Coarsening in diluted systems

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Abstract

We investigate domain growth after a quench to the unstable region of the phase diagram on a lattice lacking translational invariance due to the presence of voids: the spanning cluster of random site percolation above and at p_c. The dynamical process is analyzed using a Ginzburg-Landau model with Langevin dynamics, studied numerically by means of a cell dynamical simulation in $d = 2$ for conserved order parameter at $T_F = 0$. We find that for short times ordering proceeds as on compact substrates ($L(t) \sim t^{1/3}$); later on, when $L(t)$ becomes comparable to the typical distance separating voids, coarsening occurs only via activated processes and for $T_F = 0$ the system freezes out of equilibrium.

1 INTRODUCTION

We consider the conserved order parameter kinetics of a thermodynamic system, such as an alloy or a fluid mixture, confined to a porous medium and suddenly brought out of equilibrium by cooling it from a high temperature homogeneous state to the unstable region. We model the disordered substrate as the spanning cluster of random site percolation above and at p_c. The high temperature initial state is characterized by an average value $\phi(0)$ of the order parameter and lower the temperature. Such quench renders the system thermodynamically unstable for times $t > 0$ and a phase-separation process begins with the formation of domains whose typical size $L(t)$ grows with time. On compact substrates $L(t)$ displays the asymptotic power law behavior $t^{1/3}$.

Which behavior characterizes a non translationally invariant system?

2 MODEL

On each lattice site belonging to the percolator (Fig. 1) and whose coordinates we represent by x, we consider a scalar order parameter ϕ_x whose properties depend on a standard Ginzburg-Landau

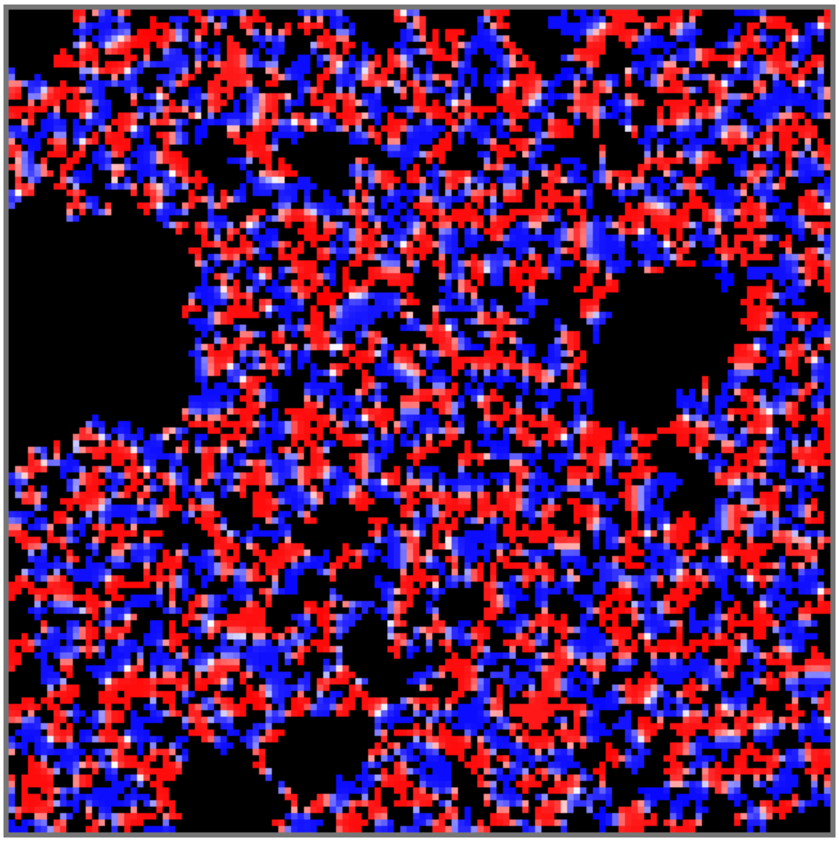

Figure 1: One realization of the system for $p = 0.63$ during the early stage of evolution. Black sites are empty. The other colors indicate the local value of the order parameter field: white regions are interfaces separating blue and red phase-separated domains.

