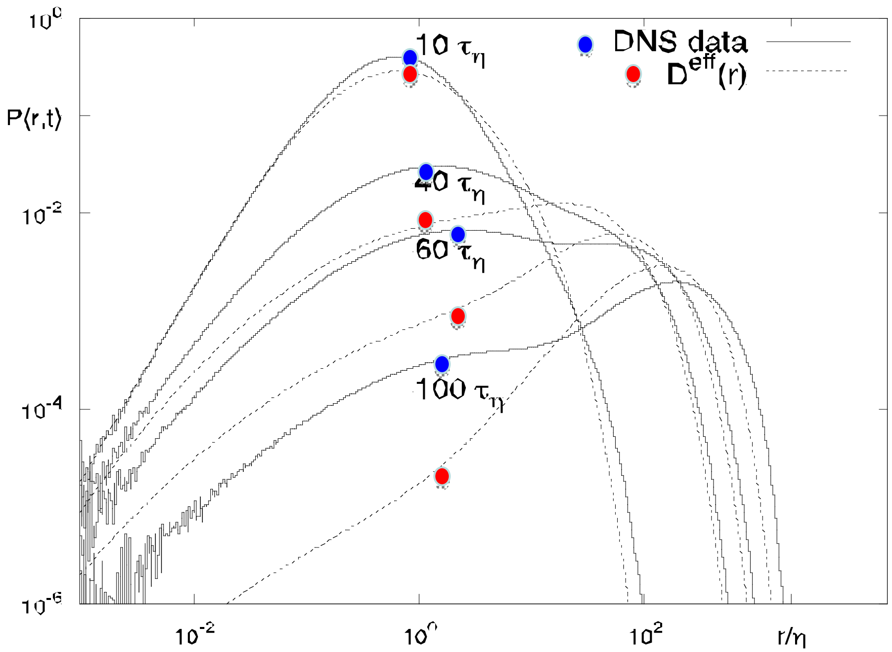
$$\partial_t P(R,t) = \frac{1}{R^2}\frac{\partial}{\partial_R}\left[D^{eff}(R,t)\, R^2 \frac{\partial P(R,t)}{\partial_R}\right]$$

$$\boxed{D^{eff}(R,t) = \tau(R)\langle\Big[(\mathbf{u}(\mathbf{x}_1+\mathbf{R}) - \mathbf{u}(\mathbf{x}_1))\cdot\hat{\mathbf{R}}\Big]^2\rangle}$$

$$\begin{cases} D^{eff}(R,t) \propto R^2 & R \ll \eta \\ D^{eff}(R,t) \propto R^{4/3} & \eta \ll R \ll L \\ D^{eff}(R,t) \propto cost. & L \ll R \end{cases}$$

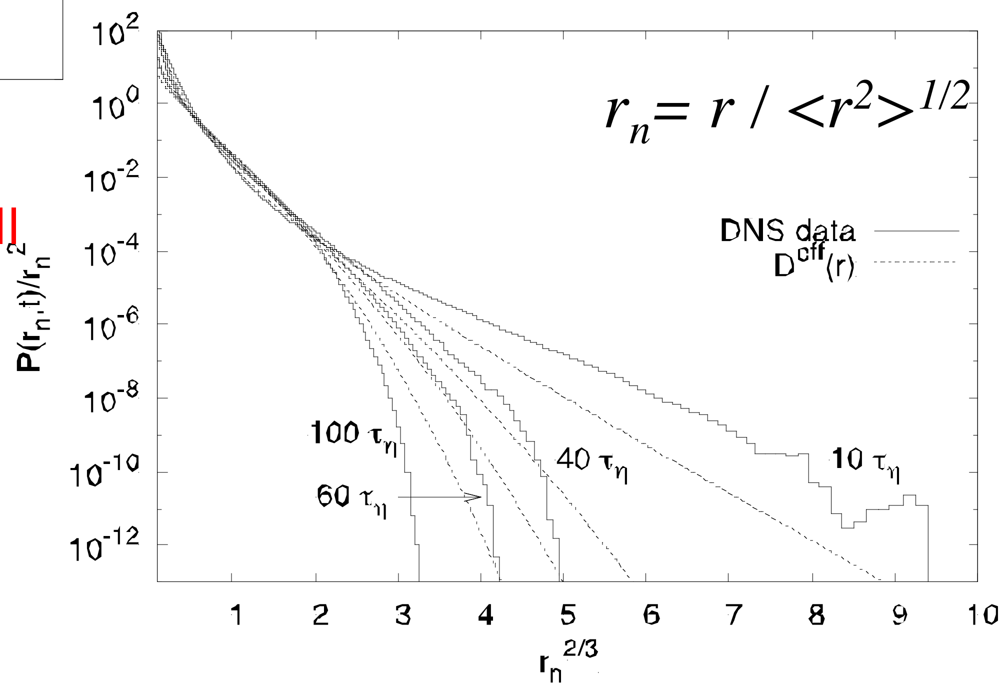


Model - NS comparison



At small & large separation, model goes in the correct direction, but it is not enough.

