

## GROWTH KINETICS IN THE $\Phi^6$ $N$ -COMPONENT MODEL, CONSERVED ORDER PARAMETER

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We extend the discussion of the growth kinetics of the large  $N$  time-dependent Ginzburg-Landau model with an order parameter described by a  $\Phi^6$  free energy functional to the conserved case. Quenches from a high temperature initial state to a zero-temperature state are studied for different selections of parameters entering the effective potential.

In all cases we obtain the asymptotic form of the structure factor. As expected in analogy with the well-studied  $\Phi^4$  model, we find multiscaling behavior whenever stable equilibrium is reached in the ordering region. On the other hand the present model also displays a novel feature, namely, the occurrence of metastable relaxation.

### 1. Introduction

In a previous paper,<sup>1</sup> hereafter referred as I, we examined the growth kinetics of an  $N$ -component field with a nonlinear local interaction of the sixth order, namely, a  $\Phi^6$  term, subjected to a sudden quench from a high temperature initial state. The equations of the large  $N$  spherical model have been deduced and solved in the case of a nonconserved order parameter (NCOP or model A) at zero temperature.

In the present paper we turn our attention to the kinetics of the conserved parameter case (COP of model B).

In contrast with the nonconserved case one can observe multiscaling<sup>2</sup> behavior whenever stable equilibrium configurations are reached in the presence of a potential with two degenerate minima (see I, Fig. 1). The breakdown of standard scaling<sup>3</sup> is due to the same mechanism as for the  $\Phi^4$  model:<sup>4</sup> two distinct dominant lengths compete in the late stage of the dynamical process, namely, the dimensional length  $L(t) = (2\Gamma t)^{1/z}$ , with  $z = 4$ , and the wavelength  $k_m^{-1}$  of the peak of the structure factor. Since these two lengths differ only by a logarithmic factor one cannot speak of a single dominant length. As a consequence, each mode  $k$  evolves with a different exponent, a behavior which has been named multiscaling.

In analogy with the NCOP, the  $\Phi^6$  model exhibits also metastable solutions which are absent in the spherical  $\Phi^4$ . In the following section, in fact, we shall extend a theorem concerning the metastability of the model which is the equivalent of the one already proved for the NCOP. This suggests that the metastability is not due to the kind of relaxation dynamics assumed, but it is a property of the functional form of the free energy.

The presence of metastability is interesting because, as stressed in I, it is commonly believed that the metastable solutions disappear when taking the limit  $N \rightarrow \infty$ .

The present paper is organized as follows. In Sec. 2 we solve the model by explicit integration of the equations of motion for the structure factor in the asymptotic regime. Since these equations have been obtained in the first part of this work we refer to I for a more detailed presentation of the model and of its large  $N$  limit, as well as for a discussion of the initial conditions. Finally in the last part we summarize, discuss some results, and conclude the work.

## 2. Solution of COP the Model

We start by considering the conserved model ( $p = 2$ ) explicitly and solving the equation of motion for the structure factor, Eq. (12) of I. In the present case, introducing the scaling function  $F(\mathbf{x}) = e^{-x^4}/x^\theta$  of the dimensionless wave vector  $\mathbf{x} = \mathbf{k}L(t)$ , Eq. (10) of I can be formally integrated, yielding

$$C(\mathbf{k}, t) = \Delta e^{-\beta(t)x^2} L^\theta(t) F(\mathbf{x}), \quad (1)$$

where

$$\beta(t) = \text{sign}(Q(t)) \left[ \frac{\mathcal{L}(t)}{L(t)} \right]^2 \quad (2)$$

is the squared rate between the two lengths  $\mathcal{L}(t) = [2\Gamma|Q(t)|]^{1/2}$  and  $L(t)$ , and  $Q(t)$  is obtained by self-consistently eliminating  $S(t)$  from Eq. (9) with the help of Eq. (10) of I. From this equation we obtain

$$S(t) = \Delta L(t)^{\theta-d} \int d^d \mathbf{x} x^{-\theta} e^{-\beta(t)x^2 - x^4}. \quad (3)$$