free energy functional:

$$H[\{\phi\}] = -\frac{1}{2}\sum_{x,y}\phi_x\Delta_{xy}\phi_y + \sum_x f(\phi_x) \tag{1}$$

where $f(\phi)$ is a function with a double well shape to account for the coexistence of the two ordered phases, while the first term is proportional to the surface energy, which in the continuum limit can be represented by a square gradient. The evolution towards equilibrium of the order parameter, ϕ_x, at the site x is described by the Ginzburg-Landau equation for a conserved order parameter:

$$\frac{\partial\phi_x(t)}{\partial t} = \Gamma_0\Delta_{xy}\frac{H[\{\phi(t)\}]}{\delta\phi_y(t)} + \eta_x(t) \tag{2}$$

Here $\eta_x(t)$ represents a Gaussian white noise with zero average and variance :

$$<\eta_x(t)\eta_y(t')> = -2T_F\Gamma_0\Delta_{xz}\delta_{zy}\delta(t-t')$$

where T_F is the temperature of the final equilibrium state while Γ_0 is a kinetic coefficient and δ_{xy} the Kronecker symbol. By substituting eq. (1) into (2) we find:

$$\frac{\partial\phi_x(t)}{\partial t} = \Gamma_0\Delta_{xy}[\Delta_{yz}\phi_z(t) + f'(\phi_y(t))] + \eta_x(t) \tag{3}$$

The diffusion operator Δ has the following representation:

$$\begin{aligned} \Delta_{xy} &= \quad 1 \qquad \text{if} \;\; x,y \;\; \text{are nearest neighbor sites} \\ \Delta_{xx} &= \; -Z_x \\ \Delta_{xy} &= \quad 0 \qquad \text{otherwise.} \end{aligned}$$

where $1 \leq Z_x \leq 4$ is the coordination number of site x. Periodic boundary conditions are assumed.