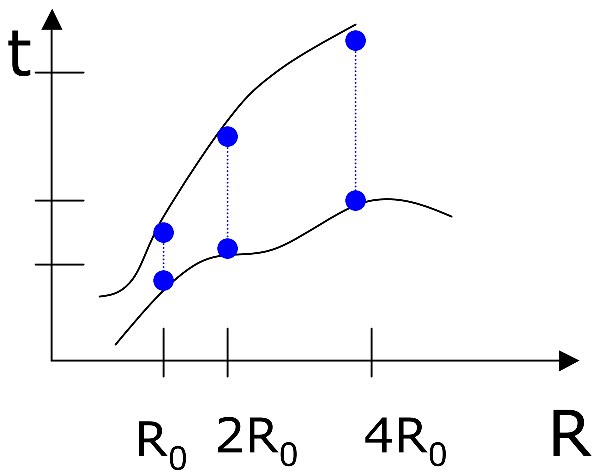
Deviations from Gaussianity at small Separations & Time Correlations at large R NEED to be modeled also.



Scatamacchia et al., PRL (2012)

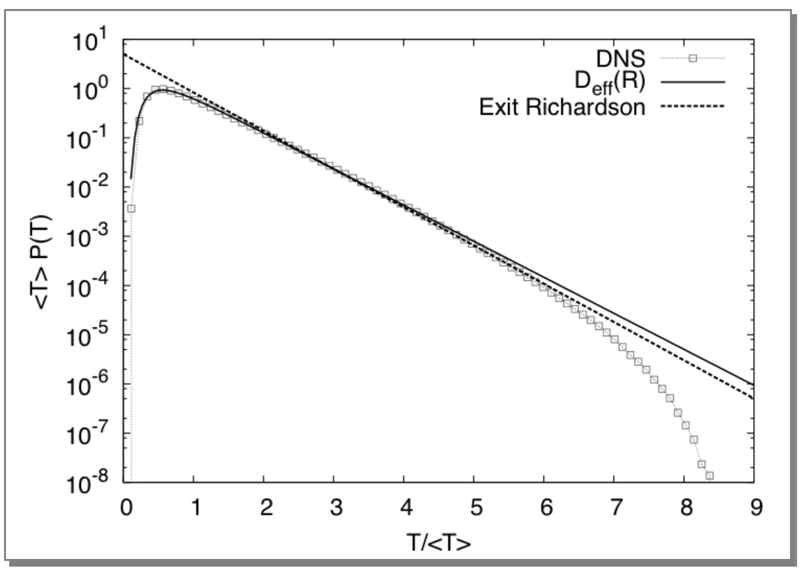
Exit time statistics

Boffetta Sokolov, PRL 2002

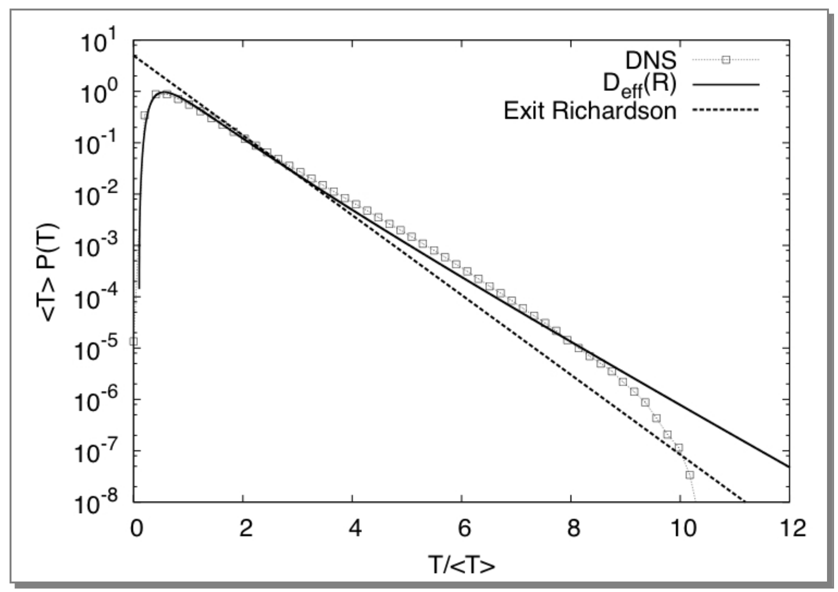


Measure statistics of time lag $T_\rho(R)$ needed to have pair distance growing from $r_0=R_0$ to $r_n = \rho^n R_0$.
An asymptotic exit time PDF can be obtained from the Richardson distribution.

$$\boxed{\mathcal{P}_{\rho,r_n}(T) \approx \exp\left(-C\,\frac{T_\rho(r_n)}{\langle T_\rho(r_n)\rangle}\right)}$$