In order to calculate the integral on the right-hand side one has to consider three different cases, according to the behavior of  $\beta(t)$  in the late stages:<sup>4</sup>

(a)  $\lim_{t \rightarrow \infty} \beta(t) = 0$

In this case  $L(t)$  prevails over  $\mathcal{L}(t)$ . To first order in  $\beta(t)$ , one obtains asymptotically

$$S(t) \simeq \Delta L(t)^{\theta-d} \int d^d \mathbf{x} x^{-\theta} [1 - \beta(t)x^2] e^{-x^4} = \Delta L(t)^{\theta-d} [I_0 - \beta(t)I_1], \quad (4)$$

where

$$I_0 = \int d^d \mathbf{x} x^{-\theta} e^{-x^4} \quad (5)$$

and

$$I_1 = \int d^d \mathbf{x} x^{2-\theta} e^{-x^4}. \quad (6)$$

(b)  $\lim_{t \rightarrow \infty} \beta(t) = \infty$

Now  $\mathcal{L}(t)$  is asymptotically dominant. Since the term  $\beta(t)x^2$  prevails over  $x^4$  in the exponential of Eq. (3), one can disregard the latter and obtain

$$S(t) \simeq \Delta L(t)^{\theta-d} \int d^d \mathbf{x} x^{-\theta} e^{-\beta(t)x^2} \sim \Delta L(t)^{\theta-d} \beta(t)^{\frac{\theta-d}{2}} \quad (7)$$

in the late stages of the dynamics.

Finally one considers

(c)  $\lim_{t \rightarrow \infty} \beta(t) = -\infty$

In this case  $S(t)$  can be evaluated asymptotically by the steepest descent technique:

$$S(t) \sim L(t)^{\theta-d} \Delta e^{\frac{\beta^2(t)}{4}} \beta(t)^{\frac{\theta-d}{2}}. \quad (8)$$

Due to this threefold possibility in the calculation of  $S(t)$  we need to know the qualitative behavior of  $S(t)$  in the late time regime. As in the nonconserved case,<sup>1</sup> it is sometimes (e.g. in cases  $\mu_3$  and  $\mu_6$ ) not *a priori* evident whether the relaxation into the metastable  $\Phi \equiv 0$  configuration is to be expected. Here we are able, as for NCOP, to produce a criterion which establishes under which conditions metastable solutions are allowed.

A necessary and sufficient condition in order to observe relaxation in the  $\Phi \equiv 0$  final state (i.e.  $\lim_{t \rightarrow \infty} S(t) = 0$ ) is to have

$$\frac{\partial^2 V(\phi)}{\partial \phi^2} \Big|_{\phi=0} \geq 0 \quad (9)$$

and

$$S(\bar{t}) \leq \frac{-g - \sqrt{g^2 - 4\lambda r}}{2\lambda}, \quad (10)$$

where  $\bar{t}$  is a generic time instant in the asymptotic region.

We outline the proof of the criterion beginning with the necessary condition Eq. (9), i.e.

$$\left\{ \lim_{t \rightarrow \infty} S(t) = 0 \right\} \Rightarrow \left\{ \frac{\partial^2 V(\Phi)}{\partial \Phi^2} \Big|_{\Phi=0} \geq 0 \right\}. \quad (11)$$

In fact, since

$$\frac{\partial^2 V(\phi)}{\partial \phi^2} \Big|_{\phi=0} = r \quad (12)$$

in the case  $r < 0$ , recalling that  $\lim_{t \rightarrow \infty} S(t) = 0$ , Eq. (9) of I reads

$$Q(\bar{t}) \simeq r \quad (13)$$

for long times. From Eq. (2) one sees that  $\lim_{t \rightarrow \infty} \beta(t) = -\infty$ . Inserting this result into Eq. (8), we obtain

$$S(t) \sim e^{\frac{r^2 L^4(t)}{4}} L(t)^{2(\theta-d)} \quad (14)$$

which is not consistent with the assumption  $\lim_{t \rightarrow \infty} S(t) = 0$ . We conclude that it must be  $r \geq 0$ . The explicit solution of the model proves that consistent solutions effectively exist in this case (see Sec. 3).