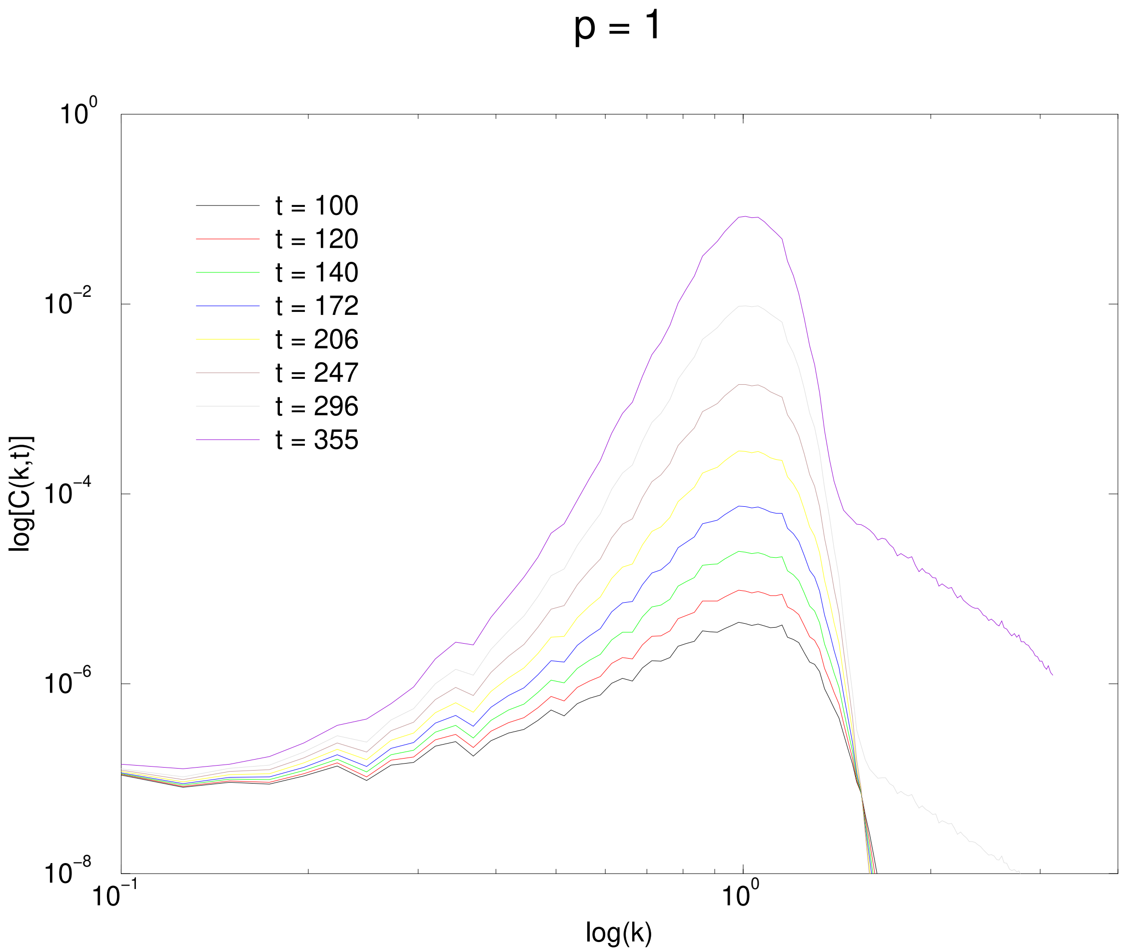

Figure 2: Evolution of the structure factor during the early stage for $p = 1$.