Exit time PDF : $r_n=10\ \eta$ and $r_{n+1}=50\ \eta$



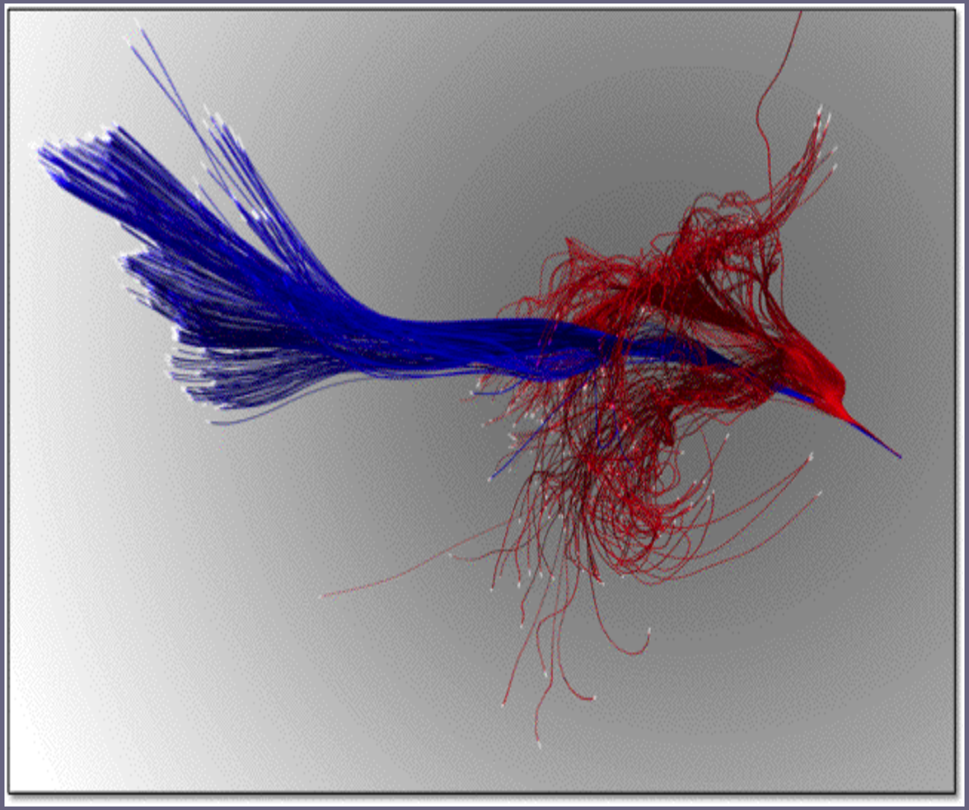
Exit time PDF : $r_n=2\ \eta$ and $r_{n+1}=10\ \eta$

POINT SOURCE EMISSION

HEAVY PARTICLES

$$\frac{d\mathbf{V}}{dt} = \frac{\mathbf{u} - \mathbf{V}}{\tau_p}$$

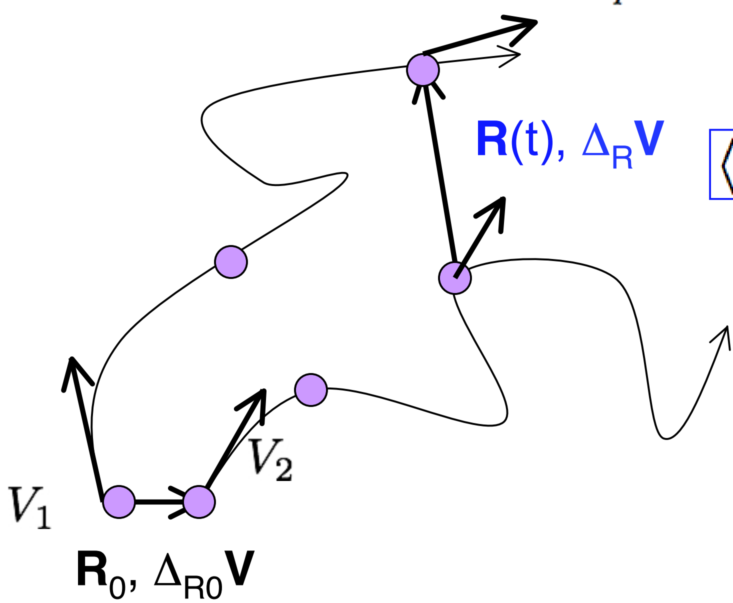
$$St = \frac{\tau_p}{\tau_\eta} = [0.6; 1; 5]$$



Modeling Relative Separations of Inertial Particles

Lagrangian framework

$$\begin{cases} \mathbf{R}(t) = \mathbf{X_1}(t) - \mathbf{X_2}(t) \\ \dot{\mathbf{R}}(t) = \mathbf{V}_1(\mathbf{X}_1, t) - \mathbf{V}_2(\mathbf{X}_2, t) = \Delta_R \mathbf{V}_{St}(\mathbf{R}(t), t) \\ \ddot{\mathbf{R}}(t) = \beta D_t \Delta_R \mathbf{u}(\mathbf{R}(t)) + \dfrac{1}{\tau_p}\left[\Delta_R \mathbf{u}(\mathbf{R}(t)) - \dot{\mathbf{R}}(t)\right] \end{cases}$$

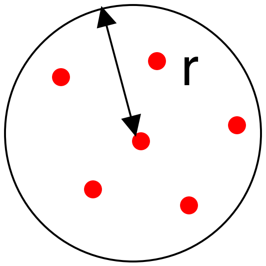


$$\boxed{\langle (R(t))^2 | R_0, t=0\rangle_{St,\beta} = \langle |\mathbf{X_1}(t) - X_2(t)|^2\rangle_{St,\beta}}$$

$$\boxed{P(R, t|R_0, t_0)_{St,\beta}}$$

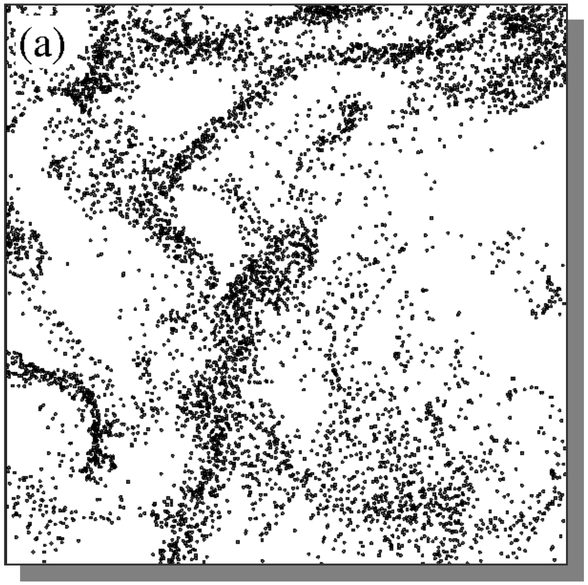
Bec et al. JFM 645 (2010); Bec et al. JFM 646(2010)