Secondly, as regards sufficiency, both Eqs. (9) and (10) are required simultaneously. In fact, from Eq. (9) of I we deduce that if the condition (10) is fulfilled, then

$$\dot{Q}(t) \geq 0. \quad (15)$$

We distinguish three asymptotic scenarios:

(a)  $\lim_{t \rightarrow \infty} \beta(t) = 0$

In this case, from Eq. (4),  $\lim_{t \rightarrow \infty} S(t) = 0$  (q.e.d.).

(b)  $\lim_{t \rightarrow \infty} \beta(t) = \infty$

Now, from Eq. (7), we have

$$\frac{\dot{S}(t)}{S(t)} = -\left(\frac{d-\theta}{2}\right) \frac{\dot{Q}(t)}{Q(t)}. \quad (16)$$

In this case, since  $S(t)$  is positively defined, from Eqs. (15) and (16), we deduce that  $\dot{S}(t) < 0$  and therefore  $\lim_{t \rightarrow \infty} S(t) = 0$  (q.e.d.).

(c)  $\lim_{t \rightarrow \infty} \beta(t) = -\infty$

In this case, from Eq. (8), we get

$$\frac{\dot{S}(t)}{S(t)} = \left(\frac{d-\theta}{2}\right) \frac{\dot{Q}(t)}{|Q(t)|} \left[ -\frac{2\Gamma Q^2(t)}{d-\theta} \left(1 + \frac{1}{2t} \frac{Q(t)}{Q(t)}\right) + 1 \right]. \quad (17)$$

In this case, since  $\lim_{t \rightarrow \infty} Q(t) = -\infty$  from Eqs. (15) and (17) we deduce that  $\dot{S}(t) < 0$  and so  $\lim_{t \rightarrow \infty} S(t) = 0$  (q.e.d.).

To summarize we observe that also in this case, as for NCOP, the possibility of metastable relaxation, established by this criterion, is due to a local property of the functional form of the potential around the metastable solution, i.e. Eq. (9), and to a dynamical property of the asymptotic regime, i.e. Eq. (10). In practice a numerical solution of Eq. (12) of I reveals that, when Eq. (9) is fulfilled, it is always possible to achieve the condition (10) by decreasing the variance of the initial condition  $S(0)$ .

We now turn to the solution of the model by considering different cases, according to the parameters  $\mu \equiv (r, g, \lambda)$  characterizing  $V(\Phi)$  (see I, Fig. 1).

Let us begin with the case of simple diffusion

$$\mu_0 \equiv (\mathbf{r} = 0, \mathbf{g} = 0, \lambda = 0) \quad (18)$$

This case is trivial to compute since  $Q(t) \equiv 0$ , but interesting because the existence of a fixed point at  $\mu_0$  affects the behavior of the dynamical processes characterized by different values of the parameters  $\mu$ . By explicitly calculating the structure factor, from Eq. (1), we find

$$C(\mathbf{k}, t) = \Delta L^\theta(t) F(\mathbf{x}). \quad (19)$$

This result, which is exact at all times (and for all  $N$ ), shows that scaling holds true from beginning to end, as in the nonconserved case.

$$\mu_1 \equiv (\mathbf{r} = 0, \mathbf{g} = 0, \lambda > 0)$$

This choice of the parameters represents the tricritical<sup>5</sup> case which can be solved by considering, firstly, the case (a)  $\lim_{t \rightarrow \infty} \beta(t) = 0$ . From Eq. (9) of I and Eq. (4), to leading order, one obtains

$$Q(t) \simeq aL(t)^{2(\theta-d+2)} + c, \quad (20)$$

where  $a = \frac{\lambda \Delta^2 J_0^2}{\Gamma(\theta-d+2)}$  and  $c$  are constants.