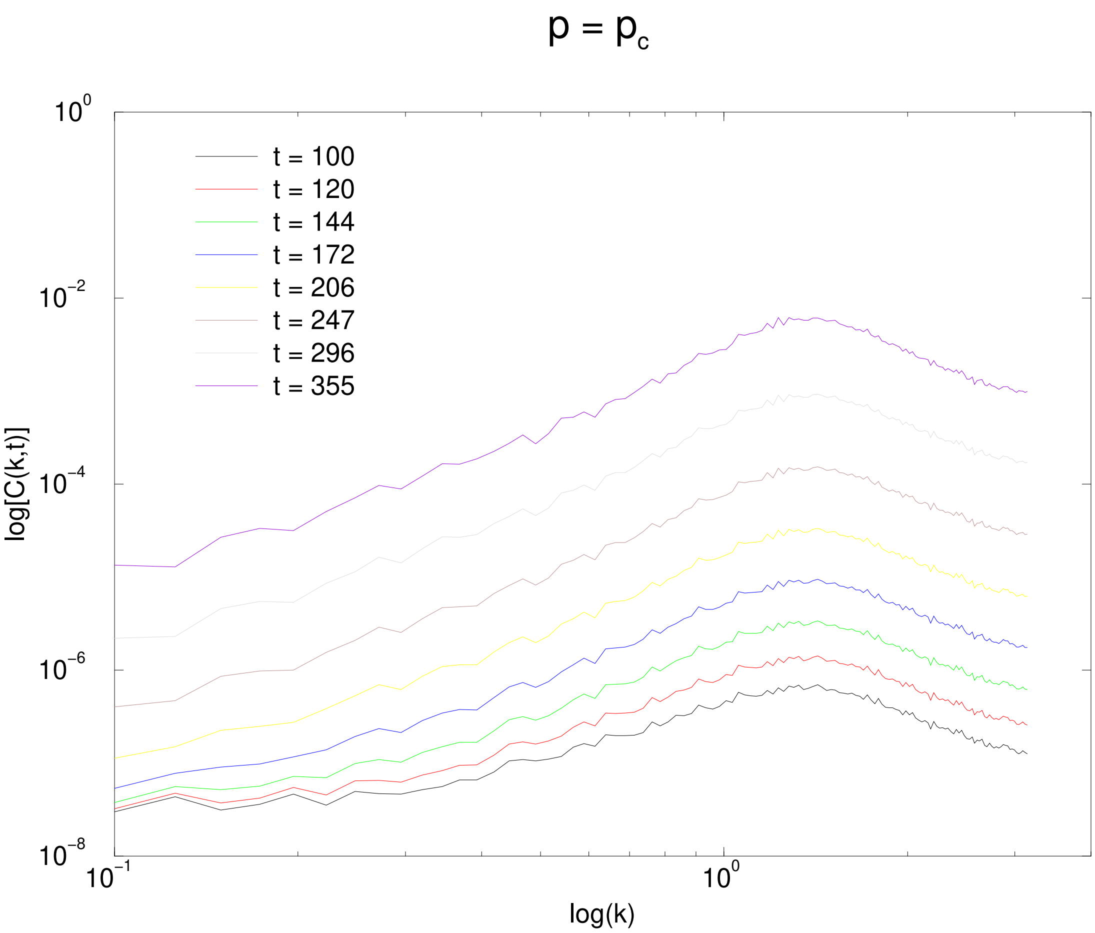

Figure 3: Evolution of the structure factor during the early stage for $p = p_c$.

3 EARLY TIME EVOLUTION

- In translationally invariant systems during the initial regime the structure factor forms a peak located at $k_m = \sqrt{-r/2}$, where $r = \partial^2 f(\phi)/\partial\phi^2|_{\phi=0}$, and growing in time as $C(k,t) = \exp(r^2 t/4)$. The wavevector $k_c = \sqrt{-r}$ discriminates between the soft growing modes and the hard damped modes (Fig. 2).

- On the percolator, instead, the peak grows around a wavevector k_m, larger and broader than the previous one, due to the randomness of the system (Fig. 3). This indicates that the presence of voids determines the existence of localized modes of low energy.

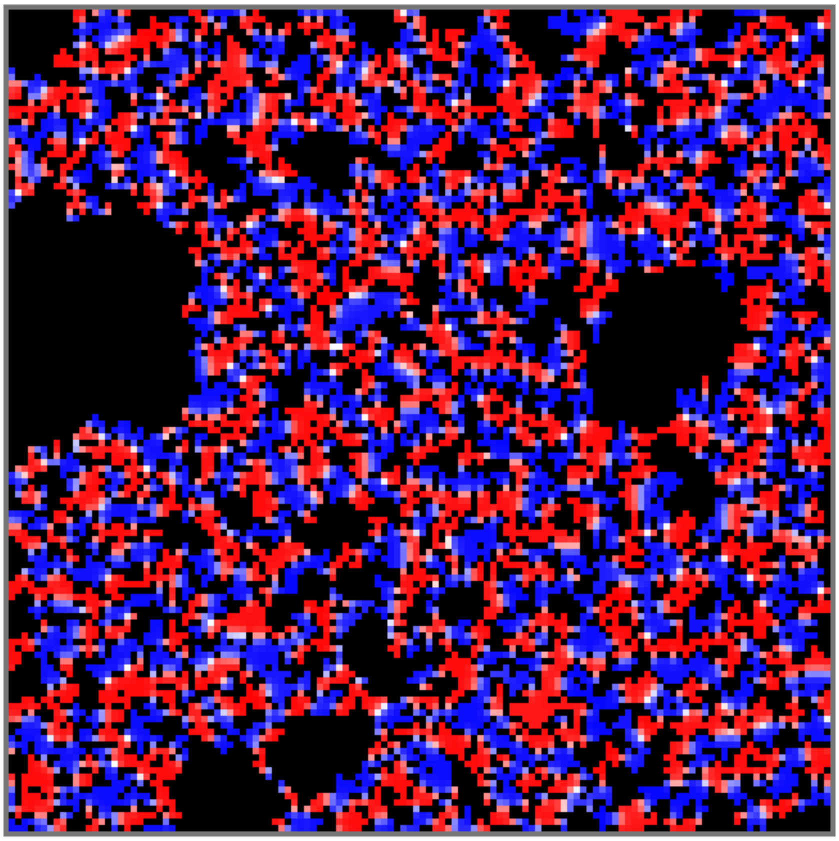

Figure 4: Frozen asymptotic configuration for $p = 0.95$.