Heavy particles *stationary* mass distribution

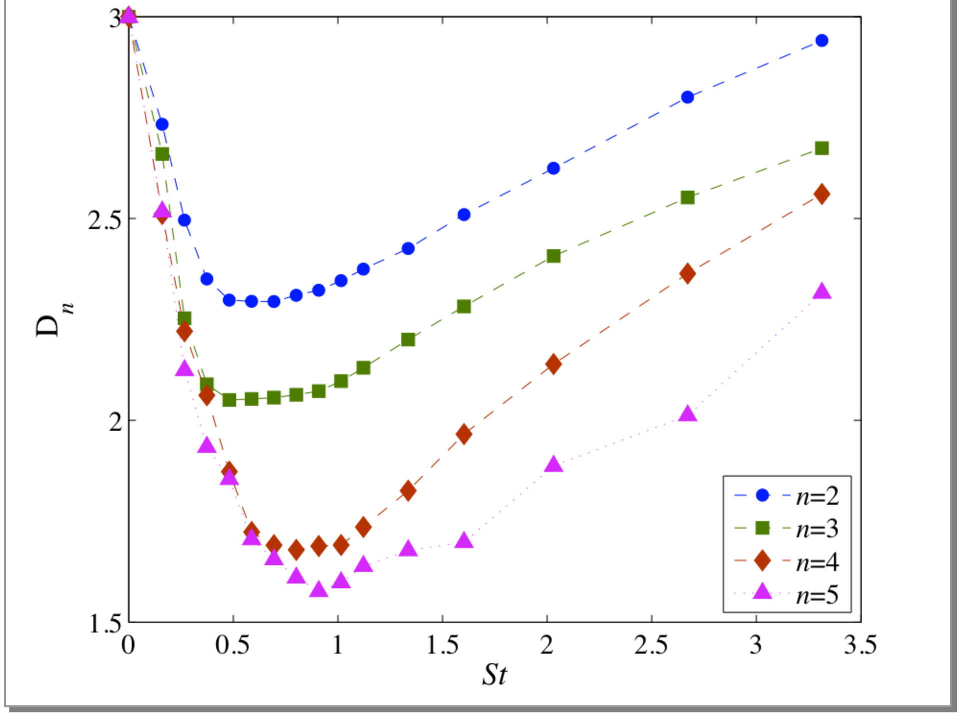


Inertial particles dynamics possess a singular measure in phase space.
By large-deviation formalism for dissipative dynamical systems,
the moments of mass distribution at separation r < η:

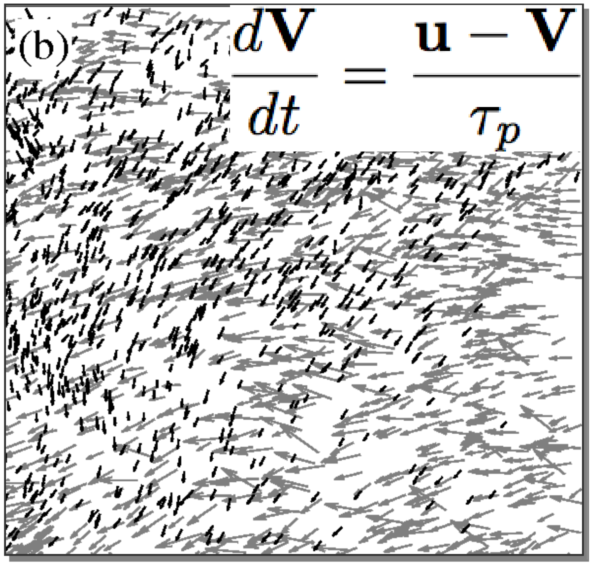
$$M_r^{(q)}(St) \equiv \langle \sum_{i=1}^{N_r(St)} m_r^q \rangle \sim r^{(q-1)D_q(St)}$$