From Eq. (19) we evaluate

$$\beta(t) \sim L(t)^{2(\theta-d+1)} + cL^{-2}(t). \quad (21)$$

Therefore this solution is consistent only for  $d > \tilde{d}_c$ , with  $\tilde{d}_c = \theta + 1$ , as for NCOP. In this case we find

$$C(\mathbf{k}, t) \simeq \Delta e^{-[aL(t)^{2(\tilde{d}_c-d)} + cL(t)^{-2}]L^\theta(t)F(\mathbf{x})}. \quad (22)$$

Hence

$$C(\mathbf{k}, t) \simeq \Delta [1 - aL(t)^{2(\tilde{d}_c-d)} x^2] L^\theta(t) F(\mathbf{x}) \quad (23)$$

for  $\tilde{d}_c < d < \tilde{d}_c + 2$ , while

$$C(\mathbf{k}, t) \simeq \Delta [1 - \tilde{c}L(t)^{-2}] L^\theta(t) F(\mathbf{x}) \quad (24)$$

for  $d \geq \tilde{d}_c + 2$  (where  $\tilde{c} = c$  for  $d > \tilde{d}_c + 2$ ).

So, for  $d > \tilde{d}_c$ , the trivial fixed point at  $\mu_0$  is still attractive and the system scales asymptotically as in the  $\mu_0$  case, with  $x$ -dependent corrections to scaling.

We now consider quenches at  $d \leq \tilde{d}_c$ . In this case  $\lim_{t \rightarrow \infty} \beta(t) = \infty$  since  $\beta(t) \rightarrow -\infty$  is not allowed for  $\mu_1$ . From Eq. (9) of I and Eq. (7) we deduce

$$Q(t) \simeq \{(d-\theta+1)[aL^4(t)+c]\}^{\frac{1}{d-\tilde{d}_c+2}} \quad (25)$$

with  $a = \frac{\lambda \Delta^2}{(2\Gamma)^{d-\tilde{d}_c+2}}$  and  $c$  is a constant. Therefore

$$\beta(t) \sim L(t)^2 \frac{d-\tilde{d}_c}{d-\tilde{d}_c+2} \quad (26)$$

which is consistent for  $d < \tilde{d}_c$ . In this case  $\mathcal{L}(t)$  prevails asymptotically over  $L(t)$  and we have to look for scaling with respect to the former length. Therefore we go back to Eq. (1), which can be written as

$$C(\mathbf{k}, t) = \Delta e^{-k^2 L^4(t)} \mathcal{L}^\theta(t) \tilde{F}(\tilde{\mathbf{x}}), \quad (26)$$

where the scaling function is now defined as

$$\tilde{F}(\tilde{\mathbf{x}}) = \frac{e^{-\tilde{\mathbf{x}}^2}}{\tilde{\mathbf{x}}^\theta} \quad (27)$$

and the dimensionless variable  $\tilde{\mathbf{x}} = k\mathcal{L}(t)$  is expressed in terms of the dominant length  $\mathcal{L}(t)$ . Substituting, with the help of Eq. (25),  $L(t)$  with  $\mathcal{L}(t)$  in Eq. (26), we obtain

$$\begin{aligned} C(\mathbf{k}, t) &\sim \Delta e^{-a\mathcal{L}^{2(d-\tilde{d}_c)\theta^4} \mathcal{L}^\theta(t) \tilde{F}(\tilde{\mathbf{x}})} \\ &\simeq \Delta [1 - a\mathcal{L}^{2(d-\tilde{d}_c)\theta^4}] \mathcal{L}^\theta(t) \tilde{F}(\tilde{\mathbf{x}}), \end{aligned} \quad (28)$$

where  $a$  is a constant. This result shows that scaling holds true even in this case, with  $\tilde{\mathbf{x}}$ -dependent corrections, but with a modified scaling function. The power growth law of the dominant length,  $\mathcal{L}(t) \sim t^{\frac{1}{2}}$ , is obeyed with an exponent  $\tilde{z} = 2(d - \tilde{d}_c + 2)$ , which depends on the space dimensionality.