4 LATE STAGE

After their formation during the early stage, domains start coarsening in order to reduce their free energy. Each portion of the interface moves with a velocity proportional to the square of its local curvature.

- In compact systems this leads for the typical domain size to a power law growth $L(t) \sim t^{1/3}$ independent of $T_F < T_c$.

- On the percolator one expects the same behavior as long as $L(t)$ is smaller than the typical distance ξ between lacunes in the substrate. Voids tend to pin interfaces in configurations which minimize locally the free energy. For $L(t) = L_0 \simeq \xi$ the system can set the driving force to zero by reaching a configuration with straight domain boundaries connecting empty sites (Fig. 4). The buildup of longer range correlations is prevented by voids, which set a scale for the system related to their typical separation.

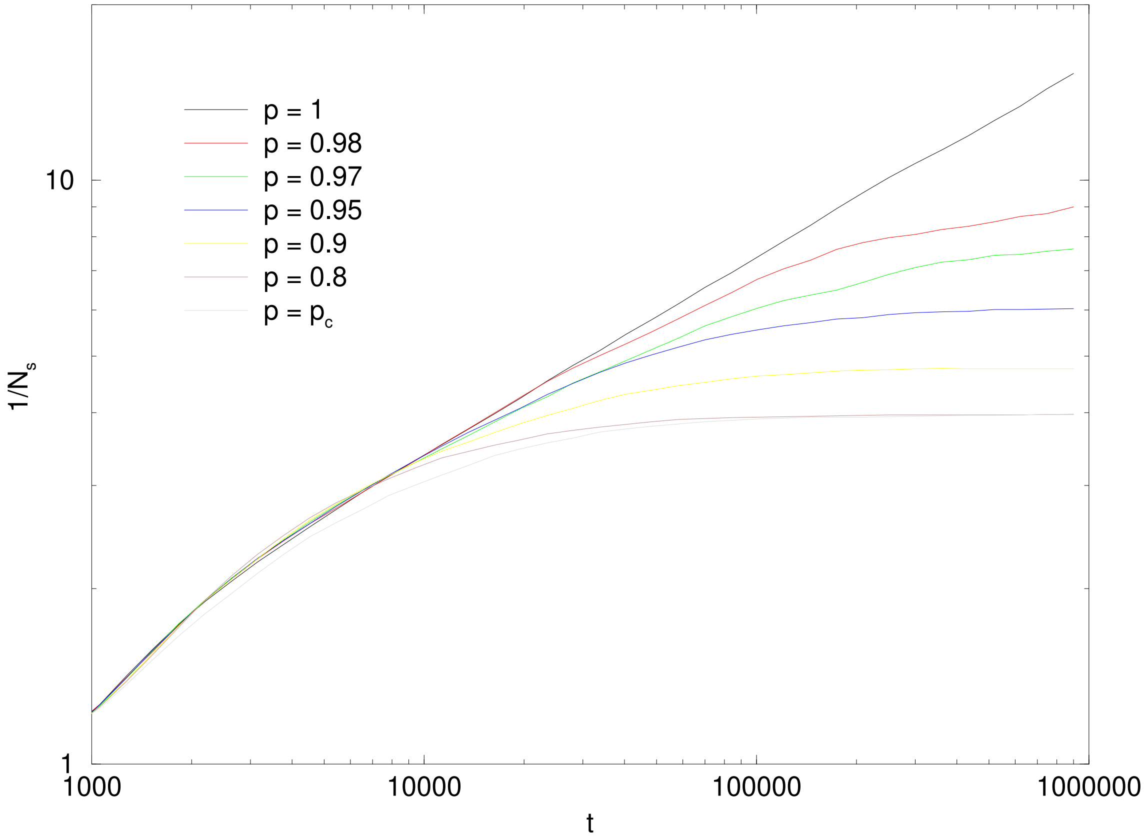

Figure 5: Time evolution of the inverse of the total interface length N_s, a measure of the typical domain size.