Generalised Fractal Dimensions



Heavy particles ***stationary*** velocity distribution

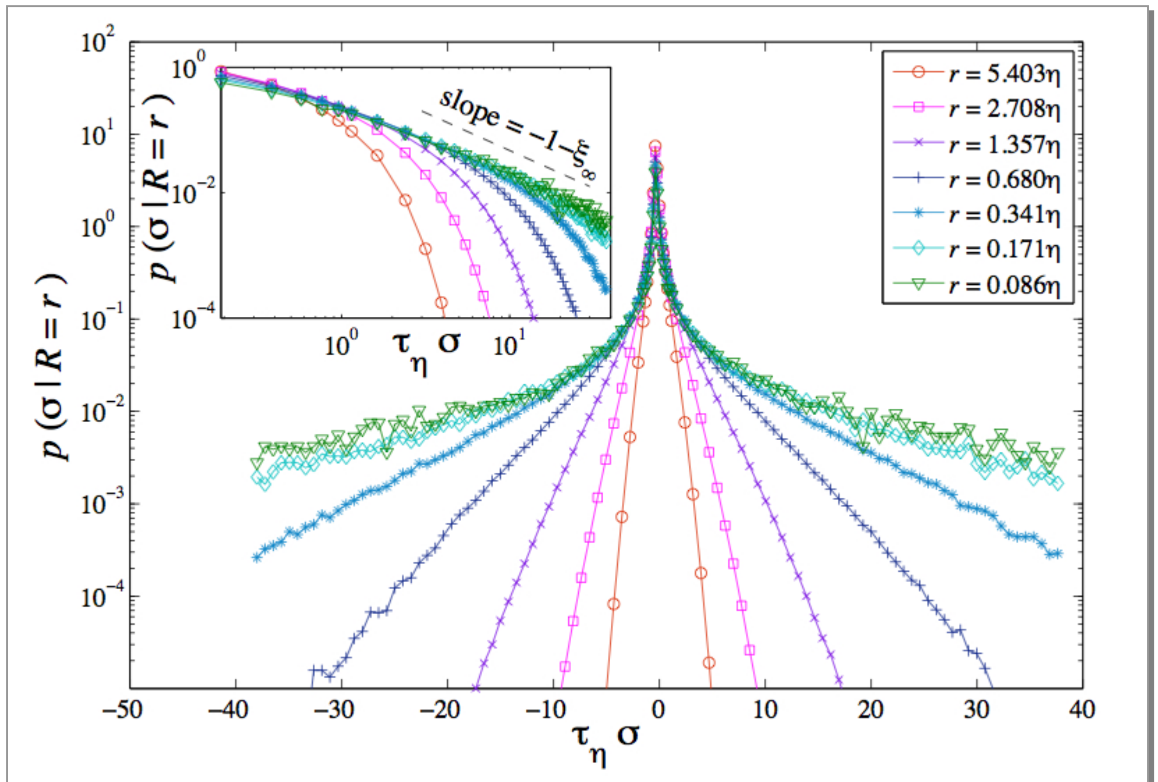


$$\frac{d\mathbf{V}}{dt} = \frac{\mathbf{u}-\mathbf{V}}{\tau_p}$$

Velocity statistics is due to the competition of *smooth fluctuations* with *quasi-singular events (caustics)* of very large velocity fluctuations at close points