$$\mu_2 \equiv (\mathbf{r} = 0, g > 0, \lambda > 0)$$

In this case, as for NCOP, we expect the parameter  $\lambda$  to be irrelevant and the asymptotic form of the structure factor to be the same as for a  $\Phi^4$  theory (with  $\mathbf{r} = 0$ ) (see Ref. 4). In fact, since  $\lim_{t \rightarrow \infty} S(t) = 0$ , solving Eq. (9) of I for long times

$$\dot{Q}(t) \simeq gS(t), \quad (29)$$

we find

$$C(\mathbf{k}, t) \sim \Delta e^{-g\Delta a L(t) \frac{2(d-\tilde{d}_c)}{2-\tilde{d}_c+4} x^2} L^\theta(t) F(\mathbf{x}) \quad (30)$$

for  $d < \tilde{d}_c = \theta + 2$ ,

$$C(\mathbf{k}, t) \sim \Delta [1 - g\Delta a L(t)^{d-\tilde{d}_c} x^2] L^\theta(t) F(\mathbf{x}) \quad (31)$$

for  $\tilde{d}_c < d < \tilde{d}_c + 2$ , and

$$C(\mathbf{k}, t) \sim \Delta [1 - cL(t)^{-2} x^2] L^\theta(t) F(\mathbf{x}) \quad (32)$$

when  $d \geq \tilde{d}_c + 2$ . In Eqs. (30)–(32),  $a$ ,  $a'$ , and  $c$  are constants and  $\tilde{d}_c = \theta + 2$  is a critical dimensionality playing a role similar to that of  $\tilde{d}_c$  in the quench at  $\mu_1$ .

As in the NCOP model, the quench at  $\mu_3$  is very peculiar because the necessary condition (9) suggests that metastable relaxation to  $\Phi(\mathbf{r}, t = \infty) \equiv 0$  is not ruled out, but the sufficient condition (10) does not apply here since it reduces to  $S(t) \leq 0$  which is never true for finite times. We will show that both the dynamics leading to stable and to metastable equilibrium are consistent with the model equations.

When stable equilibrium is reached, since in this case  $\lim_{t \rightarrow \infty} S(t) = -g/\lambda \neq 0$  from Eqs. (4), (7), and (8), we observe that only  $\lim_{t \rightarrow \infty} \beta(t) = -\infty$  is possible. Therefore, from Eq. (8), letting  $S(t) \simeq -g/\lambda$ , it is found that

$$Q(t) \sim - \left( \frac{d-\theta}{2\Gamma-t} \ln t \right)^{\frac{1}{2}} \quad (33)$$

and eventually

$$C(\mathbf{k}, t) \sim \Delta e^{[(d-\theta) \ln t]^{\frac{1}{2}} x^2} L^\theta(t) F(\mathbf{x}). \quad (34)$$

As in the  $\Phi^4$  theory standard scaling is broken in the limit of large  $N$ . Instead, a multiscaling symmetry shows up in Eq. (34). By considering the peak of the structure factor  $k_m(t)$  it has been found that

$$[k_m(t)L(t)]^4 \simeq \frac{d-\theta}{4} \ln t. \quad (35)$$

In other words in the late stages of the quench, two distinct lengths  $k_m^{-1}$  and  $L(t)$  exist and differ only by a logarithmic factor. The standard scaling symmetry is broken by this feature and, instead, multiscaling holds.<sup>2</sup>

If metastable equilibrium is approached, however, assuming  $\lim_{t \rightarrow \infty} \beta(t) = 0$ , from Eq. (4) and Eq. (9) of I, to leading order, we find

$$Q(t) \simeq \frac{g\Delta a}{2\Gamma} L(t)^{\theta-d+4} + \tilde{c}, \quad (36)$$

where  $a = \frac{J_0}{\Gamma(\theta-d+4)}$  and  $\tilde{c}$  are constants.

