Scritto del 26/2/2013

1) a)

$$P(x,y,z) = kT\, d(x,y,z)$$

$$= kT\, N \frac{\Sigma_\sigma e^{-\beta V(x,y,z,\sigma)}}{\Sigma_\sigma \int_V e^{-\beta V(x,y,z,\sigma)} dx\,dy\,dz}$$

$$= NkT \frac{e^{-\beta \frac{A}{2}(x^2+y^2)}\left(e^{\beta b z} + e^{-\beta b z}\right)}{\int\limits_{(x^2+y^2 \le R^2)} dx\,dy \int_0^L dz\, e^{-\beta\frac{A}{2}(x^2+y^2)}\left(e^{\beta b z} + e^{-\beta b z}\right)}$$

$$P(x,y,z) = NkT \frac{e^{-\frac{\beta}{2}AR^2} \cosh(\beta b L/2)}{\left(\frac{2\pi (kT)^2}{Ab}\right)\left(1 - e^{-\frac{\beta}{2}AR^2}\right)\sinh(\beta b L)}$$

$$= \frac{NAb}{2\pi kT} \frac{\cosh(\beta b L/2)}{\left(e^{\frac{\beta}{2}AR^2} - 1\right)\sinh(\beta b L)}$$

1) b)

$$p = p_1\left(0, \tfrac{L}{2}\right) = \frac{\Sigma_\sigma \int_0^{L/2} dz\, e^{-\beta b \sigma z}}{\Sigma_\sigma \int_0^L dz\, e^{-\beta b \sigma z}} = \frac{e^{\beta b \frac{L}{2}} - e^{-\beta b \frac{L}{2}}}{e^{\beta b L} - e^{-\beta b L}}$$

$$P(0) + P(1) + P(\geq 2) = 1$$

$$P(\geq 2) = 1 - P(0) - P(1)$$

$$P(0) = (p)^0 (1-p)^N \binom{N}{0} = (1-p)^N$$

$$P(1) = (p)^1 (1-p)^{N-1} \binom{N}{1} = Np(1-p)^{N-1}$$

$$P(\geq 2) = 1 - (1-p)^N \left[1 + N\left(\frac{p}{1-p}\right)\right]$$

$P(\geq 2) \to 1$ per $N \to \infty$ dato che

$(1-p)^N \to 0$ esponenzialmente in N

$N \frac{p}{1-p} \to \infty$ linearmente in N

2) a) $\quad H = \frac{\vec{p}^2}{2m} + \frac{A}{2}(x^2 + y^2)$

se $\quad (x^2+y^2)^{\frac{1}{2}} \leq \frac{R}{2}$

è anche $\quad \epsilon_F \leq \frac{A}{2}\left(\frac{R}{2}\right)^2 = \frac{AR^2}{8} = \epsilon_{F\,max}$

$$N(\epsilon) = \frac{2}{h^3} \int\limits_{\frac{p^2}{2m} + \frac{A}{2}(x^2+y^2) \leq \epsilon \left(\leq A\frac{R^2}{8}\right)} d\vec{p}\, d\vec{x}$$

$$= \frac{2L}{h^3} \int_0^{\sqrt{2\epsilon/A}} 2\pi r\, dr \int\limits_{\frac{\vec{p}^2}{2m} \leq \left(\epsilon - \frac{A}{2}r^2\right)} d\vec{p}$$

$$N(\epsilon) = \frac{8\pi^2 L}{3h^3} \int_0^{\sqrt{2\epsilon/A}} 2r\,dr\,(2m)^{3/2}\,\epsilon^{3/2}\left(1 - \frac{A}{2\epsilon}r^2\right)^{3/2}$$

$$1 - \frac{A}{2\epsilon}r^2 = t \qquad dt = -\frac{A}{\epsilon}r\,dr$$

$$N(\epsilon) = \frac{16 L\pi^2}{3h^3}(2m)^{3/2}\,\epsilon^{3/2}\left(-\frac{\epsilon}{A}\right)\int_1^0 dt\, t^{3/2}$$

$$= \frac{32}{15}\,\frac{L\pi^2}{Ah^3}(2m)^{3/2}\,\epsilon^{5/2}$$

per $\epsilon = \epsilon_{F\,max} = \frac{AR^2}{8}$

$$N(\epsilon_{F\,max}) = N_* = \frac{(mA)^{3/2} L\pi^2}{30\, h^3} R^5$$

2) b) $$G(\epsilon) = \frac{dN}{d\epsilon} = \frac{16}{3}\,\frac{L\pi^2 (2m)^{3/2}}{Ah^3}\,\epsilon^{3/2}$$

$$U_0 = \int_0^{\epsilon_F} \epsilon\, G(\epsilon)\, d\epsilon = \frac{32}{21}\,\frac{L\pi^2 (2m)^{3/2}}{Ah^3}\,\epsilon_F^{7/2}$$

per $\epsilon_F = \epsilon_{F\,max} = \frac{AR^2}{8}$ si ha

$$U_{0*} = \frac{L\pi^2 m^{3/2} A^{5/2}}{15\cdot 16\, h^3} R^7$$

2) c)

$$\mathcal{N}(\epsilon) = \left[\frac{32}{15}\frac{L\pi^2}{A h^3}(2m)^{3/2}\right]\epsilon^{5/2} \equiv \gamma\,\epsilon^{5/2}$$

solo se $\quad \epsilon < \frac{AR^2}{2}$

per cui $\quad r_{max} = \sqrt{2\epsilon/A} < R$

se $\quad \epsilon > \frac{AR^2}{2}$

$$\mathcal{N}(\epsilon) = \frac{2L}{h^3}\int_0^R 2\pi r\,dr \int_{\frac{\vec{p}^2}{2m} < \left(\epsilon - \frac{A}{2}r^2\right)} d\vec{p}$$

$$= \frac{16\,L\pi^2(2m)^{3/2}}{3h^3}\,\epsilon^{3/2}\int_0^R r\,dr\left(1 - \frac{Ar^2}{2\epsilon}\right)^{3/2}$$

$$= \frac{16\,L\pi^2}{3h^3 A}(2m)^{3/2}\,\epsilon^{5/2}\int_{\left(1-\frac{AR^2}{2\epsilon}\right)}^{1} dt\, t^{3/2}$$

$$= \frac{32}{15}\frac{L\pi^2(2m)^{3/2}}{A h^3}\,\epsilon^{5/2}\left[1 - \left(1 - \frac{AR^2}{2\epsilon}\right)^{5/2}\right]$$

$$\mathcal{N}(\epsilon) = \gamma\left[\epsilon^{5/2} - \left(\epsilon - \frac{AR^2}{2}\right)^{5/2}\right]$$

per cui

$$\begin{cases} G(\epsilon) = \frac{5}{2}\gamma\, \epsilon^{3/2} & \text{se} \quad \epsilon < \frac{AR^2}{2} \\ G(\epsilon) = \frac{5}{2}\gamma \left[\epsilon^{3/2} - \left(\epsilon - \frac{AR^2}{2}\right)^{3/2}\right] & \text{se} \quad \epsilon > \frac{AR^2}{2} \end{cases}$$

$G(\epsilon) \neq \text{cost}\ \epsilon^{\alpha}$ con α indipendente da ϵ

e <u>non</u> si può scrivere $PV = \frac{1}{\alpha+1} U$