PDF of rescaled velocity differences

$$\sigma_{St} = \frac{\Delta_R V_{\|}^{(St)}}{R}$$

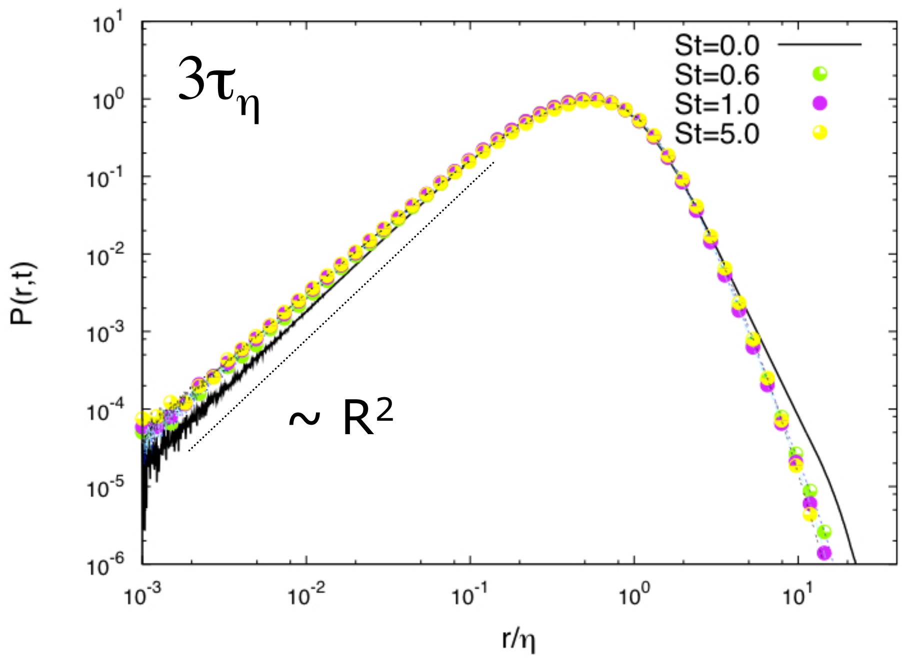


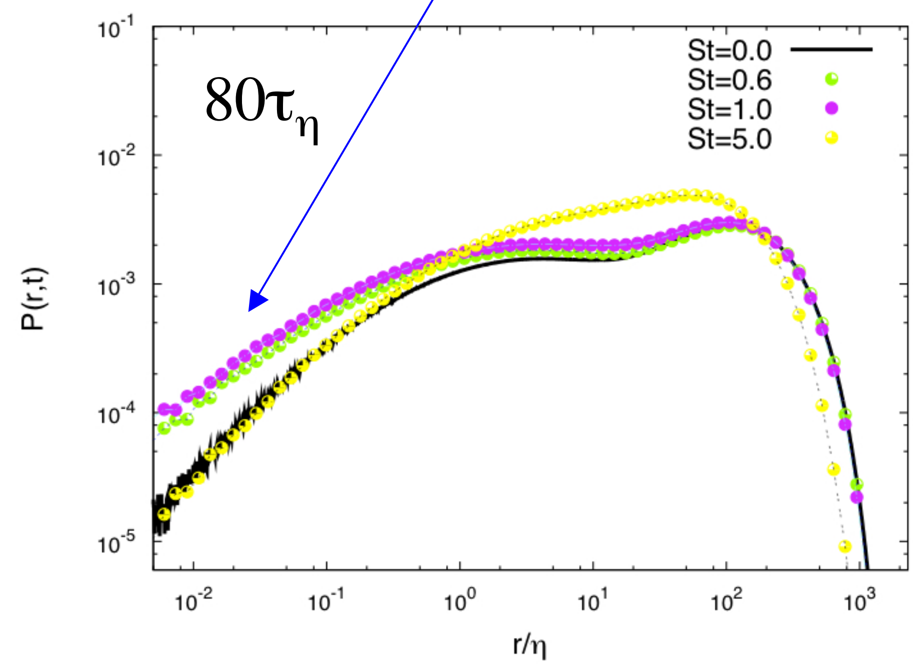
HEAVY PAIRS SEPARATION PDF

from localised source

Behaviour at small separations

preferential concentration





$$\boxed{P_{St=0}(R,t) \propto \frac{AR^2}{(\epsilon^{1/3}t)^{9/2}} \exp\left(-C\,\frac{R^{2/3}}{t}\right)}$$

tracers

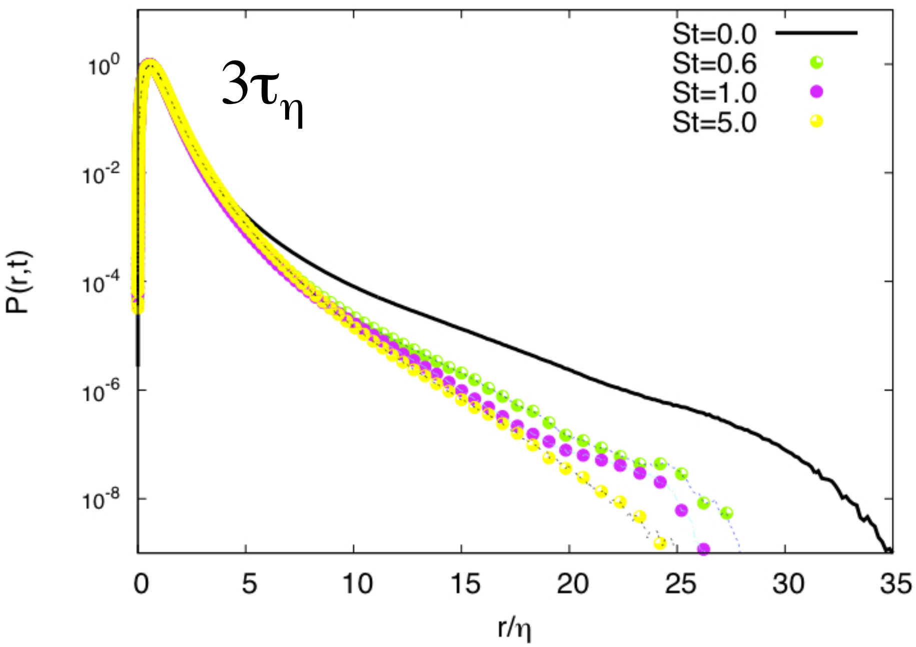
Heavy ~at small separations *like tracers* in rough & compressible Kraichnan flow

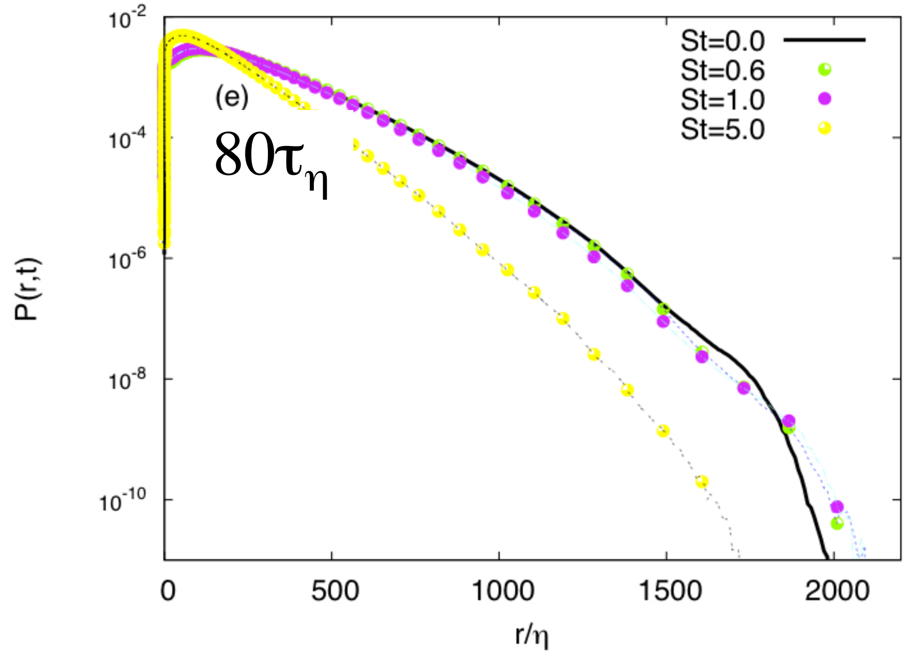
$$\boxed{\mathcal{P}_{\xi,\mu}(R,t) \propto \frac{R^{D_2-1}}{t^{(d-\mu)/(2-\xi)}} \exp\left[-A\,\frac{R^{2-\xi}}{t}\right]}$$