Computing the ratio

$$\beta(t) \sim L(t)^{\theta-d+2} + cL(t)^{-2}, \quad (37)$$

we deduce that this solution is consistent only for  $d > \tilde{d}_c = \theta + 2$ . In this case we find

$$C(\mathbf{k}, t) \simeq \Delta e^{[g\Delta a L(t)^{d-\tilde{d}_c} - cL(t)^{-2}] x^2} L^\theta(t) F(\mathbf{x}). \quad (38)$$

Hence

$$C(\mathbf{k}, t) \simeq \Delta [1 - g\Delta a L(t)^{d-\tilde{d}_c} x^2] L^\theta(t) F(\mathbf{x}) \quad (39)$$

for  $\tilde{d}_c < d < \tilde{d}_c + 2$  and

$$C(\mathbf{k}, t) \simeq \Delta [1 - bL(t)^{-2} x^2] L^\theta(t) F(\mathbf{x}) \quad (40)$$

for  $d < \tilde{d}_c + 2$  (with  $b = c$  for  $d < \tilde{d}_c + 2$  and  $b = a + c$  for  $d = \tilde{d}_c + 2$ ).

Metastable equilibrium is approached with the same asymptotic dynamics as in a quench at  $\mu_2$ , for  $d > d_c$ .

On the other hand, for  $d < d_c$ , none of the asymptotic forms (7) and (8) is consistent. In fact let us try firstly with (7):

$$S(t) \sim Q \frac{e^{-d t}}{2}. \quad (41)$$

If metastability is approached

$$\dot{Q}(t) \sim -|g|Q \frac{e^{-d t}}{2} \quad (42)$$

in the late stage. Since

$$\lim_{t \rightarrow \infty} Q(t) = +\infty, \quad (43)$$

from Eq. (42), it is

$$\lim_{t \rightarrow \infty} \dot{Q}(t) = -\infty. \quad (44)$$

Statements (43) and (44) cannot be true simultaneously.

On the other hand, even the asymptotic form (8) can never be consistent with the requirement  $S(t) \rightarrow 0$ . In this case, in fact, Eq. (9) of I would read

$$\dot{Q}(t) = -|g|S(t) \simeq -|g|Q(t) \frac{e^{-d t}}{2} e^{\frac{19d^2 t}{24}} \quad (45)$$

asymptotically. This equation, however, quickly leads to a diverging  $\dot{Q}(t)$ , which is not consistent with metastability (i.e.  $S(t) \rightarrow 0$ ).

To summarize this  $\mu_3$  case we observe that the situation is not qualitatively different from the NCOP case in that for  $d \leq d_c$  stable equilibrium is always achieved, while for  $d > d_c$ , both stable and metastable solutions are possible and one passes from the former to the latter by changing  $\Delta$ , the variance of the initial condition.

$$\mu_4 \equiv (\mathbf{r} < 0, \mathbf{g}, \lambda > 0)$$

In this case statement (9) prevents metastability. Therefore only  $\lim_{t \rightarrow \infty} \beta(t) = -\infty$  is consistent and, from Eq. (8), letting

$$S(t) \simeq \frac{-g + \sqrt{g^2 - 4\lambda r}}{2\lambda} \quad (46)$$

for long times, the same form (33) is found for  $Q(t)$ , as in the previous case. Hence

$$C(\mathbf{k}, t) \sim \Delta e^{[(d-\theta) \ln t] \frac{1}{2} x^2} L^\theta(t) F(\mathbf{x}) \quad (47)$$

as for stable relaxation at  $\mu_3$ .

$$\mu_5 \equiv \left( \mathbf{r} > 0, \mathbf{g} \geq -4\sqrt{\frac{\lambda \mathbf{r}}{3}}, \lambda > 0 \right)$$

Now  $\lim_{t \rightarrow \infty} S(t) = 0$ . Hence, solving Eq. (9) of I for long times, we find

$$Q(t) \simeq rt + c, \quad (48)$$

where  $c$  is a constant. Therefore  $\mathcal{L}(t) \sim \sqrt{r} L^2(t)$  is the dominant length and, by means of Eq. (26), we obtain

$$\begin{aligned} C(\mathbf{k}, t) &= \Delta e^{-\frac{e^4}{r c^2 t} \ln \tilde{F}(\tilde{\mathbf{x}})} \\ &\simeq \Delta \left[ 1 - \frac{\tilde{x}^4}{r c^2(t)} \right] \tilde{F}(\tilde{\mathbf{x}}). \end{aligned} \quad (49)$$

This result shows that, different from the corresponding NCOP case, scaling holds controlled by the length  $\mathcal{L}(t)$ , which grows as  $t^{\frac{1}{2}}$ , with  $\tilde{z} = 2$  in any dimension.