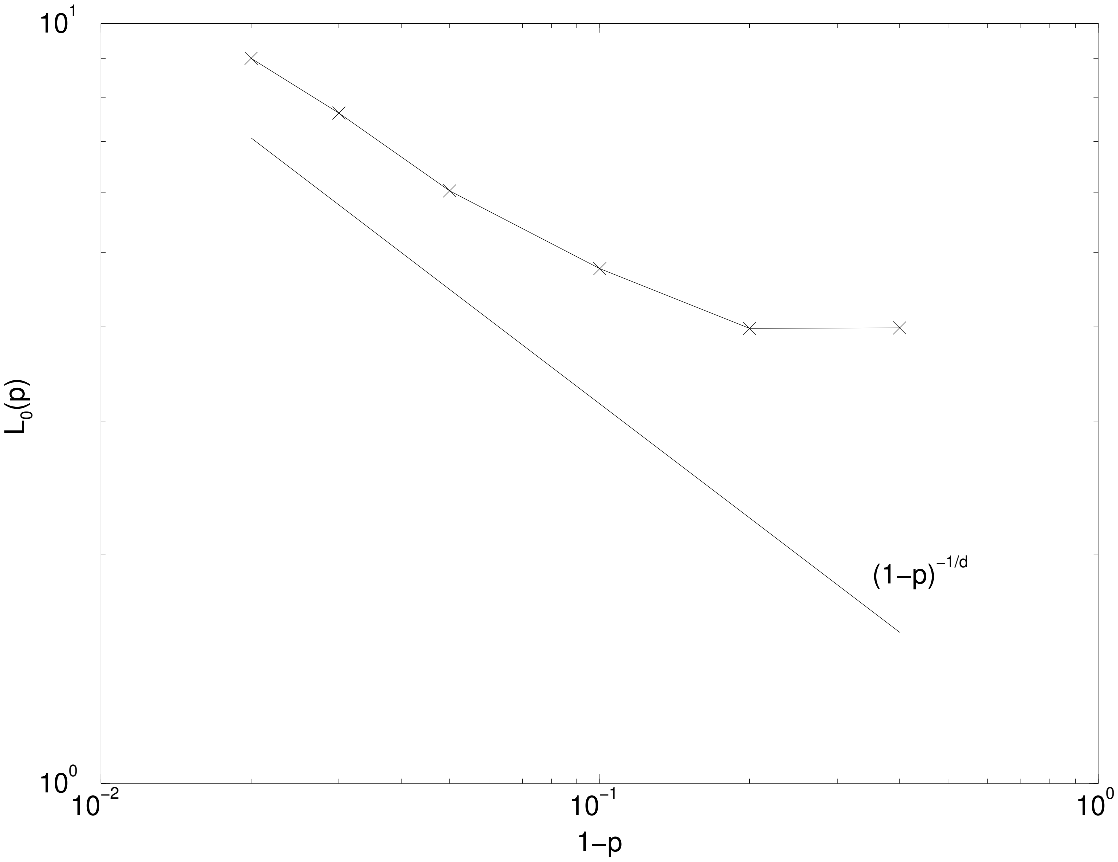

Figure 6: Asymptotic value of the inverse of N_s as a function of the dilution.

- For $T_F > 0$ thermal fluctuations can bring the system out of these metastable states; the resulting dynamics is activated and $L(t)$ grows logarithmically.

- For our simulations at $T_F = 0$ no activation process is present and the system remains frozen (Fig. 5). ¿From the dependence of ξ on p we expect $L_0 \sim (1-p)^{-1/d}$ for p close to 1. This is verified numerically (Fig. 6).

Acknowledgements

References

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