HEAVY PAIRS SEPARATION PDF

from localised source

Behaviour at large separations



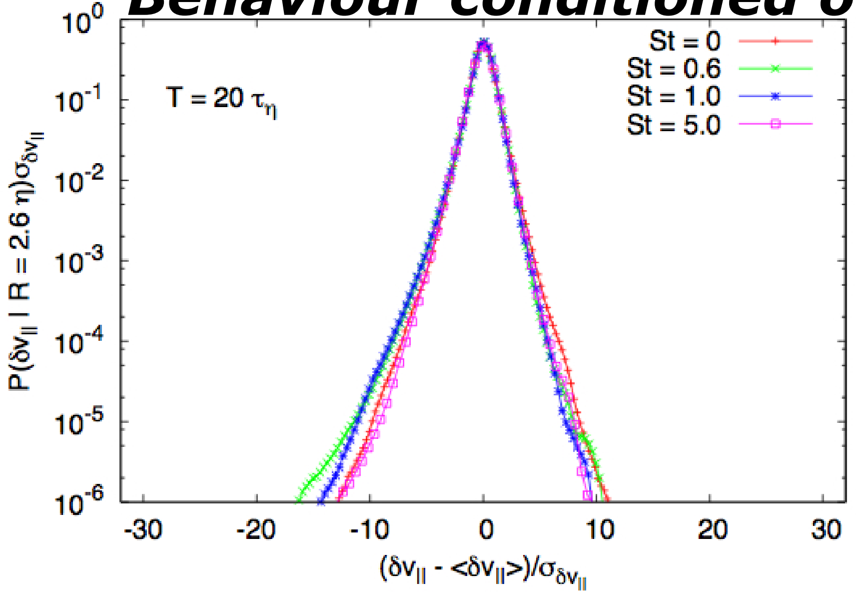


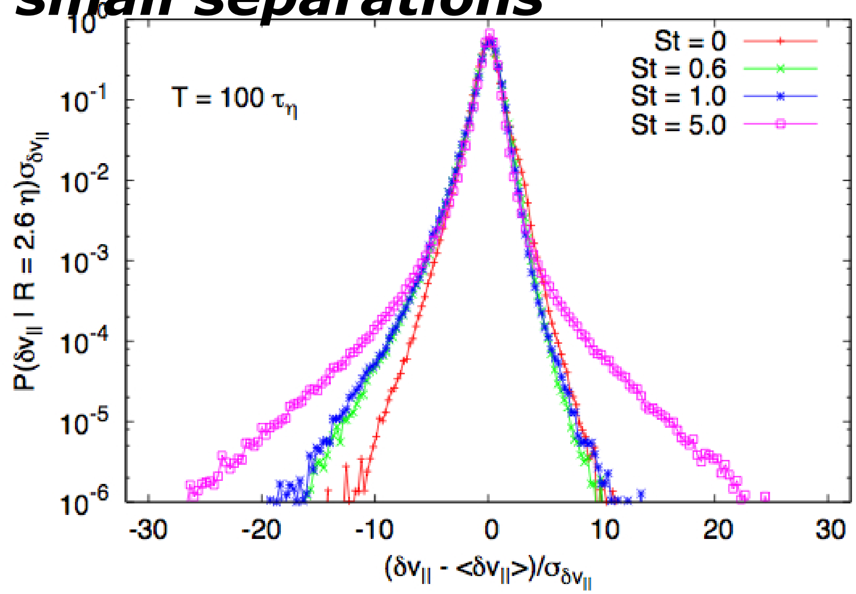
Heavy ~ for small Stokes, heavy pairs *separation dynamics relaxes onto the tracers one.*

For large Stokes, inertia prevents very large separations (i.e. intermittency is depleted)

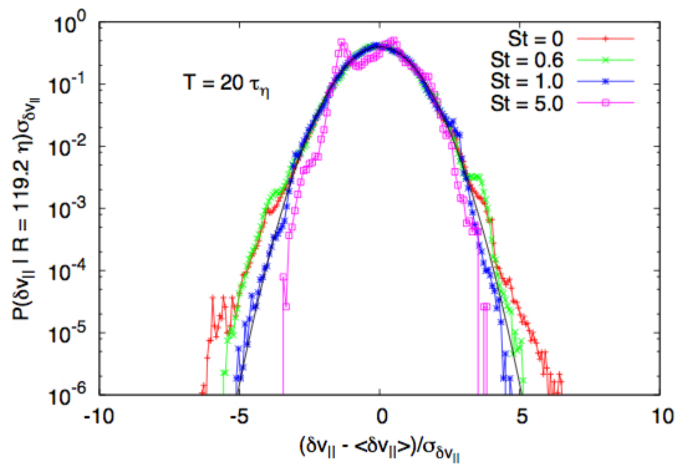
Emergence of caustics: PDF of velocity differences