$$\mu_6 \equiv \left( \mathbf{r} > 0, \mathbf{g} < -4\sqrt{\frac{\lambda \mathbf{r}}{3}}, \lambda > 0 \right)$$

According to statements (9) and (10), if the dynamics leads asymptotically the system to a state with  $S(t) \leq \frac{|g| - \sqrt{g^2 - 4\lambda r}}{2\lambda}$ , metastable relaxation in  $\Phi \equiv 0$  occurs. Practically this is always achievable by choosing an initial condition with  $S(0)$  sufficiently small. In this case the equations can be solved asymptotically as in a quench at  $\mu_6$ . The same results are obtained:

$$C(\mathbf{k}, t) \sim \Delta \left[ 1 - \frac{\tilde{x}^4}{r c^2(t)} \right] \tilde{F}(\tilde{\mathbf{x}}). \quad (50)$$

In this case, as compared to  $\mu_4$ , the stronger character of metastability eliminates the critical dimension: metastable relaxation occurs in any dimension when  $S(0)$  is small. Since (10) is only a sufficient condition, Eq. (50) can in principle hold asymptotically even for  $S(t)$  larger than  $\frac{|g| - \sqrt{g^2 - 4\lambda r}}{2\lambda}$ .

When  $S(0)$  is sufficiently large, however, stable equilibrium must be obtained. In this case, proceeding as for  $\mu_3$  or  $\mu_4$ , the same multiscaling solution (34) is obtained in the late stages:

$$C(\mathbf{k}, t) \sim \Delta e^{[(d-\theta) \ln t] \frac{1}{2} x^2} L^\theta(t) F(\mathbf{x}). \quad (51)$$

### 3. Summary

In this paper we have extended the solution of the spherical  $\Phi^6$  model to the conserved case (model B) with zero temperature.

To summarize the results of the present paper we observe that the  $\Phi^6$  model shows either scaling or multiscaling in the asymptotic regime depending on the parameters  $\mu$  of the Hamiltonian. The asymptotic multiscaling, peculiar to the large  $N$  limit, shows up, as expected, in those cases when stable equilibrium is reached in a potential with two degenerate minima (see I, Fig. 1). The standard scaling

behavior, on the other hand, is controlled either by  $L(t)$  or  $\mathcal{L}(t)$ , with different exponents and scaling functions in the two cases.

The scaling behavior controlled by  $L(t)$  is always induced by the presence of the trivial fixed point of simple diffusion at  $\mu_0$ , which can be attractive on the whole  $r = 0$  axis. For  $g \geq 0$  this line represents the edge of the sector  $\mu_5$  where  $\mathcal{L}(t)$  prevails asymptotically. As a consequence the trivial fixed point is attractive only above a dynamical critical dimension, while below the structure factor scales with  $\mathcal{L}(t)$  as at  $\mu_5$ .

The negative part of the  $g$  axis, on the contrary, is the intersection set of two sectors ( $\mu_4$  and  $\mu_6$ ) where multiscaling holds. Therefore the trivial fixed point (which represents now the metastable relaxation) competes now with the multiscaling fixed point and, again, it can be attractive only for dimensions which exceed the critical one, in analogy with static critical phenomena.

Finally, for what concerns the  $\mu_6$  region, it behaves as a transition sector between the  $\mu_4$  and  $\mu_5$  sectors of the phase diagram, in that both multiscaling and scaling induced by  $\mathcal{L}(t)$  are found (the last leads to metastable equilibrium). Since  $r > 0$  the trivial fixed point is never attractive and no critical dimension is found.

As regards metastability, it occurs in strict analogy with the nonconserved case, apart from a greater technical complexity. In both cases, in fact, a similar criterion links metastability to a local property (its curvature) of the potential around the metastable extremum. We conclude that the kind of dynamics (whether conserved or not) does not affect the occurrence of metastability which is due, on the contrary, to the analytical properties of the Hamiltonian. This is an interesting feature which suggests a unified description of physical processes involving thermodynamic metastability.

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