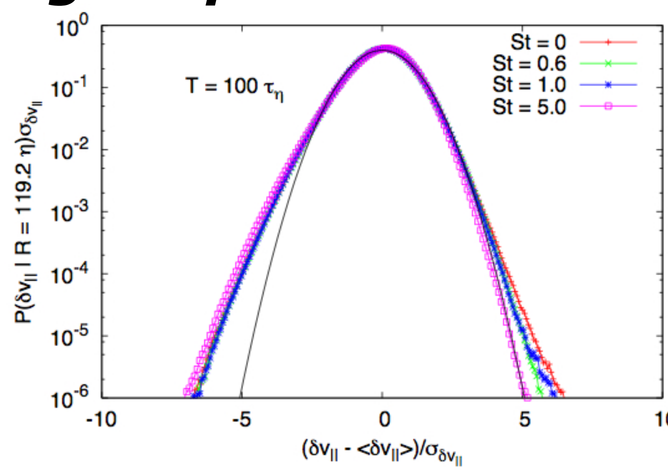
Behaviour conditioned on small separations



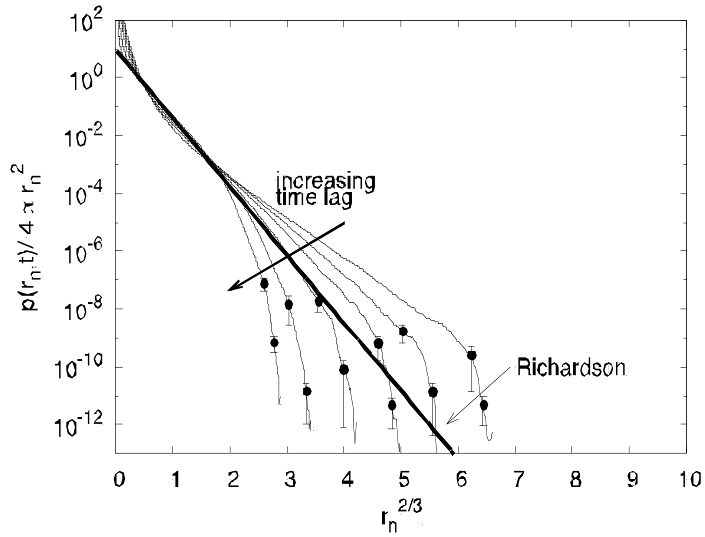


Behaviour conditioned on large separations





Conclusions



TRACERS

For tracers dispersion,

big violation to Richardson model are detectable

Finite size effects can be included
But they are not sufficient

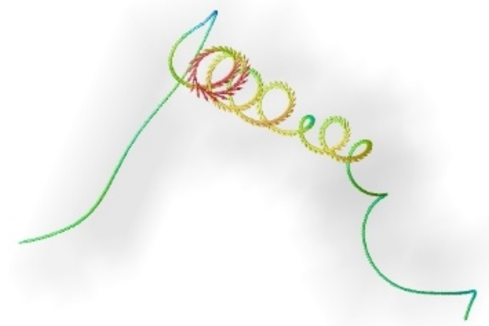
Gaussianity hypothesis at the base of $D_{eff}(R)$ can not capture stretching rate fluctuations responsible of *slowly separating pairs.*

Temporal correlations (including those associated to intermitent behaviours) are still a critical point to describe *fast separating pairs*.
(see e.g. *Bitane, Homann, Bec PRE 2012*)

INERTIAL PARTICLES

No reference theory, just observations

Thanks to



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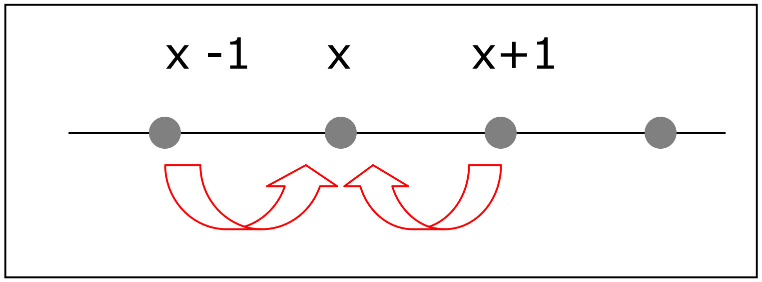
Diffusion process: basic



1D random walk: simplest case

$$p_{n+1}(x) = \frac{1}{2} p_n(x-1) + \frac{1}{2} p_n(x+1)$$

In the continuous limit, $\Delta t \to 0$ and $\Delta x \to 0$, with $D_0 = \Delta x^2/2\Delta t$

$$\boxed{\frac{\partial\, p(x,t)}{\partial t} = D_0 \frac{\partial^2 p(x,t)}{\partial x^2}}$$

Gaussian solution

$$p(x,t) = \frac{1}{\sqrt{4\pi D_0 t}}\, exp\left(-\frac{x^2}{4D_0 t}\right)$$

Diffusive flux depends on gradient particle number density

$$\Gamma(\mathbf{x},t) = -D_0 \nabla n(\mathbf{x},t)$$

Inverse Exit time moments & Eulerian intermittency

From exit-time PDF, we can measure moments dominated by fast separating pairs:

$$\langle \frac{1}{[T_\rho(R)]^p} \rangle \approx R^{-\alpha(p)} \quad ?$$

Parisi Frisch 1985; Paladin Vulpiani 1987

Using Multifractal formalism for Eulerian statistics:

$$\langle [\delta_r u(r)]^q \rangle \approx c_q r^{\zeta_q} \Rightarrow \boxed{\langle \frac{1}{[T_\rho(r)]^p} \rangle \simeq \langle \left(\frac{r}{\delta_r u}\right)^p \rangle \simeq \frac{1}{T_L^p} \left(\frac{r}{L_0}\right)^{\zeta(p)-p}}$$

Eulerian velocity intermittency clearly affects the way particles separate